10.7 Solving Quadratic Systems

What you should learn

GOAL 1 Solve systems of quadratic equations.

GOAL 2 Use quadratic systems to solve **real-life** problems, such as determining when one car will catch up to another in **Ex. 58**.

Why you should learn it

▼ To model **real-life** situations with quadratic systems, such as finding the epicenter of an earthquake in **Example 4**.

REAL LIFE

GOAL 1 SOLVING A SYSTEM OF EQUATIONS

In Lesson 3.2 you studied two algebraic techniques for solving a system of linear equations. You can use the same techniques (substitution and linear combination) to solve quadratic systems.

EXAMPLE 1 *Finding Points of Intersection*

Find the points of intersection of the graphs of $x^2 + y^2 = 13$ and $y = x + 1$.

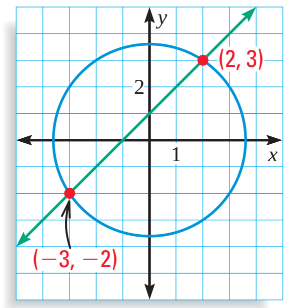
SOLUTION

To find the points of intersection, substitute $x + 1$ for y in the equation of the circle.

$x^2 + y^2 = 13$	**Equation of circle**
$x^2 + (x + 1)^2 = 13$	**Substitute $x + 1$ for y.**
$x^2 + x^2 + 2x + 1 = 13$	**Expand the power.**
$2x^2 + 2x - 12 = 0$	**Combine like terms.**
$2(x - 2)(x + 3) = 0$	**Factor.**
$x = 2$ or $x = -3$	**Zero product property**

You now know the x-coordinates of the points of intersection. To find the y-coordinates, substitute $x = 2$ and $x = -3$ into the linear equation and solve for y.

▶ The points of intersection are (2, 3) and (−3, −2).



✓**CHECK** You can check your answer algebraically by substituting the coordinates of the points into each equation. Another way to check your answer is to graph the two equations. You can see from the graph shown that the line and the circle intersect in two points, at (2, 3) and at (−3, −2).

ACTIVITY Developing Concepts

Investigating Points of Intersection

The circle and line in Example 1 intersect in two points. A circle and a line can also intersect in one point or no points. Sketch examples to illustrate the different numbers of points of intersection that the following graphs can have.

a. Circle and parabola

b. Ellipse and hyperbola

c. Circle and ellipse

d. Hyperbola and line

STUDENT HELP

Look Back
For help with solving systems, see p. 148.

EXAMPLE 2 *Solving a System by Substitution*

Find the points of intersection of the graphs in the system.

$x^2 + 4y^2 - 4 = 0$ **Equation 1**

$-2y^2 + x + 2 = 0$ **Equation 2**

SOLUTION

Because Equation 2 has no x^2-term, solve that equation for x.

$$-2y^2 + x + 2 = 0$$

$$x = 2y^2 - 2$$

Next, substitute $2y^2 - 2$ for x in Equation 1 and solve for y.

$x^2 + 4y^2 - 4 = 0$ **Equation 1**

$(2y^2 - 2)^2 + 4y^2 - 4 = 0$ **Substitute for *x*.**

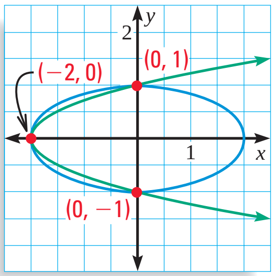
$4y^4 - 8y^2 + 4 + 4y^2 - 4 = 0$ **Expand the power.**

$4y^4 - 4y^2 = 0$ **Combine like terms.**

$4y^2(y^2 - 1) = 0$ **Factor common monomial.**

$4y^2(y - 1)(y + 1) = 0$ **Difference of squares.**

$y = 0,\ y = 1,\ \text{or } y = -1$ **Zero product property**



The corresponding x-values are $x = -2$, $x = 0$, and $x = 0$.

▶ The graphs intersect at $(-2, 0)$, $(0, 1)$, and $(0, -1)$, as shown.

STUDENT HELP

Look Back
For help with factoring, see p. 256.

EXAMPLE 3 *Solving a System by Linear Combination*

Find the points of intersection of the graphs in the system.

$x^2 + y^2 - 16x + 39 = 0$ **Equation 1**

$x^2 - y^2 - 9 = 0$ **Equation 2**

SOLUTION

You can eliminate the y^2-term by adding the two equations. The resulting equation can be solved for x because it contains no other variables.

