

12.4

Probability of Compound Events

GOAL 1 PROBABILITIES OF UNIONS AND INTERSECTIONS

What you should learn

GOAL 1 Find probabilities of unions and intersections of two events.

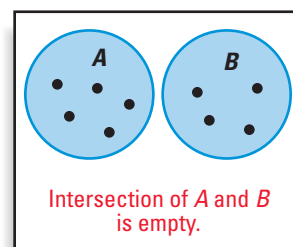
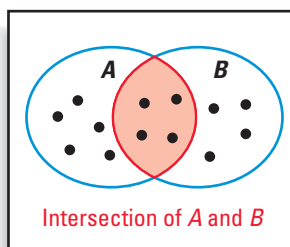
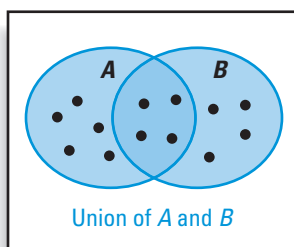
GOAL 2 Use complements to find the probability of an event, as applied in Example 5.

Why you should learn it

▼ To solve **real-life** problems, such as finding the probability that friends will be in the same college dormitory in Ex. 49.



When you consider all the outcomes for either of two events A and B , you form the *union* of A and B . When you consider only the outcomes shared by both A and B , you form the *intersection* of A and B . The union or intersection of two events is called a **compound event**.



To find $P(A \text{ or } B)$ you must consider what outcomes, if any, are in the intersection of A and B . If there are none, then A and B are **mutually exclusive events** and $P(A \text{ or } B) = P(A) + P(B)$. If A and B are not mutually exclusive, then the outcomes in the intersection of A and B are counted *twice* when $P(A)$ and $P(B)$ are added. So, $P(A \text{ and } B)$ must be subtracted *once* from the sum.

PROBABILITY OF COMPOUND EVENTS

If A and B are two events, then the probability of A or B is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are mutually exclusive, then the probability of A or B is:

$$P(A \text{ or } B) = P(A) + P(B)$$

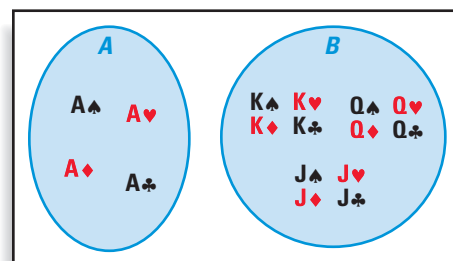
EXAMPLE 1 Probability of Mutually Exclusive Events

A card is randomly selected from a standard deck of 52 cards. What is the probability that it is an ace *or* a face card?

SOLUTION

Let event A be selecting an ace, and let event B be selecting a face card. Event A has 4 outcomes and event B has 12 outcomes. Because A and B are mutually exclusive, the probability is:

$$P(A \text{ or } B) = P(A) + P(B) = \frac{4}{52} + \frac{12}{52} = \frac{16}{52} = \frac{4}{13} \approx 0.308$$



EXAMPLE 2 Probability of a Compound Event

A card is randomly selected from a standard deck of 52 cards. What is the probability that the card is a heart *or* a face card?

SOLUTION

Let event A be selecting a heart, and let event B be selecting a face card. Event A has 13 outcomes and event B has 12 outcomes. Of these, three outcomes are common to A and B . So, the probability of selecting a heart *or* a face card is:

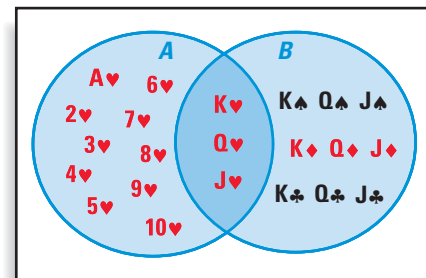
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

$$= \frac{22}{52}$$

$$= \frac{11}{26}$$

$$\approx 0.423$$



Write general formula.

Substitute known probabilities.

Combine terms.

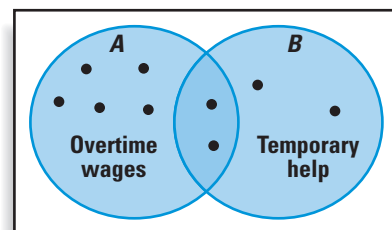
Simplify.

Use a calculator.



