

1.1

Real Numbers and Number Operations

What you should learn

GOAL 1 Use a number line to graph and order real numbers.

GOAL 2 Identify properties of and use operations with real numbers, as applied in Exs. 64 and 65.

Why you should learn it

▼ To solve **real-life** problems, such as how to exchange money in **Example 7**.



GOAL 1 USING THE REAL NUMBER LINE

The numbers used most often in algebra are the *real numbers*. Some important subsets of the real numbers are listed below.

SUBSETS OF THE REAL NUMBERS

WHOLE NUMBERS 0, 1, 2, 3, . . .

INTEGERS . . . , -3, -2, -1, 0, 1, 2, 3, . . .

RATIONAL NUMBERS Numbers such as $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{-4}{1}$ (or -4) that can be written as the ratio of two integers. When written as decimals, rational numbers terminate or repeat. For example, $\frac{3}{4} = 0.75$ and $\frac{1}{3} = 0.333 \dots$

IRRATIONAL NUMBERS Real numbers that are not rational, such as $\sqrt{2}$ and π . When written as decimals, irrational numbers neither terminate nor repeat.

The three dots in the lists of the whole numbers and the integers above indicate that the lists continue without end.

Real numbers can be pictured as points on a line called a *real number line*. The numbers increase from left to right, and the point labeled 0 is the **origin**.



The point on a number line that corresponds to a real number is the **graph** of the number. Drawing the point is called *graphing* the number or *plotting* the point. The number that corresponds to a point on a number line is the **coordinate** of the point.

EXAMPLE 1 Graphing Numbers on a Number Line

Graph the real numbers $-\frac{4}{3}$, $\sqrt{2}$, and 2.7.

SOLUTION

First, recall that $-\frac{4}{3}$ is $-1\frac{1}{3}$, so $-\frac{4}{3}$ is between -2 and -1. Then, approximate $\sqrt{2}$ as a decimal to the nearest tenth: $\sqrt{2} \approx 1.4$. (The symbol $\approx$ means *is approximately equal to*.) Finally, graph the numbers.



A number line can be used to order real numbers. The *inequality symbols* $<$, $\leq$, $>$, and $\geq$ can be used to show the order of two numbers.

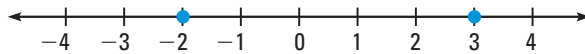
EXAMPLE 2 Ordering Real Numbers

Use a number line to order the real numbers.

- a. -2 and 3 b. -1 and -3

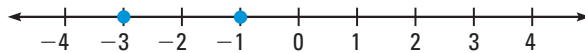
SOLUTION

- a. Begin by graphing both numbers.



Because -2 is to the left of 3 , it follows that -2 is *less than* 3 , which can be written as $-2 < 3$. This relationship can also be written as $3 > -2$, which is read as “ 3 is *greater than* -2 .”

- b. Begin by graphing both numbers.



Because -3 is to the left of -1 , it follows that -3 is *less than* -1 , which can be written as $-3 < -1$. (You can also write $-1 > -3$.)



EXAMPLE 3 Ordering Elevations

Here are the elevations of five locations in Imperial Valley, California.

Alamorio: -135 feet

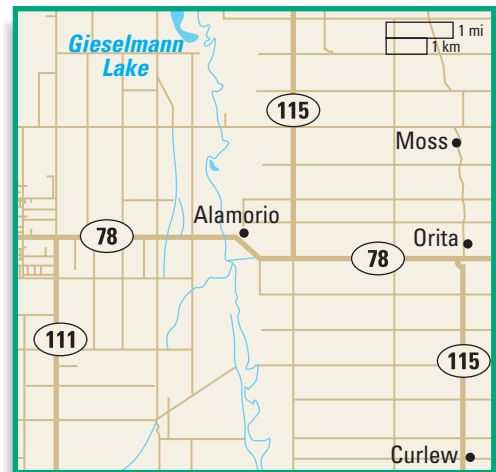
Curlew: -93 feet

Gieselmann Lake: -162 feet

Moss: -100 feet

Orita: -92 feet

- a. Order the elevations from lowest to highest.
b. Which locations have elevations below -100 feet?



SOLUTION

- a. From lowest to highest, the elevations are as follows.

