12.5

Probability of Independent and Dependent Events
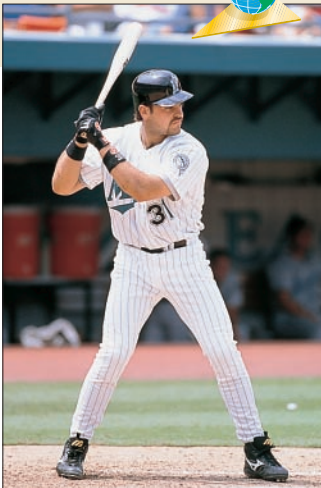
GOAL 1 PROBABILITIES OF INDEPENDENT EVENTS

Two events are **independent** if the occurrence of one has no effect on the occurrence of the other. For instance, if a coin is tossed twice, the outcome of the first toss (heads or tails) has no effect on the outcome of the second toss.

PROBABILITY OF INDEPENDENT EVENTS

If A and B are independent events, then the probability that both A and B occur is $P(A \text{ and } B) = P(A) \cdot P(B)$.

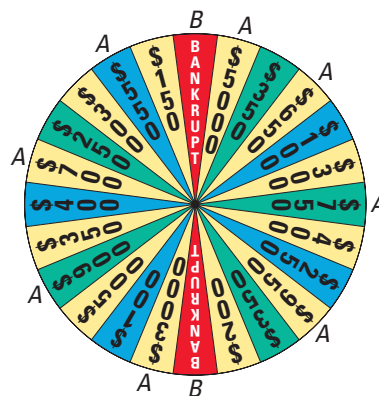### EXAMPLE 1 *Probability of Two Independent Events*

You are playing a game that involves spinning the money wheel shown. During your turn you get to spin the wheel twice. What is the probability that you get more than $500 on your first spin and then go bankrupt on your second spin?

SOLUTION Let event A be getting more than $500 on the first spin, and let event B be going bankrupt on the second spin. The two events are independent. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{8}{24} \cdot \frac{2}{24} = \frac{1}{36} \approx 0.028$$

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The formula given above for the probability of two independent events can be extended to the probability of three or more independent events.

EXAMPLE 2 *Probability of Three Independent Events*

BASEBALL During the 1997 baseball season, the Florida Marlins won 5 out of 7 home games and 3 out of 7 away games against the San Francisco Giants. During the 1997 National League Division Series with the Giants, the Marlins played the first two games at home and the third game away. The Marlins won all three games. Estimate the probability of this happening. ▶ Source: The Florida Marlins

SOLUTION Let events A, B, and C be winning the first, second, and third games. The three events are independent and have experimental probabilities based on the regular season games. So, the probability of winning the first three games is:

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C) = \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{3}{7} = \frac{75}{343} \approx 0.219$$

EXAMPLE 3 *Using a Complement to Find a Probability*

Trading Cards

You collect hockey trading cards. For one team there are 25 different cards in the set, and you have all of them except for the starting goalie card. To try and get this card, you buy 8 packs of 5 cards each. All cards in a pack are different and each of the cards is equally likely to be in a given pack. Find the probability that you will get at least one starting goalie card.
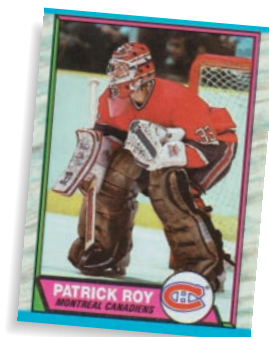
SOLUTION

In one pack the probability of *not* getting the starting goalie card is:

$$P(\text{no starting goalie}) = \frac{_{24}C_5}{_{25}C_5}$$

Buying packs of cards are independent events, so the probability of getting at least one starting goalie card in the 8 packs is:

$$P(\text{at least one starting goalie}) = 1 - P(\text{no starting goalie in any pack})$$

$$= 1 - \left(\frac{_{24}C_5}{_{25}C_5}\right)^8$$

$$\approx 0.832$$

EXAMPLE 4 *Solving a Probability Equation*

Manufacturing

A computer chip manufacturer has found that only 1 out of 1000 of its chips is defective. You are ordering a shipment of chips for the computer store where you work. How many chips can you order before the probability that at least one chip is defective reaches 50%?

