

# 12.3

## An Introduction to Probability

### GOAL 1 THEORETICAL AND EXPERIMENTAL PROBABILITY

#### What you should learn

**GOAL 1** Find theoretical and experimental probabilities.

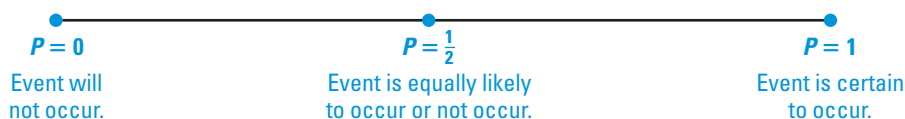
**GOAL 2** Find geometric probabilities, as applied in Example 5.

#### Why you should learn it

▼ To solve **real-life** problems, such as finding the probability that an archer hits the center of a target in Ex. 46.



The **probability** of an event is a number between 0 and 1 that indicates the likelihood the event will occur. An event that is certain to occur has a probability of 1. An event that *cannot* occur has a probability of 0. An event that is equally likely to occur or not occur has a probability of  $\frac{1}{2}$ .



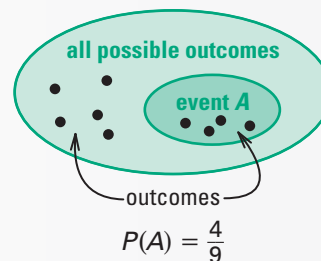
There are two types of probability: *theoretical* and *experimental*. Theoretical probability is defined below and experimental probability is defined on page 717.

### THE THEORETICAL PROBABILITY OF AN EVENT

When all outcomes are equally likely, the **theoretical probability** that an event  $A$  will occur is:

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$$

The theoretical probability of an event is often simply called the probability of the event.



### EXAMPLE 1 Finding Probabilities of Events

You roll a six-sided die whose sides are numbered from 1 through 6. Find the probability of (a) rolling a 4, (b) rolling an odd number, and (c) rolling a number less than 7.

#### SOLUTION

a. Only one outcome corresponds to rolling a 4.

$$P(\text{rolling a 4}) = \frac{\text{number of ways to roll a 4}}{\text{number of ways to roll the die}} = \frac{1}{6}$$

b. Three outcomes correspond to rolling an odd number: rolling a 1, 3, or 5.

$$P(\text{rolling an odd number}) = \frac{\text{number of ways to roll an odd number}}{\text{number of ways to roll the die}} = \frac{3}{6} = \frac{1}{2}$$

c. All six outcomes correspond to rolling a number less than 7.

$$P(\text{rolling less than 7}) = \frac{\text{number of ways to roll less than 7}}{\text{number of ways to roll the die}} = \frac{6}{6} = 1$$

You can express a probability as a fraction, a decimal, or a percent. For instance, in part (b) of Example 1 the probability of rolling an odd number can be written as  $\frac{1}{2}$ , 0.5, or 50%.



### EXAMPLE 2 Probabilities Involving Permutations or Combinations

You put a CD that has 8 songs in your CD player. You set the player to play the songs at random. The player plays all 8 songs without repeating any song.

- What is the probability that the songs are played in the same order they are listed on the CD?
- You have 4 favorite songs on the CD. What is the probability that 2 of your favorite songs are played first, in any order?

#### SOLUTION

- There are 8! different *permutations* of the 8 songs. Of these, only 1 is the order in which the songs are listed on the CD. So, the probability is:

$$P(\text{playing 8 in order}) = \frac{1}{8!} = \frac{1}{40,320} \approx 0.0000248$$

- There are  ${}_8C_2$  different *combinations* of 2 songs. Of these,  ${}_4C_2$  contain 2 of your favorite songs. So, the probability is:

$$P(\text{playing 2 favorites first}) = \frac{{}_4C_2}{{}_8C_2} = \frac{6}{28} = \frac{3}{14} \approx 0.214$$

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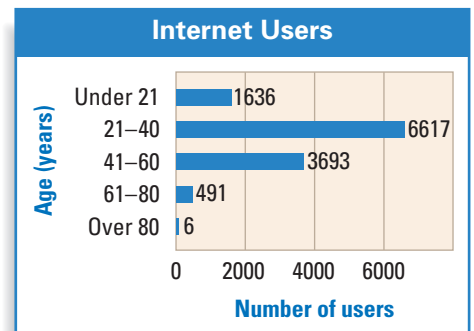
Sometimes it is not possible or convenient to find the theoretical probability of an event. In such cases you may be able to calculate an **experimental probability** by performing an experiment, conducting a survey, or looking at the history of the event.