EXAMPLE 3 Using Intersection to Find Probability

Last year a company paid overtime wages *or* hired temporary help during 9 months. Overtime wages were paid during 7 months and temporary help was hired during 4 months. At the end of the year, an auditor examines the accounting records and randomly selects one month to check the company's payroll. What is the probability that the auditor will select a month in which the company paid overtime wages *and* hired temporary help?



SOLUTION

Let event A represent paying overtime wages during a month, and let event B represent hiring temporary help during a month. From the given information you know that:

$$P(A) = \frac{7}{12}, P(B) = \frac{4}{12}, \text{ and } P(A \text{ or } B) = \frac{9}{12}$$

The probability that the auditor will select a month in which the company paid overtime wages *and* hired temporary help is $P(A \text{ and } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Write general formula.

$$\frac{9}{12} = \frac{7}{12} + \frac{4}{12} - P(A \text{ and } B)$$

Substitute known probabilities.

$$P(A \text{ and } B) = \frac{7}{12} + \frac{4}{12} - \frac{9}{12}$$

Solve for $P(A \text{ and } B)$.

$$P(A \text{ and } B) = \frac{2}{12} = \frac{1}{6} \approx 0.167$$

Simplify.

GOAL 2 USING COMPLEMENTS TO FIND PROBABILITY

The event A' , called the **complement** of event A , consists of all outcomes that are not in A . The notation A' is read as “A prime.”

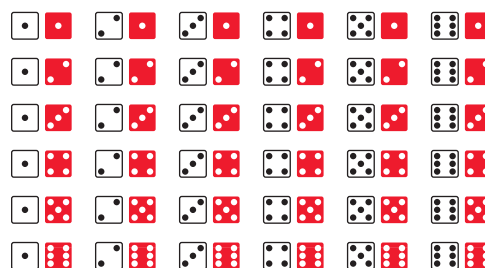
PROBABILITY OF THE COMPLEMENT OF AN EVENT

The probability of the complement of A is $P(A') = 1 - P(A)$.

EXAMPLE 4 Probabilities of Complements

When two six-sided dice are tossed, there are 36 possible outcomes as shown. Find the probability of the given event.

- The sum is not 8.
- The sum is greater than or equal to 4.



SOLUTION

$$\text{a. } P(\text{sum is not 8}) = 1 - P(\text{sum is 8})$$

$$= 1 - \frac{5}{36}$$

$$= \frac{31}{36}$$

$$\approx 0.861$$

$$\text{b. } P(\text{sum} \geq 4) = 1 - P(\text{sum} < 4)$$

$$= 1 - \frac{3}{36}$$

$$= \frac{33}{36}$$

$$= \frac{11}{12}$$

$$\approx 0.917$$

EXAMPLE 5 Using a Complement in Real Life

HOME ELECTRONICS Four houses in a neighborhood have the same model of garage door opener. Each opener has 4096 possible transmitter codes. What is the probability that at least two of the four houses have the same code?

SOLUTION

The total number of ways to assign codes to the four openers is 4096^4 . The number of ways to assign *different* codes to the four openers is $4096 \cdot 4095 \cdot 4094 \cdot 4093$. So, the probability that at least two of the four openers have the same code is:

$$P(\text{at least 2 are the same}) = 1 - P(\text{none are the same})$$

$$= 1 - \frac{4096 \cdot 4095 \cdot 4094 \cdot 4093}{4096^4}$$

$$\approx 1 - 0.99854$$

$$= 0.00146$$

STUDENT HELP

INTERNET HOMEWORK HELP
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FOCUS ON APPLICATIONS



REAL LIFE HOME ELECTRONICS

One type of garage door opener has 12 switches that can be set in one of two positions (off or on) to create a code. So for this type of garage door opener, there are 2^{12} , or 4096, possible transmitter codes.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

- Describe what it means for two events to be mutually exclusive.
- Write a formula for computing $P(A \text{ or } B)$ that applies to *any* events A and B . How can you simplify this formula when A and B are mutually exclusive?
- Are the events A and A' mutually exclusive? Explain.

Events A and B are mutually exclusive. Find $P(A \text{ or } B)$.

- $P(A) = 0.2, P(B) = 0.3$
- $P(A) = \frac{3}{8}, P(B) = \frac{1}{8}$
- $P(A) = 0.5, P(B) = 0.5$
- $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$

Find $P(A \text{ and } B)$.