Location	Gieselmann Lake	Alamorio	Moss	Curlew	Orita
Elevation (ft)	-162	-135	-100	-93	-92

- b. Gieselmann Lake and Alamorio have elevations below -100 feet.

GOAL 2 USING PROPERTIES OF REAL NUMBERS

When you add or multiply real numbers, there are several properties to remember.

CONCEPT SUMMARY

PROPERTIES OF ADDITION AND MULTIPLICATION

Let a , b , and c be real numbers.

Property	Addition	Multiplication
CLOSURE	$a + b$ is a real number.	ab is a real number.
COMMUTATIVE	$a + b = b + a$	$ab = ba$
ASSOCIATIVE	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
IDENTITY	$a + 0 = a$, $0 + a = a$	$a \cdot 1 = a$, $1 \cdot a = a$
INVERSE	$a + (-a) = 0$	$a \cdot \frac{1}{a} = 1$, $a \neq 0$
The following property involves both addition and multiplication.		
DISTRIBUTIVE	$a(b + c) = ab + ac$	

EXAMPLE 4 Identifying Properties of Real Numbers

Identify the property shown.

a. $(3 + 9) + 8 = 3 + (9 + 8)$

b. $14 \cdot 1 = 14$

SOLUTION

a. Associative property of addition

b. Identity property of multiplication

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STUDENT HELP

Study Tip

- If a is positive, then its opposite, $-a$, is negative.
- The opposite of 0 is 0.
- If a is negative, then its opposite, $-a$, is positive.

The **opposite**, or *additive inverse*, of any number a is $-a$. The **reciprocal**, or *multiplicative inverse*, of any nonzero number a is $\frac{1}{a}$. Subtraction is defined as *adding the opposite*, and division is defined as *multiplying by the reciprocal*.

$$a - b = a + (-b)$$

Definition of subtraction

$$\frac{a}{b} = a \cdot \frac{1}{b}, b \neq 0$$

Definition of division

EXAMPLE 5 Operations with Real Numbers

a. The difference of 7 and -10 is:

$$7 - (-10) = 7 + 10$$

Add 10, the opposite of -10 .

$$= 17$$

Simplify.

b. The quotient of -24 and $\frac{1}{3}$ is:

$$\frac{-24}{\frac{1}{3}} = -24 \cdot 3$$

Multiply by 3, the reciprocal of $\frac{1}{3}$.

$$= -72$$

Simplify.

STUDENT HELP



HOMESCHOOL HELP

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for extra examples.

When you use the operations of addition, subtraction, multiplication, and division in real life, you should use *unit analysis* to check that your units make sense.

EXAMPLE 6 Using Unit Analysis

Perform the given operation. Give the answer with the appropriate unit of measure.

- 345 miles $-$ 187 miles = 158 miles
- $(1.5 \text{ hours}) \left(\frac{50 \text{ miles}}{1 \text{ hour}} \right) = 75 \text{ miles}$
- $\frac{24 \text{ dollars}}{3 \text{ hours}} = 8 \text{ dollars per hour}$
- $\left(\frac{88 \text{ feet}}{1 \text{ second}} \right) \left(\frac{3600 \text{ seconds}}{1 \text{ hour}} \right) \left(\frac{1 \text{ mile}}{5280 \text{ feet}} \right) = 60 \text{ miles per hour}$

FOCUS ON APPLICATIONS

Foreign Exchange Center			We Buy At	We Sell At
Canadian	dollar		.6513	.7092
French	franc		.1529	.1717
German	mark		.5130	.5757
British	pound		1.5343	1.6938
Italian	piastre		1.2739	1.4297
Japanese	yen		.000518	.000582
Swiss	franc		.00762	.00850
Spanish	peseta		.1027	.1176
Portuguese	escudo		.00603	.00677
Dutch	guilder		.6312	.6991

EXAMPLE 7 Operations with Real Numbers in Real Life

MONEY EXCHANGE You are exchanging \$400 for Mexican pesos. The exchange rate is 8.5 pesos per dollar, and the bank charges a 1% fee to make the exchange.

- How much money should you take to the bank if you do not want to use part of the \$400 to pay the exchange fee?
- How much will you receive in pesos?
- When you return from Mexico you have 425 pesos left. How much can you get in dollars? Assume that you use other money to pay the exchange fee.