### EXAMPLE 3 Finding Experimental Probabilities

In 1998 a survey asked Internet users for their ages. The results are shown in the bar graph. Find the experimental probability that a randomly selected Internet user is (a) at most 20 years old, and (b) at least 41 years old.

▶ Source: GVU's WWW User Surveys™



**SOLUTION** The number of people surveyed was  $1636 + 6617 + 3693 + 491 + 6 = 12,443$ .

- Of the people surveyed, 1636 are at most 20 years old. So, the probability is:

$$P(\text{user is at most 20}) = \frac{1636}{12,443} \approx 0.131$$

- Of the people surveyed,  $3693 + 491 + 6 = 4190$  are at least 41 years old. So, the probability is:

$$P(\text{user is at least 41}) = \frac{4190}{12,443} \approx 0.337$$

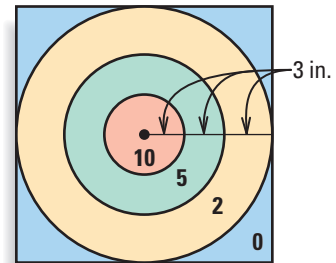
**STUDENT HELP**  
**Skills Review**  
 For help with converting decimals, fractions, and percents, see p. 906.

## GOAL 2 GEOMETRIC PROBABILITY

Some probabilities are found by calculating a ratio of two lengths, areas, or volumes. Such probabilities are called **geometric probabilities**.

### EXAMPLE 4 Using Area to Find Probability

You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board. Are you more likely to get 10 points or 0 points?



#### SOLUTION

The two probabilities are as follows.

$$\begin{aligned} P(10 \text{ points}) &= \frac{\text{area of smallest circle}}{\text{area of entire board}} \\ &= \frac{\pi \cdot 3^2}{18^2} = \frac{9\pi}{324} = \frac{\pi}{36} \approx 0.0873 \end{aligned}$$

$$\begin{aligned} P(0 \text{ points}) &= \frac{\text{area outside largest circle}}{\text{area of entire board}} \\ &= \frac{18^2 - (\pi \cdot 9^2)}{18^2} = \frac{324 - 81\pi}{324} = \frac{4 - \pi}{4} \approx 0.215 \end{aligned}$$

▶ You are more likely to get 0 points.

#### STUDENT HELP

**Skills Review**  
For help with area, see p. 914.

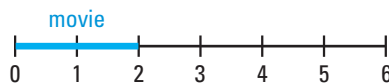


### EXAMPLE 5 Using Length to Find Probability

You have recorded a 2 hour movie at the beginning of a videocassette that has 6 hours of recording time. Starting at a random location on the videocassette, your brother records a 30 minute television show. What is the probability that your brother's television show accidentally records over part of your movie?

#### SOLUTION

You can think of the videocassette as a number line from 0 to 6. The movie can be represented as a line segment 2 units long and the television show as a line segment 0.5 unit long. Because you know the movie starts at the beginning of the videocassette, the number line is as shown.



If the 30 minute, or half hour, television show is to fit on the tape, it must start somewhere between 0 and 5.5. If it records over part of the movie, it must start somewhere between 0 and 2. So, the probability of recording over part of the movie is:

$$\begin{aligned} P(\text{recording over movie}) &= \frac{\text{length where show will record over movie}}{\text{length where show will fit on tape}} \\ &= \frac{2 - 0}{5.5 - 0} = \frac{2}{5.5} = \frac{4}{11} \approx 0.364 \end{aligned}$$

#### STUDENT HELP

**INTERNET HOMEWORK HELP**  
Visit our Web site  
[www.mcdougallittell.com](http://www.mcdougallittell.com)  
for extra examples.

# GUIDED PRACTICE

## Vocabulary Check ✓

1. Complete this statement: A probability that involves length, area, or volume is called a(n)   ?   probability.

## Concept Check ✓

2.  $P(A) = 0.2$  and  $P(B) = 0.6$ . Which event is more likely to occur? Explain.