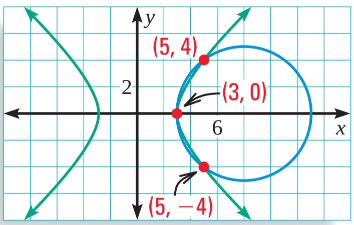
$$x^2 + y^2 - 16x + 39 = 0$$

$$x^2 - y^2 \qquad - 9 = 0$$

$2x^2 - 16x + 30 = 0$ **Add.**

$2(x - 3)(x - 5) = 0$ **Factor.**

$x = 3 \text{ or } x = 5$ **Zero product property**



The corresponding y-values are $y = 0$ and $y = \pm 4$.

▶ The graphs intersect at $(3, 0)$, $(5, 4)$, and $(5, -4)$, as shown.

FOCUS ON CAREERS

SEISMOLOGIST
A seismologist determines the location and intensity of an earthquake using an instrument which measures energy waves resulting from movements in the Earth's crust.

CAREER LINK
www.mcdougallittell.com

GOAL 2 SOLVING QUADRATIC SYSTEMS IN REAL LIFE

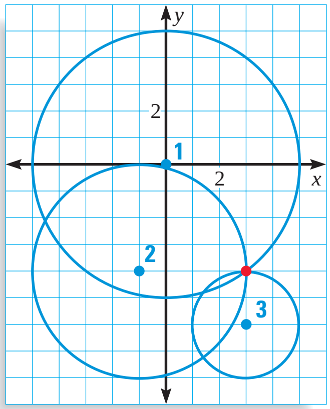
EXAMPLE 4 *Solving a System of Quadratic Models*

SEISMOLOGY A seismograph measures the intensity of an earthquake. Although a seismograph can determine the distance to the earthquake's epicenter, it cannot determine in what direction the epicenter is located. Use the following information from three seismographs to find an earthquake's epicenter.

Location 1: 500 miles from the epicenter

Location 2: 100 miles west and 400 miles south of Location 1
400 miles from the epicenter

Location 3: 300 miles east and 600 miles south of Location 1
200 miles from the epicenter



SOLUTION

Let each unit represent 100 miles. If Location 1 is at (0, 0), then Location 2 is at $(-1, -4)$ and Location 3 is at $(3, -6)$. Write the equation of each circle.

Location 1: $x^2 + y^2 = 25$

Location 2: $(x + 1)^2 + (y + 4)^2 = 16$, or
$x^2 + 2x + 1 + y^2 + 8y + 16 = 16$

Location 3: $(x - 3)^2 + (y + 6)^2 = 4$, or
$x^2 - 6x + 9 + y^2 + 12y + 36 = 4$

Subtract the equation for Location 1 from the equation for Location 2.

$$x^2 + 2x + 1 + y^2 + 8y + 16 = 16$$
$$-\,(x^2 + y^2 = 25)$$
$$2x + 8y + 17 = -9$$
$$2x + 8y = -26, \text{ or } x + 4y = -13$$

Then subtract the equation for Location 1 from the equation for Location 3.

$$x^2 - 6x + 9 + y^2 + 12y + 36 = 4$$
$$-\,(x^2 + y^2 = 25)$$
$$-6x + 12y + 45 = -21$$
$$-6x + 12y = -66, \text{ or } -x + 2y = -11$$

You are left with two linear equations. Solve this linear system to find the epicenter.

$$x + 4y = -13$$
$$-x + 2y = -11$$
$$6y = -24$$
$$y = -4$$
$$x = 3$$

▶ The epicenter of the earthquake is 300 miles east and 400 miles south of Location 1.

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete this statement: The equations $x^2 + 3y^2 - 2y = 4$ and $x^2 + y^2 = 5$ are an example of a(n) _?_ system.

Concept Check ✓

2. Sketch an example of a circle and a line intersecting in a single point.

3. Explain what method you would use to find the points of intersection of the graphs in the following system. Do not solve the system.

$4x^2 + y^2 - 16x = 0$ **Equation 1**

$x^2 - y^2 + 7 = 0$ **Equation 2**

Skill Check ✓

Find the points of intersection, if any, of the graphs in the system.

4. $x^2 + y^2 = 17$
   $y = x + 3$

5. $x^2 + y^2 + 8x - 20y + 7 = 0$
   $x^2 + 9y^2 + 8x + 4y + 7 = 0$

6. $x^2 + y^2 - 3x = 8$
   $2x^2 - y^2 = 10$

7. $x^2 - 2x + 2y + 2 = 0$
   $-x^2 + 2x - y + 3 = 0$

8. **SEISMOLOGY** Look back at Example 4. Why are three (not just two) seismographs needed to determine the location of the epicenter?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 955.

