1.7 Solving Absolute Value Equations and Inequalities

What you should learn

GOAL 1 Solve absolute value equations and inequalities.

GOAL 2 Use absolute value equations and inequalities to solve **real-life** problems, such as finding acceptable weights in **Example 4**.

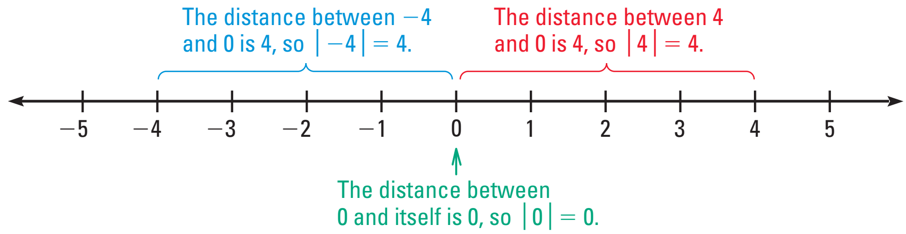
Why you should learn it

▼ To solve **real-life** problems, such as finding recommended weight ranges for sports equipment in **Ex. 72**.

REAL LIFE

GOAL 1 SOLVING EQUATIONS AND INEQUALITIES

The **absolute value** of a number x, written $|x|$, is the distance the number is from 0 on a number line. Notice that the absolute value of a number is always nonnegative.



The absolute value of x can be defined algebraically as follows.

$$|x| = \begin{cases} x, & \text{if } x \text{ is positive} \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x \text{ is negative} \end{cases}$$

To solve an absolute value equation of the form $|x| = c$ where $c > 0$, use the fact that x can have two possible values: a positive value c or a negative value $-c$. For instance, if $|x| = 5$, then $x = 5$ or $x = -5$.

SOLVING AN ABSOLUTE VALUE EQUATION

The absolute value equation $|ax + b| = c$, where $c > 0$, is equivalent to the compound statement $ax + b = c$ or $ax + b = -c$.

EXAMPLE 1 *Solving an Absolute Value Equation*

Solve $|2x - 5| = 9$.

SOLUTION

Rewrite the absolute value equation as two linear equations and then solve each linear equation.

$|2x - 5| = 9$			**Write original equation.**
$2x - 5 = 9$	or	$2x - 5 = -9$	**Expression can be 9 or −9.**
$2x = 14$	or	$2x = -4$	**Add 5 to each side.**
$x = 7$	or	$x = -2$	**Divide each side by 2.**

▶ The solutions are 7 and −2. Check these by substituting each solution into the original equation.

An absolute value inequality such as $|x - 2| < 4$ can be solved by rewriting it as a compound inequality, in this case as $-4 < x - 2 < 4$.

TRANSFORMATIONS OF ABSOLUTE VALUE INEQUALITIES

- The inequality $|ax + b| < c$, where $c > 0$, means that $ax + b$ is *between* $-c$ and c. This is equivalent to $-c < ax + b < c$.
- The inequality $|ax + b| > c$, where $c > 0$, means that $ax + b$ is *beyond* $-c$ and c. This is equivalent to $ax + b < -c$ *or* $ax + b > c$.

In the first transformation, $<$ can be replaced by $\leq$. In the second transformation, $>$ can be replaced by $\geq$.

EXAMPLE 2 *Solving an Inequality of the Form $|ax + b| < c$*

STUDENT HELP

INTERNET HOMEWORK HELP
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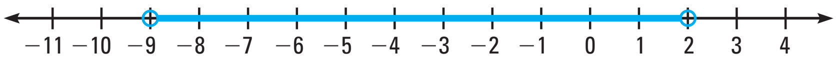
Solve $|2x + 7| < 11$.

SOLUTION

$\|2x + 7\| < 11$	**Write original inequality.**
$-11 < 2x + 7 < 11$	**Write equivalent compound inequality.**
$-18 < 2x < 4$	**Subtract 7 from each expression.**
$-9 < x < 2$	**Divide each expression by 2.**

▶ The solutions are all real numbers greater than -9 and less than 2. Check several solutions in the original inequality. The graph is shown below.