- $P(A) = 0.5, P(B) = 0.4,$
 $P(A \text{ and } B) = 0.3$
- $P(A) = \frac{2}{5}, P(B) = \frac{3}{5},$
 $P(A \text{ and } B) = \frac{1}{5}$

Find $P(A \text{ and } B)$.

- $P(A) = 0.7, P(B) = 0.2,$
 $P(A \text{ or } B) = 0.8$
- $P(A) = \frac{5}{16}, P(B) = \frac{7}{16},$
 $P(A \text{ or } B) = \frac{9}{16}$

Find $P(A')$.

- $P(A) = 0.5$
- $P(A) = 0.75$
- $P(A) = \frac{1}{3}$
- $P(A) = \frac{4}{7}$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 957.

FINDING PROBABILITIES Find the indicated probability. State whether A and B are mutually exclusive.

- $P(A) = 0.4$
 $P(B) = 0.35$
 $P(A \text{ or } B) = 0.5$
 $P(A \text{ and } B) = ?$
- $P(A) = 0.6$
 $P(B) = 0.2$
 $P(A \text{ or } B) = ?$
 $P(A \text{ and } B) = 0.1$
- $P(A) = 0.25$
 $P(B) = ?$
 $P(A \text{ or } B) = 0.70$
 $P(A \text{ and } B) = 0$
- $P(A) = \frac{13}{17}$
 $P(B) = ?$
 $P(A \text{ or } B) = \frac{14}{17}$
 $P(A \text{ and } B) = \frac{6}{17}$
- $P(A) = \frac{1}{3}$
 $P(B) = \frac{1}{4}$
 $P(A \text{ or } B) = \frac{7}{12}$
 $P(A \text{ and } B) = ?$
- $P(A) = \frac{3}{4}$
 $P(B) = \frac{1}{3}$
 $P(A \text{ or } B) = ?$
 $P(A \text{ and } B) = \frac{1}{4}$
- $P(A) = 5\%$
 $P(B) = 29\%$
 $P(A \text{ or } B) = ?$
 $P(A \text{ and } B) = 0\%$
- $P(A) = 30\%$
 $P(B) = ?$
 $P(A \text{ or } B) = 50\%$
 $P(A \text{ and } B) = 10\%$
- $P(A) = 16\%$
 $P(B) = 24\%$
 $P(A \text{ or } B) = 32\%$
 $P(A \text{ and } B) = ?$

FINDING PROBABILITIES OF COMPLEMENTS Find $P(A')$.

- $P(A) = 0.34$
- $P(A) = 0$
- $P(A) = \frac{3}{4}$
- $P(A) = 1$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–24,
29–34, 42, 43

Example 2: Exs. 16–24,
29–34, 44, 45

Example 3: Exs. 16–24,
29–34, 46, 47

Example 4: Exs. 25–28,
35–40

Example 5: Exs. 48, 49

STUDENT HELP**Look Back**

For help with a standard 52-card deck in Exs. 29–34, see p. 708.

Look Back

For help with simulations on a graphing calculator in Ex. 41, see p. 723.

CHOOSING CARDS A card is randomly drawn from a standard 52-card deck. Find the probability of the given event. (A face card is a king, queen, or jack.)

29. a queen and a heart 30. a queen or a heart 31. a heart or a diamond
32. a five or a six 33. a five and a six 34. a three or a face card

USING COMPLEMENTS Two six-sided dice are rolled. Find the probability of the given event. (Refer to Example 4 for a diagram of all possible outcomes.)

35. The sum is not 3. 36. The sum is greater than or equal to 5.
37. The sum is neither 3 nor 7. 38. The sum is less than or equal to 10.
39. The sum is greater than 2. 40. The sum is less than 8 or greater than 11.

41.  **EXPERIMENTAL PROBABILITIES** Simulate rolling two dice 120 times. Use separate lists for the results of each die, and a third list for the sum. Record the frequency of each sum in a table. Find the experimental probabilities of the events in Exercises 35–40. How do your experimental results compare with the theoretical results?