3. Explain the difference between theoretical probability and experimental probability. Give an example of each.

## Skill Check ✓

A jar contains 2 red marbles, 3 blue marbles, and 1 green marble. Find the probability of randomly drawing the given type of marble.

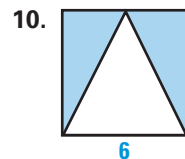
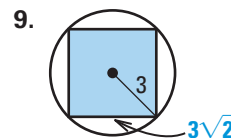
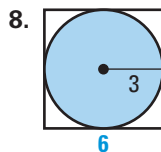
4. a red marble


5. a green marble

6. a blue or a green marble

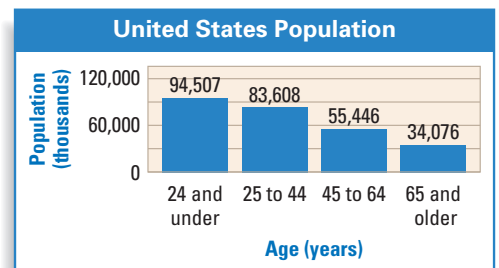
7. a red or a blue marble

Find the probability that a dart thrown at the given target will hit the shaded region. Assume the dart is equally likely to hit any point inside the target. The targets and regions within are either squares, circles, or triangles.



11.  **POPULATION** The bar graph shown gives the resident population (in thousands) of the United States in 1997. For a randomly selected person in the United States, find the probability of the given event.

 **DATA UPDATE** of *Statistical Abstract of the United States* data at [www.mcdougallittell.com](http://www.mcdougallittell.com)



- a. The person is 24 years old or under.      b. The person is at least 45 years old.

# PRACTICE AND APPLICATIONS

## STUDENT HELP

**Extra Practice**  
to help you master skills is on p. 956.

**CHOOSING NUMBERS** You have an equally likely chance of choosing any integer from 1 through 20. Find the probability of the given event.

12. An odd number is chosen.

13. A number less than 7 is chosen.

14. A perfect square is chosen.

15. A prime number is chosen.

16. A multiple of 3 is chosen.

17. A factor of 240 is chosen.

**CHOOSING CARDS** A card is drawn randomly from a standard 52-card deck. Find the probability of drawing the given card.

18. the ace of hearts

19. any ace

20. a diamond

21. a red card

22. a card other than 10

23. a face card (a king, queen, or jack)

## STUDENT HELP

**Look Back**  
For help with a standard 52-card deck, see p. 708.

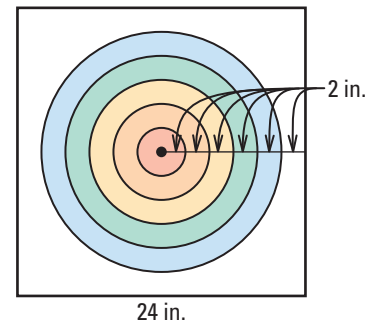
**STUDENT HELP****HOMEWORK HELP****Example 1:** Exs. 12–23**Example 2:** Exs. 35–40**Example 3:** Exs. 24–29,  
41–43**Example 4:** Exs. 30–34,  
46, 47**Example 5:** Exs. 44, 45

**ROLLING A DIE** The results of rolling a six-sided die 120 times are shown. Use the table to find the experimental probability of each event. Also find the theoretical probability. How do the probabilities compare?

| Results from Rolling a Die 120 Times |    |    |    |    |    |    |
|--------------------------------------|----|----|----|----|----|----|
| Roll                                 | 1  | 2  | 3  | 4  | 5  | 6  |
| Number of occurrences                | 15 | 18 | 20 | 17 | 24 | 26 |

24. rolling a 6  
25. rolling a 3 or 4  
26. rolling an odd number  
27. rolling an even number  
28. rolling a number greater than 2  
29. rolling anything but a 1

**GEOMETRY CONNECTION** Find the probability that a dart thrown at the square target shown will hit the given region. Assume the dart is equally likely to hit any point inside the target.



30. the red center  
31. the white border  
32. the red center or the white border  
33. the four rings or the red center  
34. the yellow or green ring

**SPEECHES** In Exercises 35 and 36, use the following information.

Your English teacher is drawing names to see who will give the first speech. There are 26 students in the class and 4 speeches will be given each day.

35. What is the probability that you will give your speech first?  
36. What is the probability that you will give your speech on the first day?

