

# 12.6

## Binomial Distributions

### What you should learn

**GOAL 1** Find binomial probabilities and analyze binomial distributions.

**GOAL 2** Test a hypothesis, as applied in Example 4.

### Why you should learn it

▼ To solve **real-life** problems, such as determining whether a computer manufacturer's claim is correct in Ex. 46.



### GOAL 1 FINDING BINOMIAL PROBABILITIES

There are many probability experiments in which the results of each trial can be reduced to two outcomes. If such an experiment satisfies the following conditions, then it is called a **binomial experiment**.

- There are  $n$  independent trials.
- Each trial has only two possible outcomes: success and failure.
- The probability of success is the same for each trial. This probability is denoted by  $p$ . The probability of failure is given by  $1 - p$ .

#### FINDING A BINOMIAL PROBABILITY

For a binomial experiment consisting of  $n$  trials, the probability of exactly  $k$  successes is

$$P(k \text{ successes}) = {}_n C_k p^k (1 - p)^{n - k}$$

where the probability of success on each trial is  $p$ .

### EXAMPLE 1 Finding a Binomial Probability

**UFOs** According to a survey taken by *USA Today*, about 37% of adults believe that Unidentified Flying Objects (UFOs) really exist. Suppose you randomly survey 6 adults. What is the probability that exactly 2 of them believe that UFOs really exist?

#### SOLUTION

Let  $p = 0.37$  be the probability that a randomly selected adult believes that UFOs really exist. By surveying 6 adults, you are conducting  $n = 6$  independent trials. The probability of getting exactly  $k = 2$  successes is:

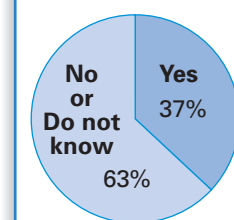
$$\begin{aligned} P(k = 2) &= {}_6 C_2 (0.37)^2 (1 - 0.37)^{6 - 2} \\ &= \frac{6!}{4! \cdot 2!} (0.37)^2 (0.63)^4 \\ &\approx 0.323 \end{aligned}$$

- The probability that exactly 2 of the people surveyed believe that UFOs really exist is about 32%.

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A **binomial distribution** shows the probabilities of all possible numbers of successes in a binomial experiment, as illustrated in Example 2 on the next page.

#### Do UFOs Exist?





## EXAMPLE 2 Constructing a Binomial Distribution

Draw a histogram of the binomial distribution for the survey in Example 1. Then find the probability that at most 2 of the people surveyed believe that UFOs really exist.

### SOLUTION

$$P(k = 0) = {}_6C_0(0.37)^0(0.63)^6 \approx 0.063$$

$$P(k = 1) = {}_6C_1(0.37)^1(0.63)^5 \approx 0.220$$

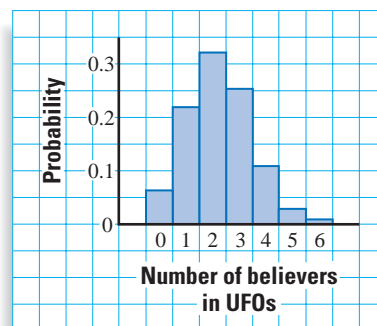
$$P(k = 2) = {}_6C_2(0.37)^2(0.63)^4 \approx 0.323$$

$$P(k = 3) = {}_6C_3(0.37)^3(0.63)^3 \approx 0.253$$

$$P(k = 4) = {}_6C_4(0.37)^4(0.63)^2 \approx 0.112$$

$$P(k = 5) = {}_6C_5(0.37)^5(0.63)^1 \approx 0.026$$

$$P(k = 6) = {}_6C_6(0.37)^6(0.63)^0 \approx 0.003$$



The probability of getting at most  $k = 2$  successes is:

$$P(k \leq 2) = P(2) + P(1) + P(0) \approx 0.323 + 0.220 + 0.063 = 0.606$$

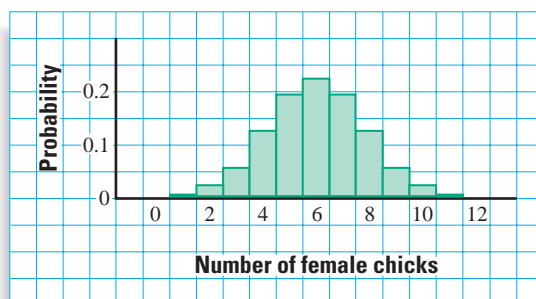
- The probability that at most 2 of the people surveyed believe that UFOs really exist is about 61%.