SOLUTION

- To find 1% of \$400, multiply to get:

$$1\% \times \$400 = 0.01 \times \$400 \quad \text{Rewrite 1\% as 0.01.}$$

$$= \$4 \quad \text{Simplify.}$$

- ▶ You need to take \$400 + \$4 = \$404 to the bank.

- To find the amount you will receive in pesos, multiply \$400 by the exchange rate.

$$(400 \text{ dollars}) \left(\frac{8.5 \text{ pesos}}{1 \text{ dollar}} \right) = (400 \times 8.5) \text{ pesos}$$

$$= 3400 \text{ pesos}$$

- ▶ You receive 3400 pesos for \$400.

- To find the amount in dollars, divide 425 pesos by the exchange rate.

$$\frac{425 \text{ pesos}}{8.5 \text{ pesos per dollar}} = (425 \text{ pesos}) \left(\frac{1 \text{ dollar}}{8.5 \text{ pesos}} \right)$$

$$= \frac{425}{8.5} \text{ dollars}$$

$$= \$50$$

- ▶ You receive \$50 for 425 pesos.

REAL LIFE MONEY

EXCHANGE In 1997, 17,700,000 United States citizens visited Mexico and spent \$7,200,000,000. That same year 8,433,000 Mexican citizens visited the United States and spent \$4,289,000,000.

STUDENT HELP



DATA UPDATE

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GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

- What is a rational number? What is an irrational number?
- Give an example of each of the following: a whole number, an integer, a rational number, and an irrational number.
- Which of the following is false? Explain.
 - No integer is an irrational number.
 - Every integer is a rational number.
 - Every integer is a whole number.

Skill Check ✓

Graph the numbers on a number line. Then decide which number is the greatest.

- $-3, 4, 0, -8, -10$
- $1, -2.5, 4.5, -0.5, 6$
- $\frac{3}{2}, -1, -\frac{5}{2}, 3, -5$
- $3.2, -0.7, \frac{3}{4}, -\frac{3}{2}, 0$

Identify the property shown.

- $5 + 2 = 2 + 5$
- $24 \cdot 1 = 24$
- $13 + 0 = 13$
- $6 + (-6) = 0$
- $8 \cdot 10 = 10 \cdot 8$
- $7\left(\frac{1}{7}\right) = 1$
- Find the product. Give the answer with the appropriate unit of measure. Explain your reasoning.

$$\left(\frac{90 \text{ miles}}{1 \text{ hour}}\right)\left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right)\left(\frac{1 \text{ hour}}{60 \text{ minutes}}\right)\left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right)$$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 940.

USING A NUMBER LINE Graph the numbers on a number line. Then decide which number is greater and use the symbol $<$ or $>$ to show the relationship.

- $\frac{1}{2}, -5$
- $4, \frac{3}{4}$
- $2.3, -0.6$
- $0.3, -2.1$
- $-\frac{5}{3}, \sqrt{3}$
- $0, -\sqrt{10}$
- $-\frac{9}{4}, -3$
- $-\frac{3}{2}, -\frac{11}{3}$
- $\sqrt{5}, 2$
- $-2, \sqrt{2}$
- $\sqrt{8}, 2.5$
- $-4.5, -\sqrt{24}$

STUDENT HELP

HOMEWORK HELP

Examples 1, 2: Exs. 15–32
Example 3: Exs. 55, 56
Example 4: Exs. 33–42
Example 5: Exs. 43–50
Example 6: Exs. 51–54
Example 7: Exs. 57–65

ORDERING NUMBERS Graph the numbers on a number line. Then write the numbers in increasing order.

- $-\frac{1}{2}, 2, \frac{13}{4}, -3, -6$
- $-\sqrt{5}, -\frac{5}{2}, 0, 3, -\frac{1}{3}$
- $0, -\frac{12}{5}, -\sqrt{12}, 0.3, -1.5$
- $\sqrt{15}, -4, -\frac{2}{9}, -1, 6$
- $\frac{1}{6}, 2.7, -1.5, -8, -\sqrt{7}$
- $0.8, \sqrt{10}, -2.4, -\sqrt{6}, \frac{9}{2}$

IDENTIFYING PROPERTIES Identify the property shown.

