1.3

What you should learn

GOAL Solve linear equations.

GOAL 2 Use linear equations to solve real-life problems, such as finding how much a broker must sell in Example 5.

Why you should learn it

To solve real-life problems, such as finding the temperature at which dry ice changes to a gas in Ex. 43.



Solving Linear Equations



SOLVING A LINEAR EQUATION

An **equation** is a statement in which two expressions are equal. A **linear equation** in one variable is an equation that can be written in the form ax = b where a and b are constants and $a \neq 0$. A number is a **solution** of an equation if the statement is true when the number is substituted for the variable.

Two equations are **equivalent** if they have the same solutions. For instance, the equations x - 4 = 1 and x = 5 are equivalent because both have the number 5 as their only solution. The following transformations, or changes, produce equivalent equations and can be used to solve an equation.

TRANSFORMATIONS THAT PRODUCE EQUIVALENT EQUATIONS

ADDITION PROPERTY	Add the same number to both sides:
OF EQUALITY	If $a = b$, then $a + c = b + c$.
SUBTRACTION PROPERTY	Subtract the same number from both sides:
OF EQUALITY	If $a = b$, then $a - c = b - c$.
MULTIPLICATION PROPERTY OF EQUALITY	<i>Multiply</i> both sides by the same nonzero number: If $a = b$ and $c \neq 0$, then $ac = bc$.
DIVISION PROPERTY	<i>Divide</i> both sides by the same nonzero number:
OF EQUALITY	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

EXAMPLE 1 Solving an Equation with a Variable on One Side

Solve $\frac{3}{7}x + 9 = 15$.

SOLUTION

Your goal is to isolate the variable on one side of the equation.

$\frac{3}{7}x + 9 = 15$	Write original equation.
$\frac{3}{7}x = 6$	Subtract 9 from each side.
$x = \frac{7}{3}(6)$	Multiply each side by $\frac{7}{3}$, the reciprocal of $\frac{3}{7}$.
x = 14	Simplify.

The solution is 14.

CHECK Check x = 14 in the original equation.

 $\frac{3}{7}(14) + 9 \stackrel{?}{=} 15$ Substitute 14 for x. $15 = 15 \checkmark$ Solution checks.

Solving an Equation with a Variable on Both Sides EXAMPLE 2



Solve 5n + 11 = 7n - 9.

SOLUTION

5n+11=7n-9	Write original equation.
11 = 2n - 9	Subtract 5 <i>n</i> from each side.
20 = 2n	Add 9 to each side.
10 = n	Divide each side by 2.

The solution is 10. Check this in the original equation.

EXAMPLE 3 Using the Distributive Property

Solve 4(3x - 5) = -2(-x + 8) - 6x.

SOLUTION

Solve $\frac{1}{3}x + \frac{1}{4} = x - \frac{1}{6}$.

4(3x - 5) = -2(-x + 8) - 6x	Write original equation.
12x - 20 = 2x - 16 - 6x	Distributive property
12x - 20 = -4x - 16	Combine like terms.
16x - 20 = -16	Add 4 <i>x</i> to each side.
16x = 4	Add 20 to each side.
$x = \frac{1}{4}$	Divide each side by 16.

The solution is $\frac{1}{4}$. Check this in the original equation.

EXAMPLE 4 Solving an Equation with Fractions

STUDENT HELP

Skills Review For help with finding the LCD, see p. 939.

SOLUTION	
$\frac{1}{3}x + \frac{1}{4} = x - \frac{1}{6}$	Write original equation.
$12\left(\frac{1}{3}x + \frac{1}{4}\right) = 12\left(x - \frac{1}{6}\right)$	Multiply each side by the LCD, 12.
4x + 3 = 12x - 2	Distributive property
3 = 8x - 2	Subtract 4x from each side.
5 = 8x	Add 2 to each side.
$\frac{5}{8} = x$	Divide each side by 8.

The solution is $\frac{5}{8}$. Check this in the original equation.

FOCUS ON





monthly payments. CAREER LINK

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EXAMPLE 6

Writing and Using a Geometric Formula

You have a 3 inch by 5 inch photo that you want to enlarge, mat, and frame. You want the width of the mat to be 2 inches on all sides. You want the perimeter of the framed photo to be 44 inches. By what percent should you enlarge the photo?

SOLUTION

Let *x* be the percent (in decimal form) of enlargement relative to the original photo. So, the dimensions of the enlarged photo (in inches) are 3x by 5x. Draw a diagram.