SOLUTION

Let n be the number of chips you order. From the given information you know that $P(\text{chip is *not* defective}) = \frac{999}{1000} = 0.999$. Use this probability and the fact that each chip ordered represents an independent event to find the value of n.

$P(\text{at least one chip is defective}) = 0.5$	**Write given assumption.**
$1 - P(\text{no chips are defective}) = 0.5$	**Use complement.**
$1 - (0.999)^n = 0.5$	**Substitute known probability.**
$-(0.999)^n = -0.5$	**Subtract 1 from each side.**
$(0.999)^n = 0.5$	**Divide each side by –1.**
$n = \dfrac{\log 0.5}{\log 0.999}$	**Solve for n.**
$n \approx 693$	**Use a calculator.**

▶ If you order 693 chips, you have a 50% chance of getting a defective chip. Therefore, you can order 692 chips before the probability that at least one chip is defective reaches 50%.

Two events A and B are **dependent events** if the occurrence of one affects the occurrence of the other. The probability that B will occur given that A has occurred is called the **conditional probability** of B given A and is written $P(B \mid A)$.

PROBABILITY OF DEPENDENT EVENTS

If A and B are dependent events, then the probability that both A and B occur is
$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

REAL LIFE

Endangered Species

EXAMPLE 5 *Finding Conditional Probabilities*

The table shows the number of endangered and threatened animal species in the United States as of November 30, 1998. Find (**a**) the probability that a listed animal is a reptile and (**b**) the probability that an endangered animal is a reptile.

▶ Source: United States Fish and Wildlife Service

	Mammals	Birds	Reptiles	Amphibians	Other
Endangered	59	75	14	9	198
Threatened	8	15	21	7	69

SOLUTION

a. $P(\text{reptile}) = \dfrac{\text{number of reptiles}}{\text{total number of animals}} = \dfrac{35}{475} \approx 0.0737$

b. $P(\text{reptile} \mid \text{endangered}) = \dfrac{\text{number of endangered reptiles}}{\text{total number of endangered animals}}$

$= \dfrac{14}{355} \approx 0.0394$

EXAMPLE 6 *Comparing Dependent and Independent Events*

You randomly select two cards from a standard 52-card deck. What is the probability that the first card is not a face card (a king, queen, or jack) and the second card is a face card if (**a**) you replace the first card before selecting the second, and (**b**) you do *not* replace the first card?

SOLUTION

a. If you replace the first card before selecting the second card, then A and B are independent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{40}{52} \cdot \frac{12}{52} = \frac{30}{169} \approx 0.178$$

b. If you do *not* replace the first card before selecting the second card, then A and B are dependent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = \frac{40}{52} \cdot \frac{12}{51} = \frac{40}{221} \approx 0.181$$

The formula for finding probabilities of dependent events can be extended to three or more events, as shown in Example 7.

EXAMPLE 7 · *Probability of Three Dependent Events*

Dining Out

You and two friends go to a restaurant and order a sandwich. The menu has 10 types of sandwiches and each of you is equally likely to order any type. What is the probability that each of you orders a different type?

SOLUTION

Let event A be that you order a sandwich, event B be that one friend orders a different type, and event C be that your other friend orders a third type. These events are dependent. So, the probability that each of you orders a different type is:

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } B)$$

$$= \frac{10}{10} \cdot \frac{9}{10} \cdot \frac{8}{10}$$

$$= \frac{18}{25} = 0.72$$

EXAMPLE 8 · *Using a Tree Diagram to Find Conditional Probabilities*

HEALTH The American Diabetes Association estimates that 5.9% of Americans have diabetes. Suppose that a medical lab has developed a simple diagnostic test for diabetes that is 98% accurate for people who have the disease and 95% accurate for people who do not have it. If the medical lab gives the test to a randomly selected person, what is the probability that the diagnosis is correct?