EXAMPLE 3 *Solving an Inequality of the Form $|ax + b| \geq c$*

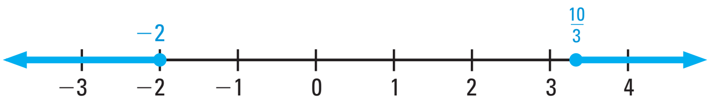
Solve $|3x - 2| \geq 8$.

SOLUTION

This absolute value inequality is equivalent to $3x - 2 \leq -8$ or $3x - 2 \geq 8$.

SOLVE FIRST INEQUALITY		SOLVE SECOND INEQUALITY
$3x - 2 \leq -8$	**Write inequality.**	$3x - 2 \geq 8$
$3x \leq -6$	**Add 2 to each side.**	$3x \geq 10$
$x \leq -2$	**Divide each side by 3.**	$x \geq \frac{10}{3}$

▶ The solutions are all real numbers less than or equal to -2 or greater than or equal to $\frac{10}{3}$. Check several solutions in the original inequality. The graph is shown below.



GOAL 2 USING ABSOLUTE VALUE IN REAL LIFE

In manufacturing applications, the maximum acceptable deviation of a product from some ideal or average measurement is called the *tolerance*.

REAL LIFE
Manufacturing

EXAMPLE 4 *Writing a Model for Tolerance*

A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for "20 ounce" boxes.

PROBLEM SOLVING STRATEGY

SOLUTION

VERBAL MODEL $\left|\boxed{\text{Actual weight}} - \boxed{\text{Ideal weight}}\right| \le \boxed{\text{Tolerance}}$

LABELS
Actual weight $= x$ (ounces)
Ideal weight $= \mathbf{20}$ (ounces)
Tolerance $= \mathbf{0.75}$ (ounces)

ALGEBRAIC MODEL
$|x - 20| \le 0.75$ **Write algebraic model.**
$-0.75 \le x - 20 \le 0.75$ **Write equivalent compound inequality.**
$19.25 \le x \le 20.75$ **Add 20 to each expression.**

▶ The weights can range between 19.25 ounces and 20.75 ounces, inclusive.

EXAMPLE 5 *Writing an Absolute Value Model*

QUALITY CONTROL You are a quality control inspector at a bowling pin company. A regulation pin must weigh between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.

SOLUTION

VERBAL MODEL $\left|\boxed{\text{Weight of pin}} - \boxed{\text{Average of extreme weights}}\right| > \boxed{\text{Tolerance}}$

LABELS
Weight of pin $= w$ (ounces)
Average of extreme weights $= \dfrac{50 + 58}{2} = \mathbf{54}$ (ounces)
Tolerance $= 58 - 54 = \mathbf{4}$ (ounces)

ALGEBRAIC MODEL $|w - 54| > 4$

▶ You should reject a bowling pin if its weight w satisfies $|w - 54| > 4$.

FOCUS ON APPLICATIONS

REAL LIFE **BOWLING** Bowling pins are made from maple wood, either solid or laminated. They are given a tough plastic coating to resist cracking. The lighter the pin, the easier it is to knock down.

Guided Practice

Vocabulary Check ✓

1. What is the absolute value of a number?

Concept Check ✓

2. The absolute value of a number cannot be negative. How, then, can the absolute value of a be $-a$?

3. Give an example of the absolute value of a number. How many other numbers have this absolute value? State the number or numbers.

Skill Check ✓

Decide whether the given number is a solution of the equation.

4. $|3x + 8| = 20; -4$
5. $|11 - 4x| = 7; 1$
6. $|2x - 9| = 11; -1$
7. $|-x + 9| = 4; -5$
8. $|6 + 3x| = 0; -2$
9. $|-5x - 3| = 8; -1$

Rewrite the absolute value inequality as a compound inequality.