VERBAL MODEL	$\boxed{\text{Perimeter}} = 2 \cdot$	Width $+2$.	Length
LABELS	Perimeter = 44	(inches)	
	Width = $4 + 3x$	(inches)	
	Length = $4 + 5x$	(inches)	
	44 = 2 (4 + 3x)	+2(4+5x)	Write lin
mobili	44 = 16 + 16x		Distribut
	28 = 16x		Subtract
	1.75 = x		Divide ea



ear equation. te and combine like terms. 16 from each side. ach side by 16.

You should enlarge the photo to 175% of its original size.

USING LINEAR EQUATIONS IN REAL LIFE



GOAL 2

Writing and Using a Linear Equation

REAL ESTATE A real estate broker's base salary is \$18,000. She earns a 4% commission on total sales. How much must she sell to earn \$55,000 total?

SOLUTION

Base +	Commission	Total
salary	rate	• sales
55,000	(dollars)	
8,000	(dollars)	
e = 0.04	(percent in decima	l form)
	(dollars)	
00 + 0.04 x	Write linear	equation.
x	Subtract 18,0)00 from each side
	Divide each	side by 0.04.
	$\frac{\text{salary}}{\text{salary}} + \frac{1}{55,000} + \frac{1}{55,$	salary+rate55,000(dollars)8,000(dollars) $e = 0.04$ (percent in decimal (dollars)00 + 0.04 xWrite linear xxSubtract 18,0 Divide each

The broker must sell real estate worth a total of \$925,000 to earn \$55,000.

GUIDED PRACTICE

Skill Check

Vocabulary Check
Concept Check

- 1. What is an equation?
- **2.** What does it mean for two equations to be equivalent? Give an example of two equivalent equations.
- **3.** How does an equation such as 2(x + 3) = 10 differ from an identity such as 2(x + 3) = 2x + 6?

ERROR ANALYSIS Describe the error(s). Then write the correct steps.



6. Describe the transformation(s) you would use to solve 2x - 8 = 14.

Solve the equation.

7. $x + 4 = 9$	8. $4x = 24$	9. $2x - 3 = 7$
10 . $0.2x - 8 = 0.6$	11. $\frac{1}{3}x + \frac{1}{2} = \frac{11}{12}$	12. $\frac{3}{4}x - \frac{2}{3} = \frac{5}{6}$
13. $1.5x + 9 = 4.5$	14. $6x - 4 = 2x + 10$	15. $2(x + 2) = 3(x - 8)$

16. Second Se

PRACTICE AND APPLICATIONS

 STUDENT HELP
 Extra Practice to help you master

skills is on p. 940.

DESCRIBING TRANSFORMATIONS Describe the transformation(s) you would use to solve the equation.

17. $x + 5 = -7$	18. $\frac{1}{6}x = 3$	19. $-\frac{4}{7}x = 6$
20. $2x - 9 = 0$	21. $\frac{x}{3} + 2 = 89$	22. $3 = -x - 5$

SOLVING EQUATIONS Solve the equation. Check your solution.

23. $4x + 7 = 27$	24. $7s - 29 = -15$
25. $3a + 13 = 9a - 8$	26. $m - 30 = 6 - 2m$
27. $15n + 9 = 21$	28. $2b + 11 = 15 - 6b$
29. $2(x+6) = -2(x-4)$	30. $4(-3x + 1) = -10(x - 4) - 14x$
31. $-(x+2) - 2x = -2(x+1)$	32. $-4(3 + x) + 5 = 4(x + 3)$

STUDENT HELP

HOMEWORK HELP Examples 1–4: Exs. 17–40 Examples 5, 6: Exs. 43–49 **SOLVING EQUATIONS** Solve the equation. Check your solution.

33. $\frac{7}{2}x - 1 = 2x + 5$ **34.** $\frac{1}{2}x - \frac{5}{3} = -\frac{1}{2}x + \frac{19}{4}$ **35.** $\frac{3}{4}(\frac{4}{5}x - 2) = \frac{11}{4}$ **36.** $-\frac{2}{3}(\frac{6}{5}x - \frac{7}{10}) = \frac{17}{20}$ **37.** 2.7n + 4.3 = 12.94**38.** -4.2n - 6.5 = -14.06**39.** 3.1(x + 2) - 1.5x = 5.2(x - 4)**40.** 2.5(x - 3) + 1.7x = 10.8(x + 1.5)

GEOMETRY CONNECTION Find the dimensions of the figure.