SOLUTION

A probability tree diagram, where the probabilities are given along the branches, can help you see the different ways to obtain a correct diagnosis. Notice that the probabilities for all branches from the same point must sum to 1.
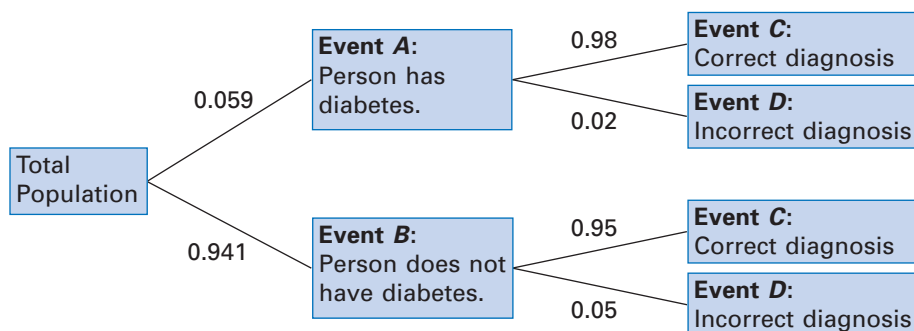
So, the probability that the diagnosis is correct is:

$P(C) = P(A \text{ and } C) + P(B \text{ and } C)$	**Follow branches leading to C.**
$= P(A) \cdot P(C \mid A) + P(B) \cdot P(C \mid B)$	**Use formula for dependent events.**
$= (0.059)(0.98) + (0.941)(0.95)$	**Substitute.**
≈ 0.952	**Use a calculator.**

GUIDED PRACTICE

1. Explain the difference between dependent events and independent events, and give an example of each.

2. If event A is drawing a queen from a deck of cards and event B is drawing a king from the remaining cards, are events A and B dependent or independent?

3. If event A is rolling a two on a six-sided die and event B is rolling a four on a different six-sided die, are events A and B dependent or independent?

Events A and B are independent. Find the indicated probability.

4. $P(A) = 0.3$
$P(B) = 0.9$
$P(A \text{ and } B) = \underline{\ ?\ }$

5. $P(A) = \underline{\ ?\ }$
$P(B) = 0.3$
$P(A \text{ and } B) = 0.06$

6. $P(A) = 0.75$
$P(B) = \underline{\ ?\ }$
$P(A \text{ and } B) = 0.15$

Events A and B are dependent. Find the indicated probability.

7. $P(A) = 0.1$
$P(B \mid A) = 0.8$
$P(A \text{ and } B) = \underline{\ ?\ }$

8. $P(A) = \underline{\ ?\ }$
$P(B \mid A) = 0.5$
$P(A \text{ and } B) = 0.25$

9. $P(A) = 0.9$
$P(B \mid A) = \underline{\ ?\ }$
$P(A \text{ and } B) = 0.54$

🌐 **READING LIST In Exercises 10 and 11, use the following information.**
Three friends are taking an English class that has a summer reading list. Each student is required to read one book from the list, which contains 3 biographies, 10 classics, and 5 historical novels.

10. Find the probability that the first friend chooses a biography, the second friend chooses a classic, and the third friend chooses a historical novel.

**11.** Find the probability that the three friends each choose a different classic.

PRACTICE AND APPLICATIONS

SPINNING A WHEEL You are playing a game that involves spinning the wheel shown. Find the probability of spinning the given colors.

12. red, then blue

13. red, then green

14. yellow, then red

15. green, then yellow

16. blue, then yellow, then green

17. green, then red, then blue
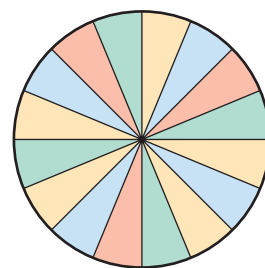
DRAWING CARDS Find the probability of drawing the given cards from a standard 52-card deck (a) with replacement and (b) without replacement.

18. a heart, then a diamond

19. a jack, then a king

20. a 2, then a face card (K, Q, or J)

21. a face card (K, Q, or J), then a 2

22. an ace, then a 2, then a 3

23. a heart, then a diamond, then another heart

24. 🌐 **GAMES** You are playing a game that involves drawing three numbers from a hat. There are 25 pieces of paper numbered 1 to 25 in the hat. Each number is replaced after it is drawn. What is the probability that each number is greater than 20 or less than 4?

