10.2 Parabolas

What you should learn

GOAL 1 Graph and write equations of parabolas.

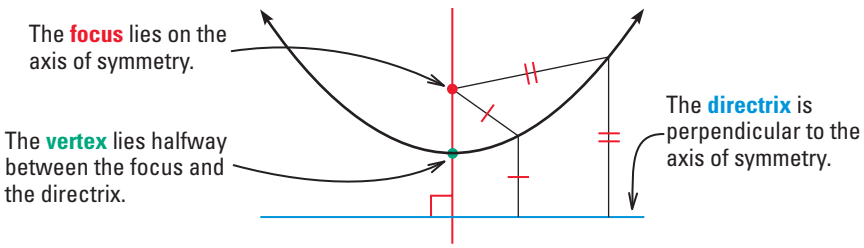
GOAL 2 Use parabolas to solve **real-life** problems, such as finding the depth of a solar energy collector in **Example 3**.

Why you should learn it

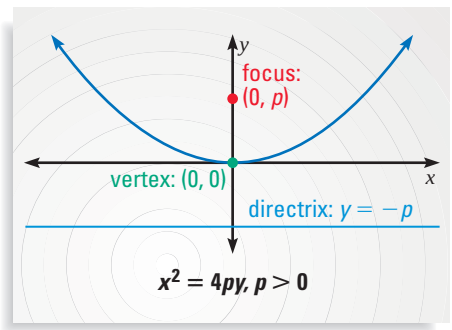
▼ To model **real-life** parabolas, such as the reflector of a car's headlight in **Ex. 79**.

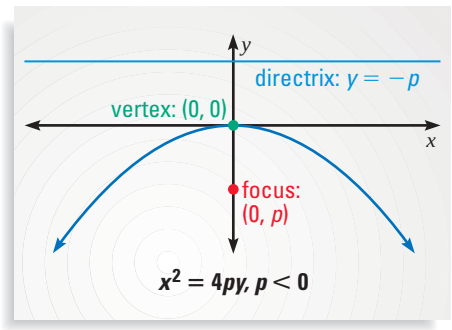
GOAL 1 GRAPHING AND WRITING EQUATIONS OF PARABOLAS

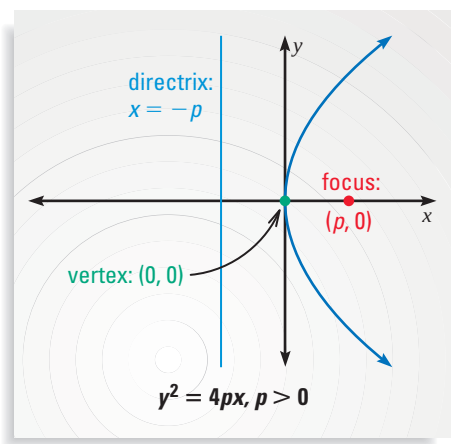
You already know that the graph of $y = ax^2$ is a parabola whose vertex $(0, 0)$ lies on its axis of symmetry $x = 0$. Every parabola has the property that any point on it is equidistant from a point called the **focus** and a line called the **directrix**.

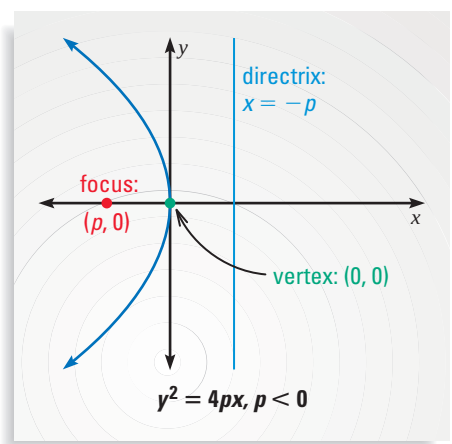


In Chapter 5 you saw parabolas that have a vertical axis of symmetry and open up or down. In this lesson you will also work with parabolas that have a horizontal axis of symmetry and open left or right. In the four cases shown below, the focus and the directrix each lie $|p|$ units from the vertex.









Characteristics of the parabolas shown above are given on the next page.

STANDARD EQUATION OF A PARABOLA (VERTEX AT ORIGIN)

The standard form of the equation of a parabola with vertex at (0, 0) is as follows.