In Exercises 43 and 44, use the following formula.

degrees Fahrenheit = $\frac{9}{5}$ (degrees Celsius) + 32

- **43.** S DRY ICE Dry ice is solid carbon dioxide. Dry ice does not melt it changes directly from a solid to a gas. Dry ice changes to a gas at -109.3° F. What is this temperature in degrees Celsius?
- **44. VETERINARY MEDICINE** The normal body temperature of a dog is 38.6°C. Your dog's temperature is 101.1°F. Does your dog have a fever? Explain.
- **45. (S) CAR REPAIR** The bill for the repair of your car was \$390. The cost for parts was \$215. The cost for labor was \$35 per hour. How many hours did the repair work take?
- **46. SUMMER JOBS** You have two summer jobs. In the first job, you work 28 hours per week and earn \$7.25 per hour. In the second job, you earn \$6.50 per hour and can work as many hours as you want. If you want to earn \$255 per week, how many hours must you work at your second job?
- **47. STOCKBROKER** A stockbroker earns a base salary of \$40,000 plus 5% of the total value of the stocks, mutual funds, and other investments that the stockbroker sells. Last year, the stockbroker earned \$71,750. What was the total value of the investments the stockbroker sold?
- **48. WORD PROCESSING** You are writing a term paper. You want to include a table that has 5 columns and is 360 points wide. (A point is $\frac{1}{72}$ of an inch.) You want the first column to be 200 points wide and the remaining columns to be equal in width. How wide should each of the remaining columns be?
- **49. WALKWAY CONSTRUCTION** You are building a walkway of uniform width around a 100 foot by 60 foot swimming pool. After completing the walkway, you want to put a fence along the outer edge of the walkway. You have 450 feet of fencing to enclose the walkway. What is the maximum width of the walkway?



FOCUS ON CAREERS



STOCKBROKER Stockbrokers buy and sell stocks, bonds, and other securities for clients as discussed in Ex. 47. Stockbrokers typically study economics in college.

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50. MULTI-STEP PROBLEM You are in charge of constructing a fence around the running track at a high school. The fence is to be built around the track so that there is a uniform gap between the outside edge of the track and the fence.



- **a**. What is the maximum width of the gap between the track and the fence if no more than 630 meters of fencing is used? (Hint: Use the equation for the circumference of a circle, $C = 2\pi r$, to help you.)
- **b.** You are charging the school \$10.50 for each meter of fencing. The school has \$5250 in its budget to spend on the fence. How many meters of fencing can you use with this budget?
- c. CRITICAL THINKING Explain whether or not it is geometrically reasonable to put up the new fence with the given budget.

† Challenge **SOLVING EQUATIONS** Solve the equation. If there is no solution, write no solution. If the equation is an identity, write all real numbers.

51. $5(x - 4) = 5x + 12$	52. $3(x + 5) = 3x + 15$
53. $7x + 14 - 3x = 4x + 14$	54. $11x - 3 + 2x = 6(x + 4) + 7x$
55. $-2(4 - 3x) + 7 = -2x + 6 + 8x$	56. $5(2 - x) = 3 - 2x + 7 - 3x$

MIXED REVIEW

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EXTRA CHALLENGE www.mcdougallittell.com

GEOMETRY CONNECTION Find the area of the figure. (Skills Review, p. 914)

57 . Circle with radius 5 inches	58 . Square with side 4 inches	
59. Circle with radius 7 inches	60. Square with side 9 inches	
EVALUATING EXPRESSIONS Evaluate the	he expression. (Review 1.2 for 1.4)	
61. 24 - (9 + 7)	62. -16 + 3(8 - 4)	
63. $-3 + 6(1 - 3)^2$	64. $2(3-5)^3 + 4(-4+7)$	
65. $2x + 3$ when $x = 4$	66. $8(x-2) + 3x$ when $x = 6$	
67. $5x - 7 + 2x$ when $x = -3$	68. $6x - 3(2x + 4)$ when $x = 5$	
SIMPLIFYING EXPRESSIONS Simplify the expression. (Review 1.2)		
69. $3(7 + x) - 8x$	70. $2(8 + x) + 2x - x$	
71. $4x - (6 - 3x)$	72. $2x - 3(4x + 7)$	
73. $3(x + 9) + 2(4 - x)$	74. $-4(x-3) - 2(x+7)$	
75. $2(x^2 + 2) - x + x^2 + 7$	76. $2(x^2 - 81) - 3x^2$	
77. $x^2 - 5x + 3(x^2 + 7x)$	78. $4x^2 - 2(x^2 - 3x) + 6x + 8$	