## EXAMPLE 3 Interpreting a Binomial Distribution

For a science project you are incubating 12 chicken eggs. The probability that a chick is female is 0.5. Draw a histogram of the binomial distribution based on the probability that exactly  $k$  of the chicks are female. Then find the most likely number of female chicks.

**SOLUTION** Begin by calculating each binomial probability using the formula  $P(k) = {}_n C_k(0.5)^k(0.5)^{n-k} = {}_n C_k(0.5)^n$  as shown in the table. Then draw the histogram.



$k$	$P(k)$
0	0.000
1	0.003
2	0.016
3	0.054
4	0.121
5	0.193
6	0.226
7	0.193
8	0.121
9	0.054
10	0.016
11	0.003
12	0.000

The most likely number of female chicks is the value of  $k$  for which  $P(k)$  is greatest. This probability is greatest for  $k = 6$ . So, the most likely number of female chicks is 6.

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The distribution in Example 3 is **symmetric**, which means that the left half of the histogram is a mirror image of the right half. A distribution that is not symmetric is called **skewed**. The distribution in Example 2 is an example of a skewed distribution.

## GOAL 2 TESTING HYPOTHESES

Example 1 presented the claim that 37% of adults believe that UFOs really exist. If you wanted to test this claim, you could use a procedure from statistics called **hypothesis testing**. Here are the basic steps.

### STUDENT HELP

#### Study Tip

Different standards are used by different statisticians, but “small” in Step 3 typically means that the probability is less than 0.1, 0.05, or 0.01.

### HYPOTHESIS TESTING

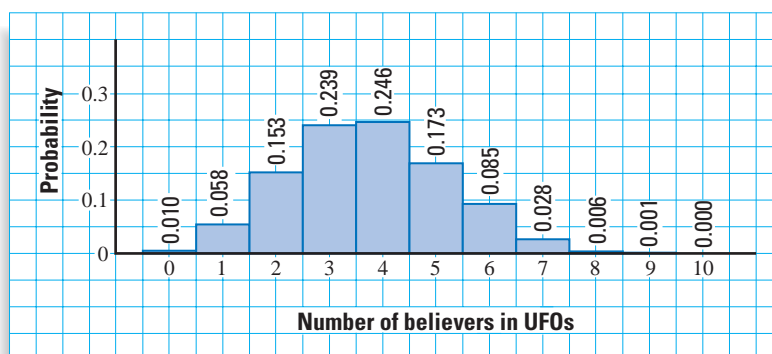
- STEP 1** State the hypothesis you are testing. The hypothesis should make a statement about some statistical measure (mean, standard deviation, or proportion) of a population.
- STEP 2** Collect data from a random sample of the population and compute the statistical measure of the sample.
- STEP 3** Assume that the hypothesis is true and calculate the resulting probability of obtaining the sample statistical measure *or a more extreme* sample statistical measure. If this probability is small, you should reject the hypothesis.

### EXAMPLE 4 Testing a Hypothesis

**UFOs** To test the claim that 37% of adults believe UFOs really exist, you conduct a survey of 10 randomly selected adults. In your survey only 2 of the adults believe that UFOs really exist. Should you reject the claim? Explain.

#### SOLUTION

- 1 Assume the claim *37% of adults believe that UFOs really exist* is true.
- 2 Form the binomial distribution for the probability of selecting exactly  $k$  people out of 10 who believe that UFOs really exist.



- 3 Calculate the probability that you could randomly select 2 or fewer adults out of 10 who believe that UFOs really exist.

$$P(k \leq 2) = 0.010 + 0.058 + 0.153 = 0.221$$

- If it is true that *37% of adults believe that UFOs really exist*, then there is a 22% probability of finding only 2 or fewer believers in a random sample of 10. With a probability this large, you should not reject the claim.

### FOCUS ON APPLICATIONS



**REAL LIFE** **UFOs** In 1938 a radio adaptation of *The War of the Worlds* caused thousands of people to believe that UFOs had landed. *The War of the Worlds* was made into a feature film in 1953.