EQUATION	FOCUS	DIRECTRIX	AXIS OF SYMMETRY
$x^2 = 4py$	$(0, p)$	$y = -p$	Vertical ($x = 0$)
$y^2 = 4px$	$(p, 0)$	$x = -p$	Horizontal ($y = 0$)

EXAMPLE 1 *Graphing an Equation of a Parabola*

STUDENT HELP

Look Back
For help with drawing parabolas, see p. 249.

Identify the focus and directrix of the parabola given by $x = -\frac{1}{6}y^2$. Draw the parabola.

SOLUTION

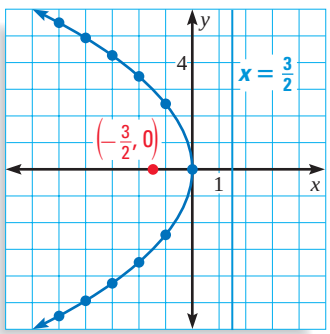
Because the variable y is squared, the axis of symmetry is horizontal. To find the focus and directrix, rewrite the equation as follows.

$x = -\frac{1}{6}y^2$ **Write original equation.**

$-6x = y^2$ **Multiply each side by −6.**

Since $4p = -6$, you know $p = -\frac{3}{2}$. The focus is $(p, 0) = \left(-\frac{3}{2}, 0\right)$ and the directrix is $x = -p = \frac{3}{2}$. To draw the parabola, make a table of values and plot points. Because $p < 0$, the parabola opens to the left. Therefore, only negative x-values should be chosen.

x	-1	-2	-3	-4	-5
y	± 2.45	± 3.46	± 4.24	± 4.90	± 5.48

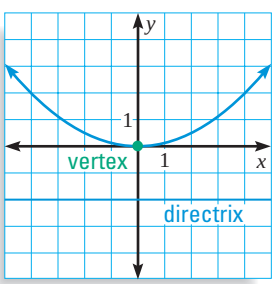


EXAMPLE 2 *Writing an Equation of a Parabola*

STUDENT HELP

INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for extra examples.

Write an equation of the parabola shown at the right.



SOLUTION

The graph shows that the vertex is (0, 0) and the directrix is $y = -p = -2$. Substitute **2** for ***p*** in the standard equation for a parabola with a vertical axis of symmetry.

$x^2 = 4py$ **Standard form, vertical axis of symmetry**

$x^2 = 4(2)y$ **Substitute 2 for *p*.**

$x^2 = 8y$ **Simplify.**

✓**CHECK** You can check this result by solving the equation for y to get $y = \frac{1}{8}x^2$ and graphing the equation using a graphing calculator.

GOAL 2 USING PARABOLAS IN REAL LIFE

Parabolic reflectors have cross sections that are parabolas. A special property of any parabolic reflector is that all incoming rays parallel to the axis of symmetry that hit the reflector are directed to the focus (Figure 1). Similarly, rays emitted from the focus that hit the reflector are directed in rays parallel to the axis of symmetry (Figure 2). These properties are the reason satellite dishes and flashlights are parabolic.

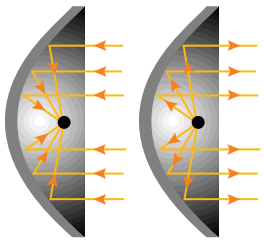


Figure 1 Figure 2

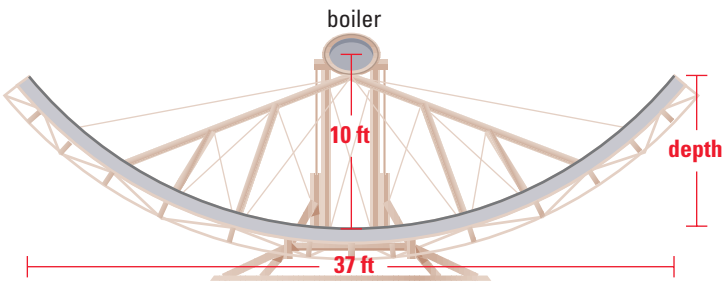
EXAMPLE 3 *Modeling a Parabolic Reflector*

SOLAR ENERGY Sunfire is a glass parabola used to collect solar energy. The sun's rays are reflected from the mirrors toward two boilers located at the focus of the parabola. When heated, the boilers produce steam that powers an alternator to produce electricity.