CHECKING POINTS OF INTERSECTION **Determine whether the given point is a point of intersection of the graphs in the system.**

9. $x^2 + y^2 = 25$
   $y = -3$
   Point: $(-3, 4)$

10. $x^2 + y^2 = 41$
    $y = -x - 1$
    Point: $(4, -5)$

11. $x^2 + 4x - 4y - 16 = 0$
    $-2x + y + 1 = 0$
    Point: $(6, 11)$

12. $3x^2 - 5y^2 + 2y = 45$
    $y = 2x + 10$
    Point: $(-3, 4)$

13. $2x^2 - 4y = 22$
    $y = -2x + 3$
    Point: $(-5, 7)$

14. $6x^2 - 5x + 8y^2 + y = 23$
    $y = x - 1$
    Point: $(2, 1)$

SOLVING SYSTEMS **Find the points of intersection, if any, of the graphs in the system.**

15. $x^2 - y = 5$
    $-3x + y = -7$

16. $x^2 + y^2 = 18$
    $x - y = 0$

17. $-3x^2 + y^2 = 9$
    $-2x + y = 0$

18. $9x^2 + 4y^2 = 36$
    $-x + y = -4$

19. $x^2 + y^2 = 5$
    $y = -2x$

20. $x + 2y^2 = -6$
    $x + 8y = 0$

21. $5x^2 + 3y^2 = 17$
    $-x + y = -1$

22. $4x^2 - 5y^2 = 16$
    $3x + y = 6$

23. $2x^2 + 2y^2 = 15$
    $x + 2y = 6$

24. $x^2 + y^2 = 1$
    $x + y = -1$

25. $x^2 + y^2 = 20$
    $y = x - 4$

26. $x^2 + y^2 = 5$
    $y = 3x + 5$

27. $x^2 = 6y$
    $y = -x$

28. $x^2 + y^2 = 9$
    $x - 3y = 3$

29. $x^2 + y^2 = 7$
    $y = x - 7$

30. $y^2 - 2x^2 = 6$
    $y = -2x$

31. $6x^2 + 3y^2 = 12$
    $y = -x + 2$

32. $3x^2 - y^2 = -6$
    $y = 2x + 1$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 9–32
Examples 2, 3: Exs. 33–51
Example 4: Exs. 52–55, 58–63

SOLVING SYSTEMS **Find the points of intersection, if any, of the graphs in the system.**

33. $x^2 + y^2 = 16$
$x^2 - 5y = 5$

34. $-3x^2 + y^2 - 3x = 0$
$x^2 - y^2 + 27 = 0$

35. $-x^2 + y^2 + 10 = 0$
$-3y^2 + x + 1 = 0$

36. $x^2 + 2y^2 - 10 = 0$
$4y^2 + x + 4 = 0$

37. $y^2 = 16x$
$4x - y = -24$

38. $10y = x^2$
$x^2 - 6 = -2y$

39. $y^2 + x = 2$
$3x + y = 8$

40. $x^2 - 16y^2 = 16$
$x^2 + y^2 = 9$

41. $x^2 + y^2 = 81$
$x + y = 0$

42. $16x^2 - y^2 + 16y - 128 = 0$
$y^2 - 48x - 16y - 32 = 0$

43. $x^2 - y^2 - 8x + 8y - 24 = 0$
$x^2 + y^2 - 8x - 8y + 24 = 0$

44. $x^2 + 4y^2 - 4x - 8y + 4 = 0$
$x^2 + 4y - 4 = 0$

45. $4x^2 - 56x + 9y^2 + 160 = 0$
$4x^2 + y^2 - 64 = 0$

46. $x^2 + y^2 - 16x + 39 = 0$
$x^2 - y^2 - 9 = 0$

47. $x^2 - 4y^2 - 20x - 64y - 172 = 0$
$4x^2 + y^2 - 80x + 16y + 400 = 0$

48. $x^2 - 2x + 4 + y^2 - 10 = 0$
$2y^2 - x + 3 = 0$

49. $4x^2 - y^2 - 8x + 6y - 9 = 0$
$2x^2 - 3y^2 + 4x + 18y - 43 = 0$

50. $10x^2 - 25y^2 - 100x = -160$
$y^2 - 2x + 16 = 0$

51. $x^2 - y - 4 = 0$
$x^2 + 3y^2 - 4y - 10 = 0$

SYSTEMS OF THREE EQUATIONS **Find the points, if any, that the graphs of all three equations have in common.**