**WORD GAMES** In Exercises 37 and 38, use the following information.

You and a friend are playing a word game that involves lettered tiles. The distribution of letters is shown at the right. At the start of the game you choose 7 letters.

| Distribution of Letters |      |      |          |
|-------------------------|------|------|----------|
| A: 9                    | H: 2 | O: 8 | V: 2     |
| B: 2                    | I: 9 | P: 2 | W: 2     |
| C: 2                    | J: 1 | Q: 1 | X: 1     |
| D: 4                    | K: 1 | R: 6 | Y: 2     |
| E: 12                   | L: 4 | S: 4 | Z: 1     |
| F: 2                    | M: 2 | T: 6 | Blank: 2 |
| G: 3                    | N: 6 | U: 4 |          |

37. What is the probability that you will choose three vowels and four consonants? (Count “Y” as a vowel.)  
38. What is the probability that you will choose the letters A, B, C, D, E, F, and G in order?

**LOTTERIES** In Exercises 39 and 40, find the probability of winning the lottery according to the given rules. Assume numbers are selected at random.

39. You must correctly select 6 out of 51 numbers. The order of the numbers is not important.  
40. You must correctly select 3 numbers, each from 0 to 9. The order of the numbers is important.

**FOCUS ON APPLICATIONS**

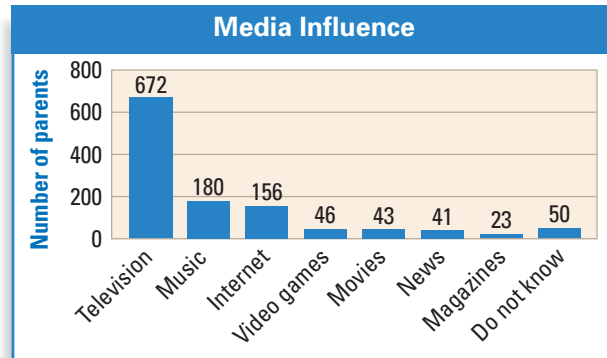


**STOCK MARKET**  
The largest one-day stock-market loss in Wall Street history occurred on October 19, 1987. That day the Dow Jones fell 508.32 points, or 22.6%. The net change for the year, however, was a growth of 20.9%.



**APPLICATION LINK**  
[www.mcdougallittell.com](http://www.mcdougallittell.com)

41. **MEDIA CONCERN** In a 1998 survey, parents were asked what media influence on their children most concerned them. The results are shown in the bar graph. Find the experimental probability that a randomly selected parent is most concerned about the given topic. **Source:** Annenberg Public Policy Center



- a. Television b. Video games

42. **HISTORY CONNECTION** The table shows how many years the stock market gained or lost a given percent over a recent 17 year period based on the Dow Jones Industrial Average. Find the experimental probability of the given event.

| Range | Lost more than 0% | Gained 0% to 9% | Gained 10% to 19% | Gained 20% to 29% | Gained more than 29% |
|-------|-------------------|-----------------|-------------------|-------------------|----------------------|
| Years | 4                 | 2               | 4                 | 4                 | 3                    |

- a. The stock market has a loss. b. The stock market gains at least 10%.

43. **STATISTICS CONNECTION** The table shows how people in the United States got to work in 1990. For a randomly selected person in the United States, find the probability that the person chose the given type of transportation.

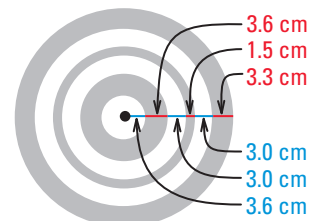
**Source:** *The World Almanac*

- a. Used public transportation  
b. Drove to work (either in an automobile or on a motorcycle)

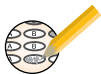
| Means of transportation | Number     |
|-------------------------|------------|
| Automobile              | 99,592,932 |
| Public transportation   | 6,069,589  |
| Motorcycle              | 237,404    |
| Bicycle                 | 466,856    |
| Other                   | 5,297,468  |
| None (work at home)     | 3,406,025  |

44. **VIDEOCASSETTES** Look back at Example 5. Suppose you recorded your movie starting 1 hour into the videocassette. What is the probability that your brother's television show accidentally records over part of your movie?
45. **CABLE INSTALLATION** You set up an appointment to have cable television installed between 12:00 P.M. and 4:00 P.M. The installer will wait 15 minutes if no one is home. Your cousin asks for a favor that would take you away from your home from 1:30 P.M. to 2:00 P.M. If you do the favor, what is the probability that you will miss the cable installer?

