1.4 Rewriting Equations and Formulas

What you should learn

GOAL 1 Rewrite equations with more than one variable.

GOAL 2 Rewrite common formulas, as applied in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems, such as finding how much you should charge for tickets to a benefit concert in **Example 4**.

GOAL 1 EQUATIONS WITH MORE THAN ONE VARIABLE

In Lesson 1.3 you solved equations with one variable. Many equations involve more than one variable. You can solve such an equation for one of its variables.

EXAMPLE 1 *Rewriting an Equation with More Than One Variable*

Solve $7x - 3y = 8$ for y.

SOLUTION

$7x - 3y = 8$ — Write original equation.

$-3y = -7x + 8$ — Subtract $7x$ from each side.

$y = \frac{7}{3}x - \frac{8}{3}$ — Divide each side by -3.

ACTIVITY Developing Concepts

Equations with More Than One Variable

Given the equation $2x + 5y = 4$, use each method below to find y when $x = -3, -1, 2$, and 6. Tell which method is more efficient.

Method 1 Substitute $x = -3$ into $2x + 5y = 4$ and solve for y. Repeat this process for the other values of x.

Method 2 Solve $2x + 5y = 4$ for y. Then evaluate the resulting expression for y using each of the given values of x.

EXAMPLE 2 *Calculating the Value of a Variable*

Given the equation $x + xy = 1$, find the value of y when $x = -1$ and $x = 3$.

SOLUTION

Solve the equation for y.

$x + xy = 1$ — Write original equation.

$xy = 1 - x$ — Subtract x from each side.

$y = \frac{1 - x}{x}$ — Divide each side by x.

Then calculate the value of y for each value of x.

When $x = -1$: $y = \frac{1 - (-1)}{-1} = -2$ **When $x = 3$:** $y = \frac{1 - 3}{3} = -\frac{2}{3}$

REAL LIFE
Benefit Concert

EXAMPLE 3 *Writing an Equation with More Than One Variable*

You are organizing a benefit concert. You plan on having only two types of tickets: adult and child. Write an equation with more than one variable that represents the revenue from the concert. How many variables are in your equation?

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Total revenue}} = \text{Adult ticket price} \cdot \text{Number of adults} + \text{Child ticket price} \cdot \text{Number of children}$$

LABELS

Total revenue = R (dollars)

Adult ticket price = p_1 (dollars per adult)

Number of adults = A (adults)

Child ticket price = p_2 (dollars per child)

Number of children = C (children)

ALGEBRAIC MODEL

$$R = p_1 A + p_2 C$$

This equation has five variables. The variables p_1 and p_2 are read as "p sub one" and "p sub two." The small lowered numbers 1 and 2 are subscripts used to indicate the two different price variables.

EXAMPLE 4 *Using an Equation with More Than One Variable*

BENEFIT CONCERT For the concert in Example 3, your goal is to sell $25,000 in tickets. You plan to charge $25.25 per adult and expect to sell 800 adult tickets. You need to determine what to charge for child tickets. How much should you charge per child if you expect to sell 200 child tickets? 300 child tickets? 400 child tickets?

SOLUTION

First solve the equation $R = p_1A + p_2C$ from Example 3 for p_2.

$$R = p_1A + p_2C \qquad \text{Write original equation.}$$

$$R - p_1A = p_2C \qquad \text{Subtract } p_1A \text{ from each side.}$$

$$\frac{R - p_1A}{C} = p_2 \qquad \text{Divide each side by } C.$$

Now substitute the known values of the variables into the equation.

If $C = 200$, the child ticket price is $p_2 = \dfrac{25{,}000 - 25.25(800)}{200} = \$24.$

If $C = 300$, the child ticket price is $p_2 = \dfrac{25{,}000 - 25.25(800)}{300} = \$16.$

If $C = 400$, the child ticket price is $p_2 = \dfrac{25{,}000 - 25.25(800)}{400} = \$12.$

FOCUS ON APPLICATIONS

REAL LIFE **BENEFIT CONCERT**
Farm Aid, a type of benefit concert, began in 1985. Since that time Farm Aid has distributed more than $13,000,000 to family farms throughout the United States.

GOAL 2 REWRITING COMMON FORMULAS

Throughout this course you will be using many formulas. Several are listed below.