25. 🌎 **LAWN CARE** The owner of a one-man lawn mowing business owns three old and unreliable riding mowers. As long as one of the three is working he can stay productive. From past experience, one of the mowers is unusable 12 percent of the time, one 6 percent of the time, and one 20 percent of the time. Find the probability that all three mowers are unusable on a given day.

26. 🌎 **TRADING CARDS** You collect movie trading cards, which have different scenes from a movie. For one movie there are 90 different cards in the set, and you have all of them except the final scene. To try and get this card, you buy 10 packs of 8 cards each. All cards in a pack are different and each of the cards is equally likely to be in a given pack. Find the probability that you will get the final scene.

27. 🌎 **FREE THROWS** Chris Mullin of the Indiana Pacers led the National Basketball Association in free-throw percentage during the 1997–1998 season. He made 93.9% of his free-throw attempts. If he attempted 10 free throws in a game, what is the probability that he missed at least one? ▶ Source: NBA

28. 🌎 **MANUFACTURING** Look back at Example 4. Suppose the computer chip manufacturer has improved quality control so that only 1 out of 10,000 of its chips is defective. Now how many chips can you order before the probability that at least one chip is defective reaches 50%?

29. 🌎 **LOTTERY** To win a state lottery, a player must correctly match six different numbers from 1 to 42. If a computer randomly assigns six numbers per ticket, how many tickets would a person have to buy to have a 1% chance of winning?

STATISTICS ▶ **CONNECTION** **In Exercises 30 and 31, use the following information.**
The table, based on a Gallup Poll, shows the number of voters (in 1000's) by party affiliation who were expected to vote for Bill Clinton and Bob Dole in the 1996 Presidential election. ▶ Source: The Gallup Organization

	Democrat	Republican	Independent
Clinton	31,378	3,340	12,685
Dole	2,092	28,386	8,721

30. Find the probability that a randomly selected person voted for Clinton.

31. Find the probability that a randomly selected Democrat voted for Clinton.

32. 🌎 **TEACHERS** In the United States during the 1993–1994 school year, 39.6% of all male teachers and 26.1% of all female teachers had twenty years or more of full-time teaching experience. That year 694,000 males and 1,867,000 females were teachers. What is the probability that a randomly chosen teacher in the United States that year was a female with twenty years or more of full-time teaching experience?

▶ INTERNET **DATA UPDATE** of *Statistical Abstract of the United States* data at www.mcdougallittell.com

33. 🌎 **COSTUMES** You and four of your friends go to the same store at different times to buy costumes for a costume party. There are 20 different costumes at the store, and the store has at least five duplicates of each costume. Find the probability that all five of you choose different costumes.

34. 🌎 **AIRPLANE MEALS** On a long flight an airline usually serves a meal. If there are 2 choices for the meal, what is the probability that all 6 people in the first row choose the same meal assuming choices are made independently?
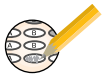
35. 🌎 **RETIREMENT PLAN** At a particular company 64% of the employees are forty years old or over. Of those employees, 83% are enrolled in the company's retirement plan. Only 61% of the employees under forty years old are enrolled in the plan. Make a probability tree diagram and use it to find the probability that a randomly selected employee is enrolled in the company's retirement plan.

36. 🌎 **FOCUS TESTING** A company is focus testing a new type of fruit drink. The focus group is 47% male. Of the males in the group, 40% said they would buy the fruit drink, and of the females, 54% said they would buy the fruit drink. Make a probability tree diagram and use it to find the probability that a randomly selected person would buy the fruit drink.

Test Preparation

QUANTITATIVE COMPARISON In Exercises 37 and 38, choose the statement that is true about the given quantities. Assume a standard 52-card deck is used.

- Ⓐ The quantity in column A is greater.
- Ⓑ The quantity in column B is greater.
- Ⓒ The two quantities are equal.
- Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
37.	The probability of drawing at least one heart when drawing 4 times with replacement	The probability of drawing at least one heart when drawing 4 times without replacement
38.	The probability of not drawing any card twice when drawing 9 times with replacement	The probability of drawing any card 2 or more times when drawing 9 times with replacement

★ **Challenge**

39. **CONDITIONAL PROBABILITY** Using the data from Example 8, find the conditional probability that a randomly selected person has diabetes given that the person is diagnosed incorrectly.