Sunfire

a. Write an equation for Sunfire's cross section.

b. How deep is the dish?



SOLUTION

a. The boilers are 10 feet above the vertex of the dish. Because the boilers are at the focus and the focus is p units from the vertex, you can conclude that $p = 10$.

Assuming the vertex is at the origin, an equation for the parabolic cross section is as follows:

$x^2 = 4py$ **Standard form, vertical axis of symmetry**

$x^2 = 4(10)y$ **Substitute 10 for *p*.**

$x^2 = 40y$ **Simplify.**

b. The dish extends $\frac{37}{2} = 18.5$ feet on either side of the origin. To find the depth of the dish, substitute 18.5 for x in the equation from part (a).

$x^2 = 40y$ **Equation for the cross section**

$(18.5)^2 = 40y$ **Substitute 18.5 for *x*.**

$8.6 \approx y$ **Solve for *y*.**

▶ The dish is about 8.6 feet deep.

FOCUS ON PEOPLE

HOWARD FRANK BROYLES is an engineer and inventor. More than 250 high school students volunteered their time to help him build Sunfire. The project took over ten years.

APPLICATION LINK
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GUIDED PRACTICE

Vocabulary Check ✓

1. Complete this statement: A parabola is the set of points equidistant from a point called the ? and a line called the ?.

Concept Check ✓

2. How does the graph of $x = ay^2$ differ from the graph of $y = ax^2$?

3. Knowing the value of a in $y = ax^2$, how can you find the focus and directrix?

Skill Check ✓

Graph the equation. Identify the focus and directrix of the parabola.

4. $x^2 = 4y$ **5.** $y = -5x^2$ **6.** $-12x = y^2$

7. $8y^2 = x$ **8.** $-6x = y^2$ **9.** $x^2 = 2y$

Write the standard form of the equation of the parabola with the given focus or directrix and vertex at (0, 0).

10. focus: $(0, 3)$ **11.** focus: $(5, 0)$ **12.** focus: $(-6, 0)$

13. directrix: $x = 4$ **14.** directrix: $x = -1$ **15.** directrix: $y = 8$

PRACTICE AND APPLICATIONS

STUDENT HELP

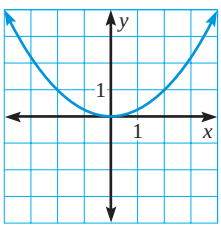
Extra Practice to help you master skills is on p. 953.

MATCHING **Match the equation with its graph.**

16. $y^2 = 4x$ **17.** $x^2 = -4y$ **18.** $x^2 = 4y$

19. $y^2 = -4x$ **20.** $y^2 = \frac{1}{4}x$ **21.** $x^2 = \frac{1}{4}y$

A.



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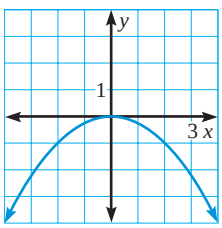

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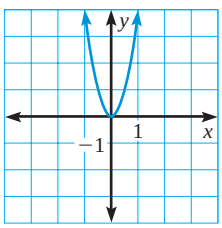
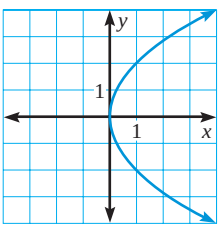
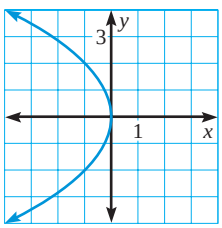


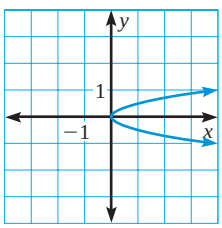
D.



E.



F.