COMMON FORMULAS

	FORMULA	VARIABLES
Distance	$d = rt$	d = distance, r = rate, t = time
Simple Interest	$I = Prt$	I = interest, P = principal, r = rate, t = time
Temperature	$F = \frac{9}{5}C + 32$	F = degrees Fahrenheit, C = degrees Celsius
Area of Triangle	$A = \frac{1}{2}bh$	A = area, b = base, h = height
Area of Rectangle	$A = \ell w$	A = area, ℓ = length, w = width
Perimeter of Rectangle	$P = 2\ell + 2w$	P = perimeter, ℓ = length, w = width
Area of Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	A = area, b_1 = one base, b_2 = other base, h = height
Area of Circle	$A = \pi r^2$	A = area, r = radius
Circumference of Circle	$C = 2\pi r$	C = circumference, r = radius

EXAMPLE 5 *Rewriting a Common Formula*

STUDENT HELP

Skills Review
For help with perimeter, see p. 914.

The formula for the perimeter of a rectangle is $P = 2\ell + 2w$. Solve for w.

SOLUTION

$P = 2\ell + 2w$ Write perimeter formula.

$P - 2\ell = 2w$ Subtract 2ℓ from each side.

$\frac{P - 2\ell}{2} = w$ Divide each side by 2.

EXAMPLE 6 *Applying a Common Formula*

REAL LIFE *Gardening*

You have 40 feet of fencing with which to enclose a rectangular garden. Express the garden's area in terms of its length only.

SOLUTION

Use the formula for the area of a rectangle, $A = \ell w$, and the result of Example 5.

$A = \ell w$ Write area formula.

$A = \ell\left(\frac{P - 2\ell}{2}\right)$ Substitute $\frac{P - 2\ell}{2}$ for w.

$A = \ell\left(\frac{40 - 2\ell}{2}\right)$ Substitute 40 for P.

$A = \ell(20 - \ell)$ Simplify.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. Complete this statement: $A = \ell w$ is an example of a(n) ? .

2. Which of the following are equations with more than one variable?

   A. $2x + 5 = 9 - 5x$ **B.** $4x + 10y = 62$ **C.** $x - 8 = 3y + 7$

3. Use the equation from Example 3. Describe how you would solve for A.

Skill Check ✓

Solve the equation for *y*.

4. $4x + 8y = 17$

5. $5x - 3y = 9$

6. $5y - 3x = 15$

7. $\frac{3}{4}x + 5y = 20$

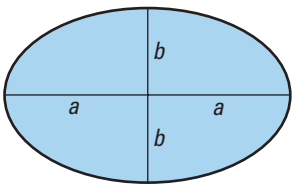
8. $xy + 2x = 8$

9. $\frac{2}{3}x - \frac{1}{2}y = 12$

In Exercises 10 and 11, use the following information.
The area A of an ellipse is given by the formula $A = \pi ab$ where a and b are half the lengths of the major and minor axes. (The longer chord is the major axis.)

10. Solve the formula for a.

11. Use the result from Exercise 10 to find the length of the major axis of an ellipse whose area is 157 square inches and whose minor axis is 10 inches long. (Use 3.14 for π.)



PRACTICE AND APPLICATIONS

STUDENT HELP
Extra Practice
to help you master skills is on p. 940.

EXPLORING METHODS **Find the value of *y* for the given value of *x* using two methods. First, substitute the value of *x* into the equation and then solve for *y*. Second, solve for *y* and then substitute the value of *x* into the equation.**

12. $4x + 9y = 30; x = 3$

13. $5x - 7y = 12; x = 1$

14. $xy + 3x = 25; x = 5$

15. $9y - 4x = -16; x = 8$

16. $-y - 2x = -11; x = -4$

17. $-x = 3y - 55; x = 20$

18. $x = 24 + xy; x = -12$

19. $-xy + 3x = 30; x = 15$

20. $-4x + 7y + 7 = 0; x = 7$

21. $6x - 5y - 44 = 0; x = 4$

22. $\frac{1}{2}x - \frac{4}{5}y = 19; x = 6$

23. $\frac{3}{4}x = -\frac{9}{11}y + 12; x = 10$

REWRITING FORMULAS **Solve the formula for the indicated variable.**

24. Circumference of a Circle
    Solve for r: $C = 2\pi r$

25. Volume of a Cone
    Solve for h: $V = \frac{1}{3}\pi r^2 h$

26. Area of a Triangle
    Solve for b: $A = \frac{1}{2}bh$

27. Investment at Simple Interest
    Solve for P: $I = Prt$

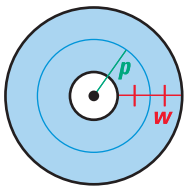
28. Celsius to Fahrenheit
    Solve for C: $F = \frac{9}{5}C + 32$