52. $x^2 + y^2 + 8x + 7 = 0$
$x^2 + y^2 + 4x + 4y - 5 = 0$
$x^2 + y^2 = 1$

53. $x^2 + y^2 - 8 = 0$
$x^2 + y^2 - 3x + y = 0$
$2x^2 + 2y^2 - 5x - 10 = 0$

54. $x^2 + 3y^2 = 16$
$3x^2 + y^2 = 16$
$y = -x$

55. $x^2 + y^2 - 4x - 4y = 26$
$x^2 + y^2 - 4x = 54$
$y = 3x - 8$

56. CRITICAL THINKING Suppose a line intersects a circle whose center is at the origin, and the line passes through the origin. If you know one of the points of intersection, how do you know what the other point of intersection is without solving the system algebraically?

57. LOGICAL REASONING Sketch examples to illustrate the different numbers of points of intersection that a circle and an ellipse can have if both are centered at the origin.

58. LAW ENFORCEMENT Suppose a car is traveling down the highway at a constant rate of 60 miles per hour. It passes a police car parked at the side of the road. To catch up to the car, the police officer accelerates at a constant rate. The distance d (in miles) the police car has traveled as a function of time t (in hours) since the other car has passed it is given by $d = 3600t^2$. Write and solve a system of equations to calculate how long it takes the police car to catch up to the other car.

59. COMMUNICATIONS The range of a radio station is bounded by a circle given by the following equation:

$$x^2 + y^2 - 1620 = 0$$

A straight highway can be modeled by the following equation:

$$y = -\frac{1}{3}x + 30$$

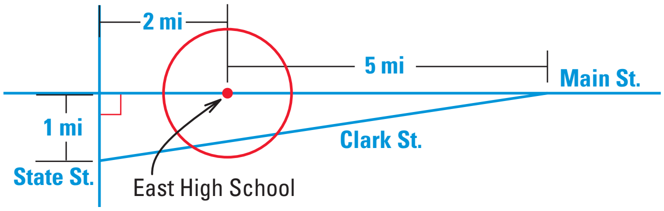
Find the length of the highway that lies within the range of the radio station.

FOCUS ON CAREERS

REAL LIFE **POLICE OFFICER** The duties of a police officer vary. An officer in a large city is often assigned to a specific type of duty, while an officer in a small community usually performs a variety of tasks.

INTERNET **CAREER LINK**
www.mcdougallittell.com

60. **BUS BOUNDARY** To be eligible to ride the school bus to East High School, a student must live at least 1 mile from the school. How long is the portion of Clark Street for which the residents are not eligible to ride the school bus? (Use a coordinate plane in which the school is at (0, 0) and each unit represents one mile.)



STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 60–62.

61. **NAVIGATION** LORAN (Long-Distance Radio Navigation) uses synchronized pulses sent out by pairs of transmitting stations. By calculating the difference in the times of arrival of the pulses from two stations, the LORAN equipment on a ship locates the ship on a hyperbola. By doing the same thing with a second pair of stations, LORAN locates the ship at the intersection of two hyperbolas. Suppose LORAN equipment indicates that a ship's location is the point of intersection of the graphs in the following system:

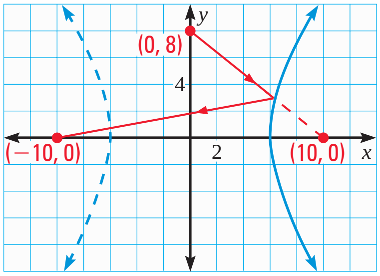
$$xy - 24 = 0$$
$$x^2 - 25y^2 + 100 = 0$$

Find the ship's location given that it is north and east of the origin.

62. **HYPERBOLIC MIRROR** In a hyperbolic mirror, light rays directed to one focus will be reflected to the other focus. The mirror shown at the right has the following equation:

$$\frac{x^2}{36} - \frac{y^2}{64} = 1$$

At which point on the mirror will light from the point (0, 8) be reflected to the focus at (−10, 0)?



63. **EARTHQUAKES** An earthquake occurred in Peru on April 18, 1993. Use the following information to approximate the location of the epicenter.
▶ Source: U.S. Department of the Interior Geological Survey

Location 1: (Cayambe, Ecuador) The epicenter was 1300 kilometers away.