10. $|x + 8| < 5$
11. $|11 - 2x| \geq 13$
12. $|9 - x| > 21$
13. $|x + 5| \leq 9$
14. $|10 - 3x| \geq 17$
15. $\left|\frac{1}{4}x + 10\right| < 18$
16. **TOLERANCE** Suppose the tolerance for the "20 ounce" cereal boxes in Example 4 is now 0.45 ounce. Write and solve an absolute value inequality that describes the new acceptable weights of the boxes.

Practice and Applications

STUDENT HELP

Extra Practice
to help you master skills is on p. 941.

REWRITING EQUATIONS **Rewrite the absolute value equation as two linear equations.**

17. $|x - 8| = 11$
18. $|5 - 2x| = 13$
19. $|6n + 1| = \frac{1}{2}$
20. $|5n - 4| = 16$
21. $|2x + 1| = 5$
22. $|2 - x| = 3$
23. $|15 - 2x| = 8$
24. $\left|\frac{1}{2}x + 4\right| = 6$
25. $\left|\frac{2}{3}x - 9\right| = 18$

CHECKING A SOLUTION **Decide whether the given number is a solution of the equation.**

26. $|4x + 1| = 11; 3$
27. $|8 - 2n| = 2; -5$
28. $\left|6 + \frac{1}{2}x\right| = 14; -40$
29. $\left|\frac{1}{5}x - 2\right| = 4; 10$
30. $|4n + 7| = 1; 2$
31. $|-3x + 5| = 7; 4$

SOLVING EQUATIONS **Solve the equation.**

32. $|11 + 2x| = 5$
33. $|10 - 4x| = 2$
34. $|22 - 3n| = 5$
35. $|2n - 5| = 7$
36. $|8x + 1| = 23$
37. $|30 - 7x| = 4$
38. $\left|\frac{1}{4}x - 5\right| = 8$
39. $\left|\frac{2}{3}x + 2\right| = 10$
40. $\left|\frac{1}{2}x - 3\right| = 2$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 17–40
Examples 2, 3: Exs. 41–58
Examples 4, 5: Exs. 65–76

REWRITING INEQUALITIES **Rewrite the absolute value inequality as a compound inequality.**

41. $|3 + 4x| \leq 15$
42. $|4n - 12| > 16$
43. $|3x + 2| < 7$
44. $|2x - 1| \geq 12$
45. $|8 - 3n| \leq 18$
46. $|11 + 4x| < 23$

SOLVING AND GRAPHING **Solve the inequality. Then graph your solution.**

47. $|x + 1| < 8$ **48.** $|12 - x| \leq 19$ **49.** $|16 - x| \geq 10$

50. $|x + 5| > 12$ **51.** $|x - 8| \leq 5$ **52.** $|x - 16| > 24$

53. $|14 - 3x| > 18$ **54.** $|4x + 10| < 20$ **55.** $|8x + 28| \geq 32$

56. $\left|20 + \frac{1}{2}x\right| > 6$ **57.** $|7x + 5| < 23$ **58.** $|11 + 6x| \leq 47$

SOLVING INEQUALITIES **Use the *Test* feature of a graphing calculator to solve the inequality. Most calculators use *abs* for absolute value. For example, you enter $|x + 1|$ as abs(x + 1).**

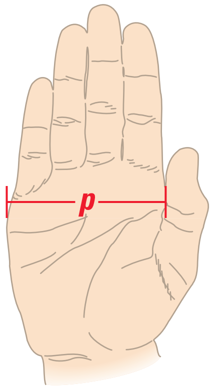
STUDENT HELP

INTERNET KEYSTROKE HELP Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

59. $|x + 1| < 3$ **60.** $\left|\frac{2}{3}x - \frac{1}{3}\right| \leq \frac{1}{3}$ **61.** $|2x - 4| > 10$

62. $\left|\frac{1}{2}x - 1\right| \leq 3$ **63.** $|4x - 10| > 6$ **64.** $|1 - 2x| \geq 13$

PALM WIDTHS **In Exercises 65 and 66, use the following information.**
In a sampling conducted by the United States Air Force, the right-hand dimensions of 4000 Air Force men were measured. The gathering of such information is useful when designing control panels, keyboards, gloves, and so on.



