CHAPTER 10

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Find the distance between two points. **(10.1)**	Find the distance a medical helicopter must travel. **(p. 593)**
Find the midpoint of the line segment connecting two points. **(10.1)**	Find the diameter of a broken dish. **(p. 591)**
Use distance and midpoint formulas in real-life situations. **(10.1)**	Design a city park. **(p. 593)**
Graph and write equations of conics.	
• parabolas **(10.2, 10.6)**	Model a solar energy collector. **(p. 597)**
• circles **(10.3, 10.6)**	Model the region lit by a lighthouse. **(p. 603)**
• ellipses **(10.4, 10.6)**	Model the shape of an Australian football field. **(p. 614)**
• hyperbolas **(10.5, 10.6)**	Model the curved sides of a sculpture. **(p. 617)**
Classify a conic using its equation. **(10.6)**	Classify mirrors in a Cassegrain telescope. **(p. 627)**
Solve systems of quadratic equations. **(10.7)**	Find the epicenter of an earthquake. **(p. 634)**
Use conics to solve real-life problems. **(10.2–10.7)**	Find the area of The Ellipse at the White House. **(p. 611)**

How does Chapter 10 fit into the BIGGER PICTURE of algebra?

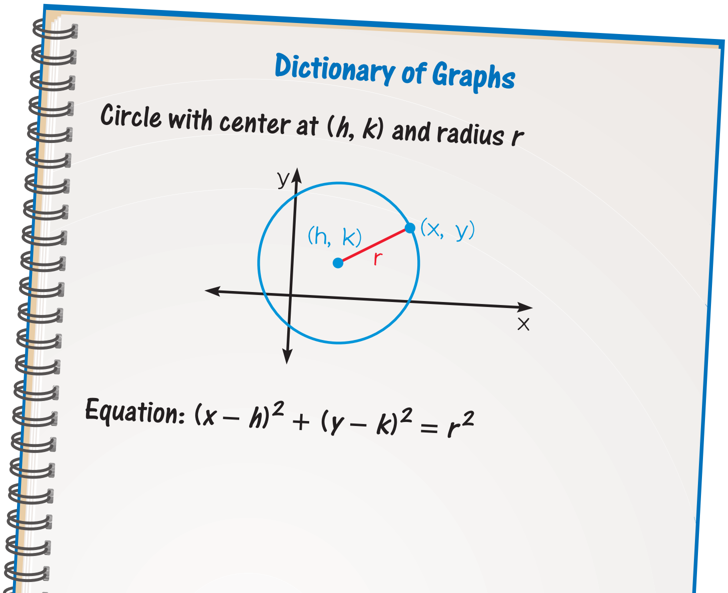
In Chapter 5 you studied parabolas as graphs of quadratic functions, and in Chapter 9 you studied hyperbolas as graphs of rational functions. In a previous course you studied circles, and possibly ellipses, in the context of geometry. In Chapter 10 you studied all four conic sections (parabolas, hyperbolas, circles, and ellipses) as graphs of equations of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

The conic sections are an important part of your study of algebra and geometry because they have many different real-life applications.

STUDY STRATEGY

How did you make and use a dictionary of graphs?

Here is an example of one entry for your dictionary of graphs, following the **Study Strategy** on page 588.



CHAPTER 10

Chapter Review

VOCABULARY

- distance formula, p. 589
- midpoint formula, p. 590
- focus, p. 595, 609, 615
- directrix, p. 595
- circle, p. 601
- center, p. 601, 609, 615
- radius, p. 601
- equation of a circle, p. 601
- ellipse, p. 609
- vertex, p. 609, 615
- major axis, p. 609
- co-vertex, p. 609
- minor axis, p. 609
- equation of an ellipse, p. 609
- hyperbola, p. 615
- transverse axis, p. 615
- equation of a hyperbola, p. 615
- conic sections, p. 623
- general second-degree equation, p. 626
- discriminant, p. 626