33. $-8 + 8 = 0$

34. $(3 \cdot 5) \cdot 10 = 3 \cdot (5 \cdot 10)$

35. $7 \cdot 9 = 9 \cdot 7$

36. $(9 + 2) + 4 = 9 + (2 + 4)$

37. $12(1) = 12$

38. $2(5 + 11) = 2 \cdot 5 + 2 \cdot 11$

LOGICAL REASONING Tell whether the statement is true for all real numbers a , b , and c . Explain your answers.

39. $(a + b) + c = a + (b + c)$

40. $(a - b) - c = a - (b - c)$

41. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

42. $(a \div b) \div c = a \div (b \div c)$

OPERATIONS Select and perform an operation to answer the question.43. What is the sum of 32 and -7 ?44. What is the sum of -9 and -6 ?45. What is the difference of -5 and 8?46. What is the difference of -1 and -10 ?47. What is the product of 9 and -4 ?48. What is the product of -7 and -3 ?49. What is the quotient of -5 and $-\frac{1}{2}$?50. What is the quotient of -14 and $\frac{7}{4}$?**UNIT ANALYSIS** Give the answer with the appropriate unit of measure.

51. $8\frac{1}{6}$ feet $+ 4\frac{5}{6}$ feet

52. $27\frac{1}{2}$ liters $- 18\frac{5}{8}$ liters

53. $(8.75 \text{ yards}) \left(\frac{\$70}{1 \text{ yard}} \right)$

54. $\left(\frac{50 \text{ feet}}{1 \text{ second}} \right) \left(\frac{1 \text{ mile}}{5280 \text{ feet}} \right) \left(\frac{3600 \text{ seconds}}{1 \text{ hour}} \right)$

55. **STATISTICS CONNECTION** The lowest temperatures ever recorded in various cities are shown. List the cities in decreasing order based on their lowest temperatures. How many of these cities have a record low temperature below -25°F ? **Source:** National Climatic Data Center

City	Low temp.	City	Low temp.
Albany, NY	-28°F	Jackson, MS	2°F
Atlanta, GA	-8°F	Milwaukee, WI	-26°F
Detroit, MI	-21°F	New Orleans, LA	11°F
Helena, MT	-42°F	Norfolk, VA	-3°F
Honolulu, HI	53°F	Seattle-Tacoma, WA	0°F

56. **MASTERS GOLF** The table shows the final scores of 10 competitors in the 1998 Masters Golf Tournament. List the players in increasing order based on their golf scores. **Source:** Sports Illustrated

Player	Score	Player	Score
Paul Azinger	-6	Lee Janzen	$+6$
Tiger Woods	-3	Jeff Maggert	$+1$
Jay Haas	-2	Mark O'Meara	-9
Jim Furyk	-7	Corey Pavin	$+9$
Vijay Singh	$+12$	Jumbo Ozaki	$+8$

FOCUS ON PEOPLE

REAL LIFE **VIJAY SINGH**, a world-class golfer from Fiji, won the 1998 PGA Championship. His final score of -9 was 2 better than that of the second-place finisher. Singh, whose first name means "victory" in Hindi, has won several PGA Tour events.

FOCUS ON APPLICATIONS



REAL LIFE **BAR CODES** Using the operations on this bar code produces $(0 + 5 + 1 + 0 + 6 + 8)(3) + (2 + 2 + 5 + 4 + 5) = 78$. The next highest multiple of 10 is 80, and $80 - 78 = 2$, which is the check digit.

STUDENT HELP

Skills Review

For help with significant digits, see p. 911.

BAR CODES In Exercises 57 and 58, use the following information.

All packaged products sold in the United States have a Universal Product Code (UPC), or bar code, such as the one shown at the left. The following operations are performed on the first eleven digits, and the result should equal the twelfth digit, called the *check digit*.

- Add the digits in the odd-numbered positions. Multiply by 3.
- Add the digits in the even-numbered positions.
- Add the results of the first two steps.
- Subtract the result of the previous step from the next highest multiple of 10.

57. Does a UPC of 0 76737 20012 9 check? Explain.

58. Does a UPC of 0 41800 48700 3 check? Explain.

59. SOCIAL STUDIES CONNECTION Two of the tallest buildings in the world are the Sky Central Plaza in Guangzhou, China, which reaches a height of 1056 feet, and the Petronas Tower I in Kuala Lumpur, Malaysia, which reaches a height of 1483 feet. Find the heights of both buildings in yards, in inches, and in miles. Give your answers to four significant digits.