MIXED REVIEW

HISTOGRAMS Using the given intervals, make a frequency distribution of the data set. Then draw a histogram of the data set. (Review 7.7 for 12.6)

40. Use five intervals beginning with 51–60.
56, 68, 73, 79, 82, 82, 83, 85, 85, 87, 88, 89, 90, 90, 91, 93, 95, 100

41. Use four intervals beginning with 0.01–0.25.
0.3, 0.3, 0.4, 0.4, 0.5, 0.6, 0.6, 0.6, 0.7, 0.9, 0.9, 0.9, 0.9, 0.9

SOLVING EQUATIONS Solve the equation. (Review 8.8)

42. $\dfrac{5}{1 + 2e^{-x}} = 4$

43. $\dfrac{10}{1 + 6e^{-x}} = 8$

44. $\dfrac{9}{1 + 3e^{-3x}} = 6$

45. $\dfrac{12}{1 + e^{-2x}} = 4$

46. $\dfrac{1}{1 + 3e^{-5x}} = \dfrac{1}{2}$

47. $\dfrac{70}{1 + 12e^{-10x}} = \dfrac{2}{9}$

BINOMIAL THEOREM Expand the power of the binomial. (Review 12.2 for 12.6)

**48.** $(x + 1)^5$ **49.** $(2x - 1)^7$ **50.** $(x - 3y)^4$ **51.** $(x - 1)^6$

A jar contains 6 blue marbles, 12 green marbles, and 7 yellow marbles. Find the probability of randomly drawing the given marble. **(Lesson 12.3)**

1. a green marble   **2.** a blue marble   **3.** a green or a blue marble

Find the probability that a dart thrown at the circular target shown will hit the given region. Assume the dart is equally likely to hit any point inside the target. **(Lesson 12.3)**
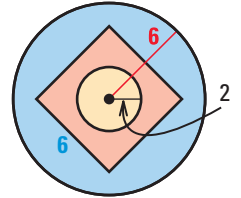
4. the center circle   **5.** outside the square

6. the area inside the square but outside the center circle

Find the indicated probability. **(Lesson 12.4)**

7. $P(A) = 0.7$
$P(B) = 0.2$
$P(A \text{ or } B) = \underline{\ ?\ }$
$P(A \text{ and } B) = 0.1$

8. $P(A) = 0.5$
$P(B) = 0.4$
$P(A \text{ or } B) = 0.9$
$P(A \text{ and } B) = \underline{\ ?\ }$

9. $P(A) = 0.25$
$P(A') = \underline{\ ?\ }$

10. 🌍 **FRUIT** You and four friends are in line at lunch and are each selecting a piece of fruit to eat. If there are 5 types of fruit available, what is the probability that you each select a different type? **(Lesson 12.5)**

MATH & History
Probability Theory

THEN

IN 1654 Blaise Pascal and Pierre Fermat solved the first probability problem, which asked how to divide the stakes of an interrupted game of chance between two players of equal ability. Suppose Player A has 2 points, Player B has 1 point, and the game is won by the first player to score 4 out of a possible 7 points.

1. If the stakes are divided based on which player is closest to winning, what fraction of the stakes should each player receive?

2. If the stakes are divided based on the number of points each player has so far, what fraction of the stakes should each player receive?

3. What is the probability Player A will win the game? What is the probability Player B will win the game? Based on these probabilities, what fraction of the stakes should each player receive?

NOW

TODAY probability theory is used to decide more than just games of chance. Actuaries, for example, use probability theory to design financial plans, to calculate insurance rates, and to price corporate securities offerings.
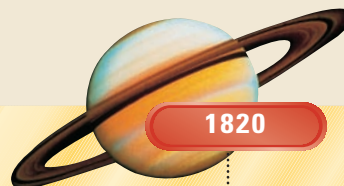
Actuary is named the best job in America.

Dice are in use.

c. 2000 B.C.

c. A.D. 1450

Playing cards are in use.

1820

Probability theory is extended to astronomy.

JOB RANKINGS
1. ACTUARY
2. SOFTWARE ENGINEER
3. COMPUTER SYSTEMS ANALYST
4. ACCOUNTANT
5. PARALEGAL ASSISTANT
6. MATHEMATICIAN

1995