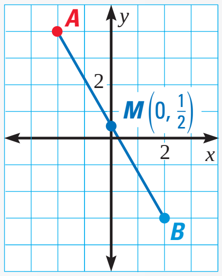
10.1 THE DISTANCE AND MIDPOINT FORMULAS

Examples on pp. 589–591

EXAMPLES Let $A = (-2, 4)$ and $B = (2, -3)$.

$$\text{Distance between } A \text{ and } B = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - (-2))^2 + (-3 - 4)^2}$$
$$= \sqrt{16 + 49} = \sqrt{65} \approx 8.06$$

$$\text{Midpoint of } \overline{AB} = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{(-2) + 2}{2}, \frac{4 + (-3)}{2}\right) = \left(0, \frac{1}{2}\right)$$



Find the distance between the two points. Then find the midpoint of the line segment connecting the two points.

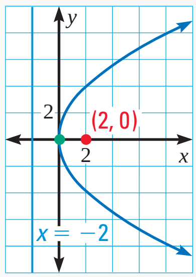
1. $(-2, -3), (4, 2)$ **2.** $(-5, 4), (10, -3)$ **3.** $(0, 0), (-4, 4)$ **4.** $(-2, 0), (0, -8)$

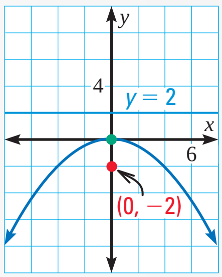
10.2 PARABOLAS

Examples on pp. 595–597

EXAMPLES The parabola with equation $y^2 = 8x$ has **vertex (0, 0)** and a horizontal axis of symmetry. It opens to the right. Note that $y^2 = 4px = 8x$, so $p = 2$. The **focus** is $(p, 0) = (2, 0)$, and the **directrix** is $x = -p = -2$.

The parabola with equation $x^2 = -8y$ has **vertex (0, 0)** and a vertical axis of symmetry. It opens down. Note that $x^2 = 4py = -8y$, so $p = -2$. The **focus** is $(0, p) = (0, -2)$, and the **directrix** is $y = -p = 2$.





Identify the focus and directrix of the parabola. Then draw the parabola.

5. $x^2 = 4y$ **6.** $x^2 = -2y$ **7.** $6x + y^2 = 0$ **8.** $y^2 - 12x = 0$

Write the equation of the parabola with the given characteristic and vertex (0, 0).

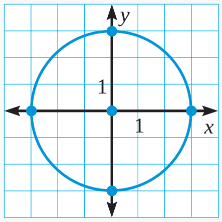
9. focus: $(4, 0)$ **10.** focus: $(0, -3)$ **11.** directrix: $y = -2$ **12.** directrix: $x = 1$

10.3 CIRCLES

Examples on pp. 601–603

EXAMPLE The circle with equation $x^2 + y^2 = 9$ has center at (0, 0) and radius $r = \sqrt{9} = 3$.

Four points on the circle are (3, 0), (0, 3), (−3, 0), and (0, −3).



Graph the equation.

13. $x^2 + y^2 = 16$ **14.** $x^2 + y^2 = 64$ **15.** $x^2 + y^2 = 6$ **16.** $3x^2 + 3y^2 = 363$

Write the standard form of the equation of the circle that has the given radius or passes through the given point and whose center is the origin.

17. radius: 5 **18.** radius: $\sqrt{10}$ **19.** point: (−2, 3) **20.** point: (1, 8)

10.4 ELLIPSES

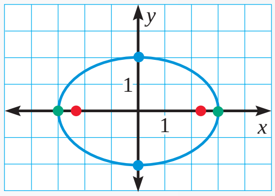
Examples on pp. 609–611

EXAMPLE The ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ has a horizontal major axis because $9 > 4$.

Since $\sqrt{9} = 3$, the vertices are at (−3, 0) and (3, 0).

Since $\sqrt{4} = 2$, the co-vertices are at (0, −2) and (0, 2).

Since $9 - 4 = 5$, the foci are at $(-\sqrt{5}, 0)$ and $(\sqrt{5}, 0)$.