► Source: Council on Tall Buildings and Urban Habitat



Petronas Tower I

60. ELEVATOR SPEED The elevator in the Washington Monument takes 75 seconds to travel 500 feet to the top floor. What is the speed of the elevator in miles per hour? Give your answer to two significant digits.

► Source: National Park Service

TRAVEL In Exercises 61–63, use the following information.

You are taking a trip to Switzerland. You are at the bank exchanging \$600 for Swiss francs. The exchange rate is 1.5 francs per dollar, and the bank charges a 1.5% fee to make the exchange.

- 61.** You brought \$10 extra with you to pay the exchange fee. Do you have enough to pay the fee?
- 62.** How much will you receive in Swiss francs for your \$600?
- 63.** After your trip, you have 321 Swiss francs left. How much is this amount in dollars? Assume that you use other money to pay the exchange fee.

HISTORY CONNECTION In Exercises 64 and 65, use the following information.

In 1862, James Glaisher and Henry Coxwell went up too high in a hot-air balloon. At 25,000 feet, Glaisher passed out. To get the balloon to descend, Coxwell grasped a valve, but his hands were too numb to pull the cord. He was able to pull the cord with his teeth. The balloon descended, and both men made it safely back. The temperature of air drops about 3°F for each 1000 foot increase in altitude.

- 64.** How much had the temperature dropped from the sea level temperature when Glaisher and Coxwell reached an altitude of 25,000 feet?
- 65.** If the temperature at sea level was 60°F , what was the temperature at 25,000 feet?

Test Preparation

- 66. MULTI-STEP PROBLEM** You are taking a trip through the provinces of Alberta and British Columbia in Canada. You are at Quesnel Lake when you decide to visit some of the national parks. You visit the following places in order: Kamloops, Revelstoke, Lethbridge, and Red Deer. After you visit Red Deer, you return to Quesnel Lake.



- Using the scale on the map, estimate the distance traveled (in kilometers) for the entire trip. Approximately where was the “halfway point” of your trip?
- Your car gets 12 kilometers per liter of gasoline. If your gas tank holds 60 liters and the cost of gasoline is \$.29 per liter, about how much will you spend on gasoline for the entire trip? How many times will you have to stop for gasoline if you begin the trip with a full tank?
- If you drive at an average speed of 88 kilometers per hour, how many hours will you spend driving on your trip?

★ Challenge

- 67. LOGICAL REASONING** Show that $a + (a + 2) = 2(a + 1)$ for all values of a by justifying the steps using the properties of addition and multiplication.

$a + (a + 2) = (a + a) + 2$	a. <u>?</u>
$= (1 \cdot a + 1 \cdot a) + 2$	b. <u>?</u>
$= (1 + 1)a + 2$	c. <u>?</u>
$= 2a + 2 \cdot 1$	d. <u>?</u>
$= 2(a + 1)$	e. <u>?</u>

EXTRA CHALLENGE

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MIXED REVIEW

OPERATIONS WITH SIGNED NUMBERS Perform the operation.
(Skills Review, p. 905)

- | | | | |
|----------------------|------------------------|---------------------------|--------------------------|
| 68. $4 - 12$ | 69. $(-7)(-9)$ | 70. $-20 \div 5$ | 71. $6(-5)$ |
| 72. $-14 + 9$ | 73. $6 - (-13)$ | 74. $56 \div (-7)$ | 75. $-16 + (-18)$ |

ALGEBRAIC EXPRESSIONS Write the given phrase as an algebraic expression.
(Skills Review, p. 929 for 1.2)

- | | |
|---------------------------------|--------------------------------------|
| 76. 7 more than a number | 77. 3 less than a number |
| 78. 6 times a number | 79. $\frac{1}{4}$ of a number |

GEOMETRY CONNECTION Find the area of the figure. (Skills Review, p. 914)

- Triangle with base 6 inches and height 4 inches
- Triangle with base 7 inches and height 3 inches
- Rectangle with sides 5 inches and 7 inches
- Rectangle with sides 25 inches and 30 inches