65. Ninety-five percent of the palm widths p were within 0.26 inch of 3.49 inches. Write an absolute value inequality that describes these values of p. Graph the inequality.

66. Ninety-nine percent of the palm widths p were within 0.37 inch of 3.49 inches. Write an absolute value inequality that describes these values of p. Graph the inequality.

67. **ACCURACY OF MEASUREMENTS** Your woodshop instructor requires that you cut several pieces of wood within $\frac{3}{16}$ inch of his specifications. Let p represent the specification and let x represent the length of a cut piece of wood. Write an absolute value inequality that describes the acceptable values of x. One piece of wood is specified to be $p = 9\frac{1}{8}$ inches. Describe the acceptable lengths for the piece of wood.

FOCUS ON APPLICATIONS

REAL LIFE **BODY TEMPERATURE**
Doctors routinely use ear thermometers to measure body temperature. The first ear thermometers were used in 1990. The thermometers use an infrared sensor and microprocessors.

68. **BASKETBALL** The length of a standard basketball court can vary from 84 feet to 94 feet, inclusive. Write an absolute value inequality that describes the possible lengths of a standard basketball court.

69. **BODY TEMPERATURE** Physicians consider an adult's normal body temperature to be within 1°F of 98.6°F, inclusive. Write an absolute value inequality that describes the range of normal body temperatures.

WEIGHING FLOUR **In Exercises 70 and 71, use the following information.**
A 16 ounce bag of flour probably does not weigh exactly 16 ounces. Suppose the actual weight can be between 15.6 ounces and 16.4 ounces, inclusive.

70. Write an absolute value inequality that describes the acceptable weights for a "16 ounce" bag of flour.

71. A case of flour contains 24 of these "16 ounce" bags. What is the greatest possible weight of the flour in a case? What is the least possible weight? Write an absolute value inequality that describes the acceptable weights of a case.

SPORTS EQUIPMENT **In Exercises 72 and 73, use the table giving the recommended weight ranges for the balls from five different sports.**

Sport	Weight range of ball used
Volleyball	260–280 grams
Basketball	600–650 grams
Water polo	400–450 grams
Lacrosse	142–149 grams
Football	14–15 ounces

72. Write an absolute value inequality for the weight range of each ball.

73. For each ball, write an absolute value inequality describing the weights of balls that are *outside* the recommended range.

74. **SCIENCE CONNECTION** Green plants can live in the ocean only at depths of 0 feet to 100 feet. Write an absolute value inequality describing the range of possible depths for green plants in an ocean.

75. **BOTTLING** A juice bottler has a tolerance of 9 milliliters in a two liter bottle, of 5 milliliters in a one liter bottle, and of 2 milliliters in a 500 milliliter bottle. For each size of bottle, write an absolute value inequality describing the capacities that are outside the acceptable range.

76. **SCIENCE CONNECTION** To determine height from skeletal remains, scientists use the equation $H = 2.26f + 66.4$ where H is the person's height (in centimeters) and f is the skeleton's femur length (in centimeters). The equation has a margin of error of ± 3.42 centimeters. Suppose a skeleton's femur length is 51.6 centimeters. Write an absolute value inequality that describes the person's height. Then solve the inequality to find the range of possible heights.

Test Preparation

77. **MULTIPLE CHOICE** Which of the following are solutions of $|3x - 7| = 14$?

Ⓐ $x = \frac{7}{3}$ or $x = 7$　　Ⓑ $x = -\frac{7}{3}$ or $x = 7$

Ⓒ $x = \frac{7}{3}$ or $x = -7$　　Ⓓ $x = -\frac{7}{3}$ or $x = -7$

78. **MULTIPLE CHOICE** Which of the following is equivalent to $|2x - 9| < 3$?