Graph the equation.

21. $4x^2 + 81y^2 = 324$ **22.** $-9x^2 - 4y^2 = -36$ **23.** $49x^2 + 36y^2 = 1764$

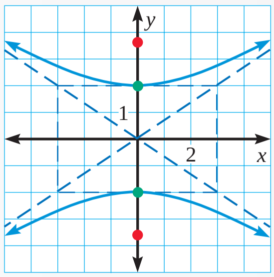
Write an equation of the ellipse with the given characteristics and center at (0, 0).

24. Vertex: (0, 5), Co-vertex: (1, 0) **25.** Vertex: (4, 0), Focus: (−3, 0)

10.5 HYPERBOLAS

Examples on pp. 615–617

EXAMPLE The hyperbola with equation $\frac{y^2}{4} - \frac{x^2}{9} = 1$ has a vertical transverse axis because the y^2-term is positive.
Since $\sqrt{4} = 2$, vertices are (0, −2) and (0, 2).
Since $4 + 9 = 13$, foci are $(0, -\sqrt{13})$ and $(0, \sqrt{13})$.
Asymptotes are $y = \frac{2}{3}x$ and $y = -\frac{2}{3}x$.



10.5 continued

Graph the hyperbola.

26. $\frac{x^2}{100} - \frac{y^2}{64} = 1$ **27.** $16y^2 - 9x^2 = 144$ **28.** $y^2 - 4x^2 = 4$

Write an equation of the hyperbola with the given foci and vertices.

29. Foci: $(0, -3), (0, 3)$
Vertices: $(0, -1), (0, 1)$

30. Foci: $(0, -4), (0, 4)$
Vertices: $(0, -2), (0, 2)$

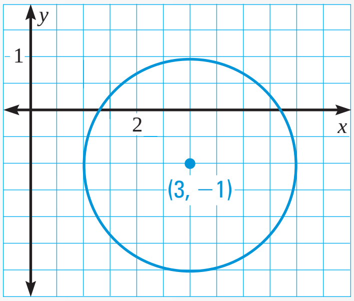
31. Foci: $(-5, 0), (5, 0)$
Vertices: $(-3, 0), (3, 0)$

10.6 Graphing and Classifying Conics

Examples on pp. 623–627

EXAMPLE You can use the discriminant $B^2 - 4AC$ to classify a conic.

For the equation $x^2 + y^2 - 6x + 2y + 6 = 0$, the discriminant is $B^2 - 4AC = 0^2 - 4(1)(1) = -4$. Because $B^2 - 4AC < 0$, $B = 0$, and $A = C$, the equation represents a circle.



To graph the circle, complete the square as follows.

$$x^2 + y^2 - 6x + 2y + 6 = 0$$

$$(x^2 - 6x + 9) + (y^2 + 2y + 1) = -6 + 9 + 1$$

$$(x - 3)^2 + (y + 1)^2 = 4$$

The center of the circle is at $(h, k) = (3, -1)$ and $r = \sqrt{4} = 2$.

Classify the conic section and write its equation in standard form. Then graph the equation.

32. $x^2 + 8x - 8y + 16 = 0$ **33.** $x^2 + y^2 - 10x + 2y - 74 = 0$

34. $9x^2 + y^2 + 72x - 2y + 136 = 0$ **35.** $y^2 - 4x^2 - 18y - 8x + 76 = 0$

10.7 Solving Quadratic Systems

Examples on pp. 632–634

EXAMPLE You can solve systems of quadratic equations algebraically.

$y^2 - 2x - 10y + 31 = 0$
$x - y + 2 = 0$ — **Solve the second equation for y: $y = x + 2$.**

$(x + 2)^2 - 2x - 10(x + 2) + 31 = 0$ — **Substitute into the first equation.**

$x^2 - 8x + 15 = 0$, so $x = 3$ or $x = 5$. — **Simplify and solve.**

The points of intersection of the graphs of the system are (3, 5) and (5, 7).

Find the points of intersection, if any, of the graphs in the system.