DIRECTION **Tell whether the parabola opens *up*, *down*, *left*, or *right*.**

22. $y = -3x^2$ **23.** $-9x^2 = 2y$ **24.** $2y^2 = -6x$ **25.** $x = 7y^2$

26. $x^2 = 16y$ **27.** $-3y^2 = 8x$ **28.** $-5x = -y^2$ **29.** $x^2 = \frac{4}{3}y$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–53
Example 2: Exs. 54–77
Example 3: Exs. 78–81

FOCUS AND DIRECTRIX **Identify the focus and directrix of the parabola.**

30. $3x^2 = -y$ **31.** $2y^2 = x$ **32.** $x^2 = 8y$ **33.** $y^2 = -10x$

34. $y^2 = -16x$ **35.** $x^2 = -36y$ **36.** $-4x + 9y^2 = 0$ **37.** $-28y + x^2 = 0$

GRAPHING **Graph the equation. Identify the focus and directrix of the parabola.**

38. $y^2 = 12x$ **39.** $x^2 = -6y$ **40.** $y^2 = -2x$ **41.** $y^2 = 24x$

42. $x^2 = 8y$ **43.** $y^2 = -14x$ **44.** $x^2 = -20y$ **45.** $x^2 = 18y$

46. $x^2 = -4y$ **47.** $x^2 = 16y$ **48.** $y^2 = 9x$ **49.** $y^2 = -3x$

50. $x^2 - 40y = 0$ **51.** $x + \frac{1}{20}y^2 = 0$ **52.** $3x^2 = 4y$ **53.** $x - \frac{1}{8}y^2 = 0$

WRITING EQUATIONS **Write the standard form of the equation of the parabola with the given focus and vertex at (0, 0).**

54. $(4, 0)$ **55.** $(-2, 0)$ **56.** $(-3, 0)$ **57.** $(0, 1)$

58. $(0, 4)$ **59.** $(0, -3)$ **60.** $(0, -4)$ **61.** $(-5, 0)$

62. $\left(-\frac{1}{4}, 0\right)$ **63.** $\left(0, -\frac{3}{8}\right)$ **64.** $\left(0, \frac{1}{2}\right)$ **65.** $\left(\frac{5}{12}, 0\right)$

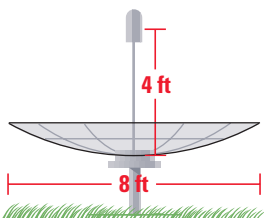
WRITING EQUATIONS **Write the standard form of the equation of the parabola with the given directrix and vertex at (0, 0).**

66. $y = 2$ **67.** $y = -3$ **68.** $x = -4$ **69.** $x = 6$

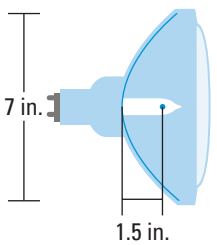
70. $x = -5$ **71.** $y = -1$ **72.** $x = 2$ **73.** $y = 4$

74. $x = -\frac{1}{2}$ **75.** $x = \frac{3}{4}$ **76.** $y = \frac{5}{8}$ **77.** $y = -\frac{1}{12}$

78. **COMMUNICATIONS** The cross section of a television antenna dish is a parabola. For the dish at the right, the receiver is located at the focus, 4 feet above the vertex. Find an equation for the cross section of the dish. (Assume the vertex is at the origin.) If the dish is 8 feet wide, how deep is it?



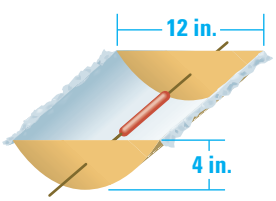
79. **AUTOMOTIVE ENGINEERING** The filament of a lightbulb is a thin wire that glows when electricity passes through it. The filament of a car headlight is at the focus of a parabolic reflector, which sends light out in a straight beam. Given that the filament is 1.5 inches from the vertex, find an equation for the cross section of the reflector. If the reflector is 7 inches wide, how deep is it?



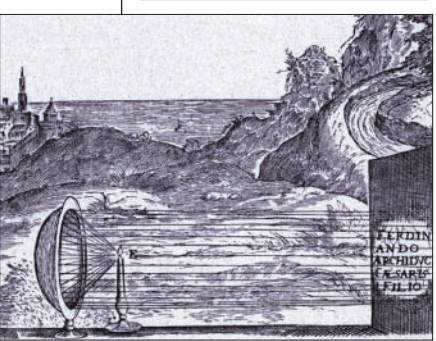
80. **HISTORY CONNECTION** In the drawing shown at the left, the rays of the sun are lighting a candle. If the candle flame is 12 inches from the back of the parabolic reflector and the reflector is 6 inches deep, then what is the diameter of the reflector?