29. Area of a Trapezoid
    Solve for b_2: $A = \frac{1}{2}(b_1 + b_2)h$

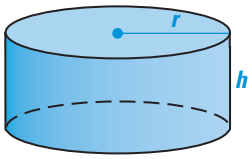
STUDENT HELP
HOMEWORK HELP
Examples 1, 2: Exs. 12–23
Examples 3, 4: Exs. 33–39
Examples 5, 6: Exs. 24–32, 40–42

GEOMETRY CONNECTION **In Exercises 30–32, solve the formula for the indicated variable. Then evaluate the rewritten formula for the given values. (Include units of measure in your answer.)**

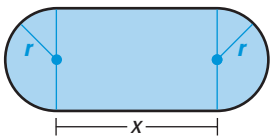
30. Area of a circular ring: $A = 2\pi pw$
Solve for p. Find p when $A = 22$ cm^2 and $w = 2$ cm.



31. Surface area of a cylinder: $S = 2\pi rh + 2\pi r^2$
Solve for h. Find h when $S = 105$ in.^{2} and $r = 3$ in.



32. Perimeter of a track: $P = 2\pi r + 2x$
Solve for r. Find r when $P = 440$ yd and $x = 110$ yd.



HONEYBEES **In Exercises 33 and 34, use the following information.**
A forager honeybee spends about three weeks becoming accustomed to the immediate surroundings of its hive and spends the rest of its life collecting pollen and nectar. The total number of miles T a forager honeybee flies in its lifetime L (in days) can be modeled by $T = m(L - 21)$ where m is the number of miles it flies each day.

33. Solve the equation $T = m(L - 21)$ for L.

34. A forager honeybee's flight muscles last only about 500 miles; after that the bee dies. Some forager honeybees fly about 55 miles per day. Approximately how many days do these bees live?

BASEBALL **In Exercises 35 and 36, use the following information.**
The Pythagorean Theorem of Baseball is a formula for approximating a team's ratio of wins to games played. Let R be the number of runs the team scores during the season, A be the number of runs allowed to opponents, W be the number of wins, and T be the total number of games played. Then the formula

$$\frac{W}{T} \approx \frac{R^2}{R^2 + A^2}$$

approximates the team's ratio of wins to games played. ▶ Source: *Inside Sports*

35. Solve the formula for W.

36. The 1998 New York Yankees scored 965 runs and allowed 656. How many of its 162 games would you estimate the team won?

FUNDRAISER **In Exercises 37–39, use the following information.**
Your tennis team is having a fundraiser. You are going to help raise money by selling sun visors and baseball caps.

37. Write an equation that represents the total amount of money you raise.

38. How many variables are in the equation? What does each represent?

39. Your team raises a total of $4480. Give three possible combinations of sun visors and baseball caps that could have been sold if the price of a sun visor is $3.00 and the price of a baseball cap is $7.00.

40. **GEOMETRY CONNECTION** The formula for the area of a circle is $A = \pi r^2$. The formula for the circumference of a circle is $C = 2\pi r$. Write a formula for the area of a circle in terms of its circumference.

FOCUS ON CAREERS

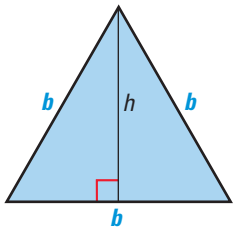
REAL LIFE SPORTS STATISTICIANS are employed by many professional sports teams, leagues, and news organizations. They collect and analyze team and individual data on items such as scoring.

INTERNET CAREER LINK
www.mcdougallittell.com

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 41 and 42.

41. **GEOMETRY CONNECTION** The formula for the height h of an equilateral triangle is $h = \frac{\sqrt{3}}{2}b$ where b is the length of a side.

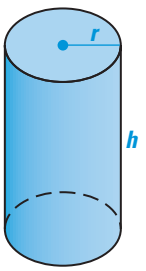


Write a formula for the area of an equilateral triangle in terms of the following.

a. the length of a side only

b. the height only

42. **GEOMETRY CONNECTION** The surface area S of a cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$. The height h of the cylinder shown at the right is 5 more than 3 times its radius r.



a. Write a formula for the surface area of the cylinder in terms of its radius.

b. Find the surface area of the cylinder for $r = 3$, 4, and 6.