36. $x^2 + y^2 - 18x + 24y + 200 = 0$
$4x + 3y = 0$

37. $5x^2 + 3x - 8y + 2 = 0$
$3x + y - 6 = 0$

38. $4x^2 + y^2 - 48x - 2y + 129 = 0$
$x^2 + y^2 - 2x - 2y - 7 = 0$

39. $9x^2 - 16y^2 + 18x + 153 = 0$
$9x^2 + 16y^2 + 18x - 135 = 0$

CHAPTER 10

Chapter Test

Find the distance between the two points. Then find the midpoint of the line segment connecting the two points.

1. $(1, 9), (5, 3)$
2. $(-8, 3), (4, 7)$
3. $(-4, -2), (3, 10)$
4. $(-11, -5), (-3, 7)$
5. $(-1, 6), (2, 8)$
6. $(3, -2), (4, 9)$

Graph the equation.

7. $x^2 + y^2 = 36$
8. $y^2 = 16x$
9. $9y^2 - 81x^2 = 729$
10. $25x^2 + 9y^2 = 225$
11. $(x - 4)^2 = y + 7$
12. $(x - 3)^2 + (y + 2)^2 = 1$
13. $\frac{(x + 6)^2}{4} + \frac{(y - 7)^2}{1} = 1$
14. $\frac{(x - 4)^2}{16} - \frac{(y + 4)^2}{16} = 1$
15. $\frac{(y + 2)^2}{4} - \frac{(x + 1)^2}{16} = 1$

Write an equation for the conic section.

16. Parabola with vertex at $(0, 0)$ and directrix $x = 5$
17. Parabola with vertex at $(3, -6)$ and focus at $(3, -4)$
18. Circle with center at $(0, 0)$ and passing through $(4, 6)$
19. Circle with center at $(-8, 3)$ and radius 5
20. Ellipse with center at $(0, 0)$, vertex at $(4, 0)$, and co-vertex at $(0, 2)$
21. Ellipse with vertices at $(3, -5)$ and $(3, -1)$ and foci at $(3, -4)$ and $(3, -2)$
22. Hyperbola with vertices at $(-7, 0)$ and $(7, 0)$ and foci at $(-9, 0)$ and $(9, 0)$
23. Hyperbola with vertex at $(4, 2)$, focus at $(4, 4)$, and center at $(4, -1)$

Classify the conic section and write its equation in standard form.

24. $x^2 + 4y^2 - 2x - 3 = 0$
25. $2x^2 + 20x - y + 41 = 0$
26. $5x^2 - 3y^2 - 30 = 0$
27. $x^2 + y^2 - 12x + 4y + 31 = 0$
28. $y^2 - 8x - 4y + 4 = 0$
29. $-x^2 + y^2 - 6x - 6y - 4 = 0$
30. $x^2 - 8x + 4y + 16 = 0$
31. $3x^2 + 3y^2 - 30x + 59 = 0$
32. $x^2 + 2y^2 - 8x + 7 = 0$
33. $4x^2 - y^2 + 16x + 6y - 3 = 0$
34. $3x^2 + y^2 - 4y + 3 = 0$
35. $x^2 + y^2 - 2x + 10y + 1 = 0$

Find the points of intersection, if any, of the graphs in the system.

36. $x^2 + y^2 = 64$
$x - 2y = 17$

37. $x^2 + y^2 = 20$
$x^2 + 4y^2 - 2x - 2 = 0$

38. $x^2 = 8y$
$x^2 = 2y + 12$

39. **ARCHITECTURE** The Royal Albert Hall in London is nearly elliptical in shape, about 230 feet long and 200 feet wide. Write an equation for the shape of the hall, assuming its center is at $(0, 0)$. Then graph the equation.

40. **SEARCH TEAM** A search team of three members splits to search an area in the woods. Each member carries a family service radio with a circular range of 3 miles. They agree to communicate from their bases every hour. One member sets up base 2 miles north of the first member. Where should the other member set up base to be as far east as possible but within range of communication?