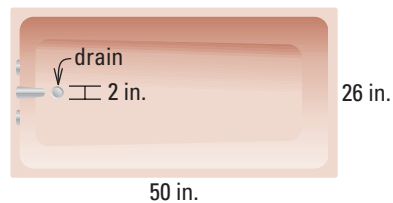
46. **KYUDO** Kyudo is a form of Japanese archery. The most common target is shown. Find the probability that an arrow shot at the target will hit the center circle. Assume the arrow is equally likely to hit any point inside the target.



## Test Preparation



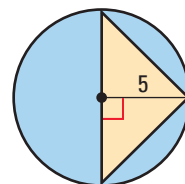
47. **CONTACT LENSES** You have just stepped into the tub to take a shower when one of your contact lenses falls out. (You have not yet turned on the shower.) Assuming that the lens is equally likely to land anywhere on or inside of the tub, what is the probability that it landed in the drain?



48. **MULTIPLE CHOICE** On a multiple choice question, you know that the answer is not B or D, but you are not sure about answers A, C, or E. What is the probability that you will get the right answer if you guess?

(A)  $\frac{1}{5}$       (B)  $\frac{4}{5}$       (C)  $\frac{1}{3}$       (D)  $\frac{2}{3}$       (E)  $\frac{3}{5}$

49. **MULTIPLE CHOICE** A dart thrown at the circular target shown is equally likely to hit any point inside the target. What is the probability that it hits the region outside the triangle?



(A) 0.5      (B) 0.75      (C) 0.32  
(D) 0.47      (E) 0.68

## ★ Challenge

50. **PROBABILITY** Find the probability that the graph of  $y = x^2 - 4x + c$  intersects the  $x$ -axis if  $c$  is a randomly chosen integer from 1 to 6.

## MIXED REVIEW

**DETERMINANTS** Evaluate the determinant of the matrix. (Review 4.3)

51.  $\begin{bmatrix} 2 & 7 \\ 5 & 9 \end{bmatrix}$

52.  $\begin{bmatrix} 6 & 0 \\ 1 & -3 \end{bmatrix}$

53.  $\begin{bmatrix} 3 & 8 \\ -2 & 1 \end{bmatrix}$

54.  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

55.  $\begin{bmatrix} 0 & 1 & 5 \\ -3 & 4 & 7 \\ 2 & 1 & -4 \end{bmatrix}$

56.  $\begin{bmatrix} -1 & -2 & 2 \\ 4 & 3 & -4 \\ 2 & 4 & 6 \end{bmatrix}$

**MULTIPLYING** Multiply the rational expressions. Simplify the result. (Review 9.4)

57.  $\frac{6xy^2}{5x^3y} \cdot \frac{10y^4}{9xy}$

58.  $\frac{x^2 + 3x + 2}{x^2 - x - 6} \cdot \frac{x^2 - 3x}{x^2 - x - 2}$

59.  $\frac{25x^2 - 16}{5x - 4} \cdot \frac{x^2 - 4x - 21}{5x^3 - 31x^2 - 28x}$

60.  $\frac{4x^2 - 12x}{27 - x^3} \cdot (x^2 + 3x + 9)$

**WRITING TERMS** Write the first five terms of the sequence. (Review 11.5)

61.  $a_0 = 3$   
 $a_n = a_{n-1} + 7$

62.  $a_0 = -1$   
 $a_n = 3 \cdot a_{n-1}$

63.  $a_0 = 2$   
 $a_n = (a_{n-1})^3$

64.  $a_0 = 1$   
 $a_1 = 1$   
 $a_n = a_{n-1} + a_{n-2}$

65.  $a_0 = -2$   
 $a_1 = 0$   
 $a_n = a_{n-1} - a_{n-2}$

66.  $a_0 = 1$   
 $a_1 = -2$   
 $a_n = a_{n-1} \cdot a_{n-2}$

67. **TRADE SHOWS** You are attending a trade show that has booths from 20 different vendors. You hope to visit at least 5 of the booths. How many combinations of booths can you visit? (Review 12.2 for 12.4)