Ⓐ $-3 \le x \le 6$　　Ⓑ $3 < x < 6$

Ⓒ $3 \le x \le 6$　　Ⓓ $-3 < x < -6$

79. **MULTIPLE CHOICE** Which of the following is equivalent to $|3x + 5| \ge 19$?

Ⓐ $x \le -\frac{14}{3}$ or $x \ge 8$　　Ⓑ $x < -8$ or $x > \frac{14}{3}$

Ⓒ $x \le -8$ or $x \ge \frac{14}{3}$　　Ⓓ $x < -\frac{14}{3}$ or $x > 8$

★ Challenge

SOLVING INEQUALITIES **Solve the inequality. If there is no solution, write *no solution*.**

80. $|2x + 3| \ge -13$　　**81.** $|5x + 2| \le -2$

82. $|3x - 8| < -10$　　**83.** $|4x - 2| > -6$

84. $|6 - 2x| > -8$　　**85.** $|7 - 3x| \le -14$

SOLVING INEQUALITIES **Solve for *x*. Assume *a* and *b* are positive numbers.**

86. $|x + a| < b$　　**87.** $|x - a| > b$

88. $|x + a| \ge a$　　**89.** $|x - a| \le a$

EXTRA CHALLENGE
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MIXED REVIEW

LOGICAL REASONING **Tell whether the statement is *true* or *false*. If the statement is false, explain why.** (Skills Review, p. 926)

90. A triangle is a right triangle if and only if it has a right angle.

91. $2x = 14$ if and only if $x = -7$.

92. All rectangles are squares.

EVALUATING EXPRESSIONS **Evaluate the expression for the given value(s) of the variable(s).** (Review 1.2 for 2.1)

93. $5x - 9$ when $x = 6$

94. $-2y + 4$ when $y = 14$

95. $11c + 6$ when $c = -3$

96. $-8a - 3$ when $a = -4$

97. $a - 11b + 2$ when $a = 61$ and $b = 7$

98. $15x + 8y$ when $x = \frac{1}{2}$ and $y = \frac{1}{3}$

99. $\frac{1}{5}\left(8g + \frac{1}{3}h\right)$ when $g = 6$ and $h = 6$

100. $\frac{1}{5}(p + q) - 7$ when $p = 5$ and $q = 3$

SOLVING INEQUALITIES **Solve the inequality.** (Review 1.6)

101. $6x + 9 > 11$

102. $15 - 2x \geq 45$

103. $-3x - 5 \leq 10$

104. $13 + 4x < 9$

105. $-18 < 2x + 10 < 6$

106. $x + 2 \leq -1$ or $4x \geq 8$

QUIZ 3

Self-Test for Lessons 1.6 and 1.7

Solve the inequality. Then graph your solution. (Lesson 1.6)

1. $4x - 3 \leq 17$

2. $2y - 9 > 5y + 12$

3. $-8 < 3x + 4 < 22$

4. $3x - 5 \leq -11$ or $2x - 3 > 3$

Solve the equation. (Lesson 1.7)

5. $|x + 5| = 4$

6. $|x - 3| = 2$

7. $|6 - x| = 9$

8. $|4x - 7| = 13$

9. $|3x + 4| = 20$

10. $|15 - 3x| = 12$

Solve the inequality. Then graph your solution. (Lesson 1.7)

11. $|y + 2| \geq 3$

12. $|x + 6| < 4$

13. $|x - 3| > 7$

14. $|2y - 5| \leq 3$

15. $|2x - 3| > 1$

16. $|4x + 5| \geq 13$

17. **FUEL EFFICIENCY** Your car gets between 20 miles per gallon and 28 miles per gallon of gasoline and has a 16 gallon gasoline tank. Write a compound inequality that represents your fuel efficiency. How many miles can you travel on one tank of gasoline? (Lesson 1.6)

18. **MANUFACTURING TOLERANCE** The ideal diameter of a certain type of ball bearing is 30 millimeters. The manufacturer has a tolerance of 0.045 millimeter. Write an absolute value inequality that describes the acceptable diameters for these ball bearings. Then solve the inequality to find the range of acceptable diameters. (Lesson 1.7)