Location 2: (Cocohabamba, Bolivia, 1200 kilometers east and 1900 kilometers south of Cayambe) The epicenter was 1300 kilometers away.

Location 3: (Cerro El Oso, Venezuela, 1100 kilometers east and 1000 kilometers north of Cayambe) The epicenter was 2500 kilometers away.

Test Preparation

64. **MULTIPLE CHOICE** How many points of intersection do the equations $x^2 + y^2 = 6$ and $2x^2 + 4y^2 = 7$ have?

Ⓐ 0　Ⓑ 1　Ⓒ 2　Ⓓ 3　Ⓔ 4

65. **MULTIPLE CHOICE** Which of the following is a point of intersection of the graphs of $25x^2 + 36y^2 - 900 = 0$ and $-2x^2 + y + 5 = 0$?

Ⓐ (−5, 0)　Ⓑ (0, 5)　Ⓒ (2, 5)　Ⓓ (1, 5)　Ⓔ (0, −5)

★ **Challenge**

66. **CRITICAL THINKING** Write equations for three different conics that all intersect at the point (−4, 6).

MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression for the given value of *x*.** **(Review 1.2 for 11.1)**

67. $2x + 5$ when $x = 4$

68. $\frac{1}{x^3} - 1$ when $x = 2$

69. $(-2)^{x-1}$ when $x = 5$

70. $\frac{3}{(-3)^{x-2}}$ when $x = 4$

WRITING FUNCTIONS **Write a polynomial function of least degree that has real coefficients, the given zeros, and a leading coefficient of 1.** **(Review 6.7)**

71. $3, -3, 1$

72. $0, 2, 2, 4$

73. $2i, -2i$

74. $3 + i, 3 - i$

75. $2, -1, -1 - i$

76. $-2, -3, i, i$

GRAPHING **Graph the function. Then state the domain and range.** **(Review 7.5)**

77. $f(x) = \sqrt{2x + 3}$

78. $f(x) = 5\sqrt{x - 8}$

79. $f(x) = -(x + 4)^{1/2} + 2$

80. $f(x) = -3\sqrt[3]{x + 1}$

81. $f(x) = \sqrt[3]{4x + 1} + 2$

82. $f(x) = 5(x - 1)^{1/3}$

CLASSIFYING CONICS **Classify the conic section.** **(Review 10.6)**

83. $3x^2 + y^2 + 2x + 2y = 0$

84. $4x^2 - y^2 - 8x + 4y - 9 = 0$

85. $x^2 + 6x - 2y + 13 = 0$

86. $x^2 + y^2 - 2x + 6y + 9 = 0$

QUIZ 3

Self-Test for Lessons 10.6 and 10.7

Write an equation for the conic section. **(Lesson 10.6)**

1. Circle with center at $(-3, -5)$ and radius 8

2. Ellipse with vertices at $(-7, 2)$ and $(6, 2)$ and foci at $(4, 2)$ and $(-5, 2)$

3. Parabola with vertex at $(4, -1)$ and focus at $(7, -1)$

4. Hyperbola with foci at $(2, -1)$ and $(2, 8)$ and vertices at $(2, 3)$ and $(2, 4)$

Classify the conic section. **(Lesson 10.6)**

5. $x^2 + 4y^2 - 8x + 3y + 12 = 0$

6. $-3x^2 - 3y^2 + 6x + 4y + 1 = 0$

7. $-2y^2 + x + 5y + 26 = 0$

8. $-6x^2 + 4y^2 + 2x + 9 = 0$

Find the points of intersection, if any, of the graphs in the system. **(Lesson 10.7)**

9. $3x^2 - 4x - y + 2 = 0$
$y = -5x + 4$

10. $-x^2 + y^2 + 4x - 6y + 4 = 0$
$x^2 + y^2 - 4x - 6y + 12 = 0$

11. $x^2 + y^2 + 4y - 12 = 0$
$x^2 - 16y^2 - 64y - 80 = 0$

12. $y^2 - 6x - 2y - 3 = 0$
$2y^2 - 4y + x + 6 = 0$

13. **SEISMOLOGY** A seismograph records the epicenter of an earthquake 50 miles away. A second seismograph, 50 miles west and 35 miles north of the first, records the epicenter as being 35 miles away. A third seismograph, 80 miles due west of the first, records the epicenter 30 miles away. Where was the earthquake's epicenter in relation to the first seismograph? **(Lesson 10.7)**