81. **CAMPING** You can make a solar hot dog cooker using foil-lined cardboard shaped as a parabolic trough. The drawing at the right shows how to suspend a hot dog with a wire through the focus of each end piece. If the trough is 12 inches wide and 4 inches deep, how far from the bottom should the wire be placed?

▶ Source: *Boys' Life*



FOCUS ON APPLICATIONS



REAL LIFE MATH IN HISTORY Man has been aware of the reflective properties of parabolas for two thousand years. The above illustration of Archimedes' burning mirror was taken from a book printed in the 17th century.

Test Preparation

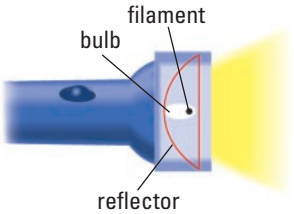
82. *Writing* For an equation of the form $y = ax^2$, discuss what effect increasing $|a|$ has on the focus and directrix.

83. MULTI-STEP PROBLEM A flashlight has a parabolic reflector. An equation for the cross section of the reflector is $y^2 = \frac{32}{7}x$. The depth of the reflector is $\frac{3}{2}$ inches.

a. *Writing* Explain why the value of p must be less than the depth of the reflector of a flashlight.

b. How wide is the beam of light projected by the flashlight?

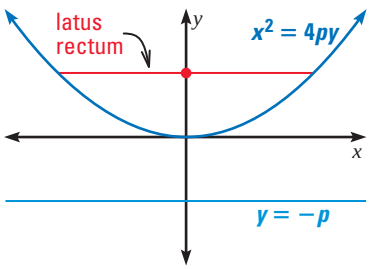
c. Write an equation for the cross section of a reflector having the same depth but a wider beam than the flashlight shown. How wide is the beam of the new reflector?



d. Write an equation for the cross section of a reflector having the same depth but a narrower beam than the flashlight shown. How wide is the beam of the new reflector?

Challenge

84. GEOMETRY CONNECTION The *latus rectum* of a parabola is the line segment that is parallel to the directrix, passes through the focus, and has endpoints that lie on the parabola. Find the length of the latus rectum of a parabola given by $x^2 = 4py$.



EXTRA CHALLENGE
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MIXED REVIEW

LOGARITHMIC AND EXPONENTIAL EQUATIONS **Solve the equation. Check for extraneous solutions.** (Review 8.6)

85. $8^{5x} = 16^{2x+1}$ **86.** $3^x = 15$ **87.** $5^x = 7$

88. $10^{3x+1} + 4 = 33$ **89.** $\log_7 (3x - 5) = \log_7 8x$ **90.** $\log_3 (4x - 3) = 3$

OPERATIONS WITH RATIONAL EXPRESSIONS **Perform the indicated operation and simplify.** (Review 9.4 and 9.5)

91. $\frac{3xy^3}{x^3y} \cdot \frac{y}{6x}$ **92.** $\frac{3xy^3}{2x} \div \frac{2xy^3}{3x}$ **93.** $\frac{x^2 - 9}{x^2 - x - 6} \cdot (x + 2)$

94. $\frac{-3x}{x + 2} + \frac{4x}{x - 1}$ **95.** $\frac{x + 1}{6x^2} - \frac{x + 1}{6x^2 + 6x}$ **96.** $\frac{x^2 - 3x + 2}{x - 1} - \frac{x^2 - 4}{x - 2}$

FINDING A DISTANCE **Find the distance between the two points.** (Review 10.1 for 10.3)

97. (3, 4), (6, 7) **98.** (−3, 7), (−7, 3) **99.** (18, −4), (−2, 9)

100. (3.7, 5.1), (2, 5) **101.** (−9, −31), (8, 7) **102.** (8.8, 3.3), (1.2, 6)

103. CONSUMER ECONOMICS The amount A (in dollars) you pay for grapes varies directly with the amount P (in pounds) that you buy. Suppose you buy $1\frac{1}{2}$ pounds for \$2.25. Write a linear model that gives A as a function of P. (Review 2.4)