**APPLICATION LINK**  
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check

- 1. Explain the difference between a binomial experiment and a binomial distribution.
- 2. You draw ten cards from a standard 52-card deck without replacement. Is this a binomial experiment? Explain.
- 3. Consider the binomial distribution shown. Is the distribution skewed or symmetric? Explain.

k	P(k)
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

Ex. 3

Concept Check

Skill Check

Calculate the probability of  $k$  successes for a binomial experiment consisting of  $n$  trials with probability  $p$  of success on each trial.

- 4.  $k = 7, n = 12, p = 0.7$
- 5.  $k \leq 3, n = 14, p = 0.45$

A binomial experiment consists of  $n$  trials with probability  $p$  of success on each trial. Draw a histogram of the binomial distribution that shows the probability of exactly  $k$  successes. Then find the most likely number of successes.

- 6.  $n = 6, p = 0.5$
- 7.  $n = 8, p = 0.33$
- 8.  $n = 10, p = 0.25$

- 9. **CLASS RINGS** You read an article that claims only 30% of graduating seniors will buy a class ring. To test this claim you survey 15 randomly selected seniors in your school and find that 4 are planning to buy class rings. Should you reject the claim? Explain.   
▶ Source: *America by the Numbers*

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 957.

**CALCULATING PROBABILITIES** Calculate the probability of tossing a coin 20 times and getting the given number of heads.

- 10. 1
- 11. 3
- 12. 5
- 13. 9
- 14. 10
- 15. 11
- 16. 15
- 17. 17

**BINOMIAL PROBABILITIES** Calculate the probability of randomly guessing the given number of correct answers on a 30-question multiple-choice exam that has choices A, B, C, and D for each question.

- 18. 0
- 19. 2
- 20. 5
- 21. 10
- 22. 15
- 23. 20
- 24. 25
- 25. 30

**BINOMIAL DISTRIBUTIONS** Calculate the probability of  $k$  successes for a binomial experiment consisting of  $n$  trials with probability  $p$  of success on each trial.

- 26.  $k \geq 3, n = 5, p = 0.2$
- 27.  $k \leq 2, n = 6, p = 0.5$
- 28.  $k \leq 1, n = 9, p = 0.15$
- 29.  $k \leq 5, n = 12, p = 0.64$

**HISTOGRAMS** A binomial experiment consists of  $n$  trials with probability  $p$  of success on each trial. Draw a histogram of the binomial distribution that shows the probability of exactly  $k$  successes. Then find the most likely number of successes.

- 30.  $n = 2, p = 0.4$
- 31.  $n = 4, p = 0.7$
- 32.  $n = 5, p = 0.17$
- 33.  $n = 8, p = 0.92$
- 34.  $n = 9, p = 0.125$
- 35.  $n = 12, p = 0.033$

STUDENT HELP

HOMEWORK HELP

- Example 1: Exs. 10–25, 36, 37
- Example 2: Exs. 26–29, 38–41
- Example 3: Exs. 30–35, 42–45
- Example 4: Exs. 46–48

# FOCUS ON PEOPLE



**DR. CHARLES DREW** was a leading expert in blood collection and storage, discussed in Exs. 42 and 43. Although he died in 1950 at the age of 45, his contributions to medicine continue to save lives.

36. **COURT CASE** In a particular court case, legal analysts determine from the evidence presented that there is about a 90% probability a juror will vote that the defendant is guilty. Find the probability that all 12 jurors vote that the defendant is guilty.
37. **VIDEOCASSETTES** A product-quality researcher runs a study on a particular brand of videocassette tape and discovers that on a random basis one out of 400 tapes is defective. Find the probability that you buy 20 of these tapes and exactly 2 are defective.

**PUPPIES** In Exercises 38 and 39, use the following information.

A dog gives birth to a litter of 8 puppies. Assume that the probability of a puppy being female is 0.5.

38. Draw a histogram of the binomial distribution for the number of female puppies in the litter.
39. What is the probability of having at least 5 female puppies?

**EGGS** In Exercises 40 and 41, use the following information.

A study states 34% of people prefer their eggs scrambled. You randomly select 7 people who had eggs for breakfast ▶ Source: *America by the Numbers*

40. Draw a histogram of the binomial distribution for the number of people who prefer their eggs scrambled.
41. What is the probability that at most 3 of these people prefer scrambled eggs?

**BLOOD DRIVE** In Exercises 42 and 43, use the following information.

A hospital is having a blood drive. The hospital is desperately in need of type O– blood, which 7% of the people in the United States have. Each hour at the hospital, an average of 12 people give blood. ▶ Source: American Red Cross

42. Draw a histogram of the binomial distribution for the number of people each hour who give type O– blood.
43. What is the most likely number of people each hour who give type O– blood?