42.  **COMPANY MOVE** An employee of a large national company is promoted to management and will be moved within six months. The employee is told that there is a 33% probability of being moved to Denver, Colorado, and a 50% probability of being moved to Dallas, Texas. What is the probability that the employee will be moved to Dallas or Denver?
43.  **CLASS ELECTIONS** You and your best friend are among several candidates running for class president. You estimate that there is a 40% chance you will win the election and a 35% chance your best friend will win. What is the probability that either you or your best friend wins the election?
44.  **PARAKEETS** A pet store contains 35 light green parakeets (14 females and 21 males) and 44 sky blue parakeets (28 females and 16 males). You randomly choose one of the parakeets. What is the probability that it is a female or a sky blue parakeet?

45.  **HONORS BANQUET** Of 162 students honored at an academic awards banquet, 48 won awards for mathematics and 78 won awards for English. Fourteen of these students won awards for both mathematics and English. One of the 162 students is chosen at random to be interviewed for a newspaper article. What is the probability that the student interviewed won an award for English or mathematics?
46.  **SCIENCE CONNECTION** A tree in a forest is not growing properly. A botanist determines that there is an 85% probability the tree has a disease or is being damaged by insects, a 45% probability it has a disease, and a 50% probability it is being damaged by insects. What is the probability that the tree both has a disease and is being damaged by insects?

47.  **RAIN** A weather forecaster says that the probability it will rain on Saturday or Sunday is 50%, the probability it will rain on Saturday is 20%, and the probability it will rain on Sunday is 40%. What is the probability that it will rain on both Saturday and Sunday?
48.  **POTLUCK DINNER** The organizer of a potluck dinner sends 5 people a list of 8 different recipes and asks each person to bring one of the items on the list. If all 5 people randomly choose a recipe from the list, what is the probability that at least 2 will bring the same thing?

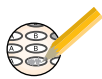
FOCUS ON CAREERS**BOTANIST**

A botanist studies plants and their environment. Some botanists specialize in the causes and cures of plant illnesses, as discussed in Ex. 46.

**CAREER LINK**

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49. **CAMPUS HOUSING** Four high school friends will all be attending the same university next year. There are 14 dormitories on campus. Find the probability that at least 2 of the friends will be in the same dormitory.
50. **CRITICAL THINKING** What is the complement of A' ? Explain.
51. **MULTI-STEP PROBLEM** Follow the steps below to explore a famous probability problem called the *birthday problem*.
- Suppose that 5 people are chosen at random. Find the probability that at least two share the same birthday (Assume that there are 365 possible birthdays).
 - Suppose that 10 people are chosen at random. Find the probability that at least two of the people share the same birthday.
 - Generalize the results from parts (a) and (b) by writing a formula that gives the probability $P(n)$ that in a group of n people at least two people share the same birthday. (Hint: Use ${}_nP_r$ notation.)
 - LOGICAL REASONING** Enter the formula for $P(n)$ from part (c) into a graphing calculator put in sequence mode. Use the *Table* feature to make a table of values for $P(n)$. How large must a group be if the probability that at least two of the people share the same birthday exceeds 50%?

ODDS The odds in favor of an event occurring are the ratio of the probability that the event *will* occur to the probability that the event *will not* occur. The reciprocal of this ratio represents the odds *against* the event occurring.

52. Five marbles in a jar are green. The odds against choosing a green marble are “4 to 1.” How many marbles are in the jar?
53. If a jar contains 4 red marbles and 7 blue marbles, what are the odds in favor of choosing a red marble? What are the odds against choosing a red marble?
54. Write a formula that converts the odds in favor of an event to the probability of the event.
55. Write a formula that converts the probability of an event to the odds in favor of the event.

MIXED REVIEW

SOLVING EXPONENTIAL EQUATIONS Solve the equation. (Review 8.6 for 12.5)

56. $7^{2x} = 49^{16}$ 57. $9^x = 3^{x+1}$ 58. $2^{4x+8} = 32^{19}$
59. $5^x = 21$ 60. $10^{3x-1} - 13 = 8$ 61. $72 = 91e^{-0.023x} + 50$

WRITING EQUATIONS Write the standard form of the equation of the circle that passes through the given point and whose center is the origin. (Review 10.3)

62. (0, 7) 63. (3, 4) 64. (-1, 6) 65. (8, -2)
66. (4, 4) 67. (3, 10) 68. (-4, -2) 69. (16, 0)

LICENSE PLATES For the given configuration, determine how many different license plates are possible if (a) digits and letters can be repeated, and (b) digits and letters cannot be repeated. (Review 12.1 for 12.5)

70. 3 letters followed by 4 digits 71. 4 letters followed by 3 digits