Test Preparation

QUANTITATIVE COMPARISON **In Exercises 43 and 44, choose the statement that is true about the given quantities.**

(A) The quantity in column A is greater.

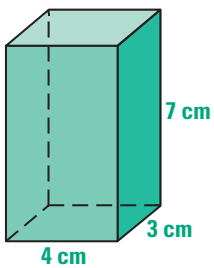
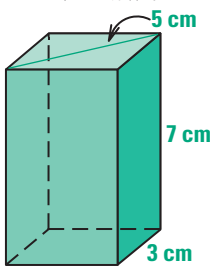
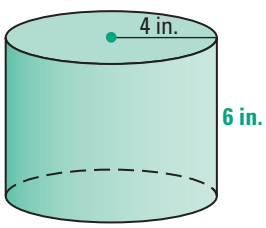
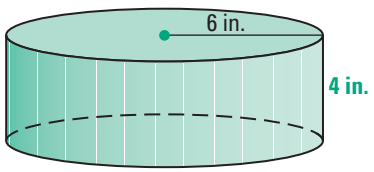
(B) The quantity in column B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the given information.

STUDENT HELP

Skills Review
For help with the Pythagorean theorem, see p. 917.

	Column A	Column B
43.	$V = \ell wh$ 	$V = \ell wh$ 
44.	$V = \pi r^2 h$ 	$V = \pi r^2 h$ 

★ Challenge

45. FUEL EFFICIENCY The more aerodynamic a vehicle is, the less fuel the vehicle's engine must use to overcome air resistance. To design vehicles that are as fuel efficient as possible, automotive engineers use the formula

$$R = 0.00256 \times D_C \times F_A \times s^2$$

where R is the air resistance (in pounds), D_C is the drag coefficient, F_A is the frontal area of the vehicle (in square feet), and s is the speed of the vehicle (in miles per hour). The formula assumes that there is no wind.

a. Rewrite the formula to find the drag coefficient in terms of the other variables.

EXTRA CHALLENGE
www.mcdougallittell.com

b. Find the drag coefficient of a car when the air resistance is 50 pounds, the frontal area is 25 square feet, and the speed of the car is 45 miles per hour.

MIXED REVIEW

WRITING EXPRESSIONS **Write an expression to answer the question.** (Skills Review, p. 929)

46. You buy x birthday cards for \$1.85 each. How much do you spend?

47. You have \$30 and spend x dollars. How much money do you have left?

48. You drive 55 miles per hour for x hours. How many miles do you drive?

49. You have \$250 in your bank account and you deposit x dollars. How much money do you now have in your account?

50. You spend \$42 on x music cassettes. How much does each cassette cost?

51. A certain ball bearing weighs 2 ounces. A box contains x ball bearings. What is the total weight of the ball bearings?

UNIT ANALYSIS **Give the answer with the appropriate unit of measure.** (Review 1.1)

52. $\left(\frac{7 \text{ meters}}{1 \text{ minute}}\right)(60 \text{ minutes})$

53. $\left(\frac{168 \text{ hours}}{1 \text{ week}}\right)(52 \text{ weeks})$

54. $4\frac{1}{4}$ feet $+ 7\frac{3}{4}$ feet

55. $13\frac{1}{4}$ liters $- 8\frac{7}{8}$ liters

56. $\left(\frac{3 \text{ yards}}{1 \text{ second}}\right)(12 \text{ seconds}) - 10 \text{ yards}$

57. $\left(\frac{15 \text{ dollars}}{1 \text{ hour}}\right)(8 \text{ hours}) + 45 \text{ dollars}$

SOLVING EQUATIONS **Solve the equation. Check your solution.** (Review 1.3)

58. $3d + 16 = d - 4$

59. $5 - x = 23 + 2x$

60. $10(y - 1) = y + 4$

61. $p - 16 + 4 = 4(2 - p)$

62. $-10x = 5x + 5$

63. $12z = 4z - 56$

64. $\frac{2}{3}x - 7 = 1$

65. $-\frac{3}{4}x + 19 = -11$

66. $\frac{1}{4}x + \frac{3}{8} = \frac{1}{5} - \frac{1}{5}x$

67. $\frac{5}{4}x - \frac{3}{4} = \frac{5}{6}x + \frac{1}{2}$