**WORK HOURS** In Exercises 44 and 45, use the following information.

A recent survey found that 25% of women wish they had more flexible work hours. A small company has 24 female employees. ▶ Source: *America by the Numbers*

44. Draw a histogram of the binomial distribution for the number of female employees who want more flexible work hours.
45. At the small company, what is the most likely number of female employees who want more flexible work hours?

46. **PERSONAL COMPUTERS** A manufacturer of personal computers claims that under normal work use only 5% of its computers will fail to operate at some point in a month. A small business firm uses 30 of the manufacturer's computers under normal work use and has 2 failures in a month. Would you reject the manufacturer's claim? Explain.

47. **STUDENT ATHLETES** In an article about athletes at a particular college, a journalist claims that 75% of the athletes who were given scholarships would still be attending the college if they had not been given scholarships. Curious, a professor polls a random sample of 10 students with sports scholarships. Three of the students said they had plans to attend the college with or without a scholarship. Would you reject the journalist's claim? Explain.

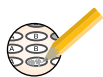
## STUDENT HELP



### HOMEWORK HELP

Visit our Web site  
[www.mcdougallittell.com](http://www.mcdougallittell.com)  
for help with problem  
solving in Ex. 47.

## Test Preparation



48. **BOTTLED JUICES** A company that makes bottled juices has created a new brand of apple juice. The company claims 80% of people prefer the new apple juice over a competitor's apple juice. A taste test is conducted to test this claim. Of 20 people sampled, 12 preferred the new apple juice. Would you reject the company's claim? Explain.
49. **MULTIPLE CHOICE** A baseball player's batting average is .310. What is the probability that the player will get 3 or more hits in a game in which the player has 5 official at-bats?
- (A) 0.500      (B) 0.142      (C) 0.035      (D) 0.177      (E) 0.600
50. **MULTIPLE CHOICE** Based on sales figures it is assumed that 60% of the students in a particular high school prefer Beverage A to Beverage B. In a survey of 10 randomly selected students, 3 preferred Beverage A. What is the probability of obtaining this sample result *or a more extreme* sample result?
- (A) 0.947      (B) 0.043      (C) 0.055      (D) 0.300      (E) 0.200
51. **FINDING A FORMULA** The mean of a binomial distribution is the sum of the products of the numbers of successes and the corresponding probabilities. Mathematically, the mean can be represented by:

$$\sum_{k=0}^n k \cdot P(k)$$

Find the mean of the binomial distribution in Example 2. Then divide the mean by  $n$ . What significance does this number have relative to the probability of success on any trial? Use this result to find a simpler formula for the mean of a binomial distribution.

## ★ Challenge

### EXTRA CHALLENGE

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## MIXED REVIEW

**MEASURES OF DISPERSION** Find the range and standard deviation of the data set. (Review 7.7 for 12.7)

52. 8, 9, 9, 9, 10

53. 16, 18, 19, 21, 25, 27, 27

54. 1.2, 1.3, 1.4, 1.7, 1.8

55. 81, 87, 88, 91, 99, 100

**SOLVING SYSTEMS** Find the points of intersection, if any, of the graphs in the system. (Review 10.7)

56.  $x^2 + y^2 = 1$   
 $y = x$

57.  $x^2 + y^2 = 74$   
 $x - y = -2$

58.  $x^2 + 4y^2 = 1$   
 $y = x + 1$

59.  $5x^2 + y^2 = 10$   
 $x^2 + y^2 = 9$

60.  $x^2 - y^2 = 49$   
 $y = 7$

61.  $2x^2 - 3y^2 = 6$   
 $y = 3x + 1$

**WRITING RULES** Write a recursive rule for the sequence. The sequence may be arithmetic, geometric, or neither. (Review 11.5)

62. 3, 5, 9, 15, ...

63. 4, 40, 400, 4000, ...

64. 80, 60, 40, 20, ...

65. 1, 3, 3, 9, ...

66. 160, 40, 10, 2.5, ...

67. 1, 2, 3, 5, ...

68. **GEOMETRY CONNECTION** The radius  $r$  of a sphere can be approximated by  $r = 0.62\sqrt[3]{V}$  where  $V$  is the volume of the sphere. Graph the model. Then determine the volume of a sphere with a radius of 3 inches. (Lesson 7.5)