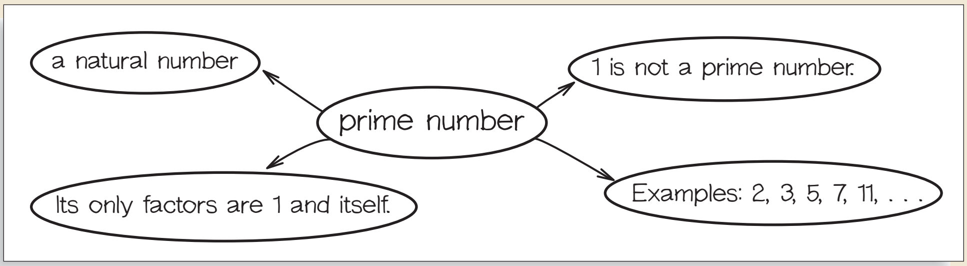
Skills Review Handbook

FACTORS AND MULTIPLES

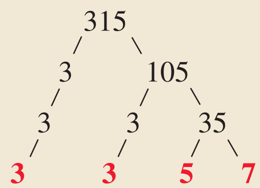
The **natural numbers** are all the numbers in the sequence 1, 2, 3, 4, 5, ... Natural numbers that are multiplied are **factors**. For example, **3** and **7** are factors of 21, because **3** • **7** = 21. A **prime number** is a natural number that has exactly two factors, itself and 1.



To write the **prime factorization** of a number, write the number as a product of prime numbers.

EXAMPLE Write the prime factorization of 315.

SOLUTION Use a tree diagram to factor the number until all factors are prime numbers. To determine the factors, test the prime numbers in order.



▶ The prime factorization of 315 is 3 • 3 • 5 • 7. This may also be written as $3^2 \cdot 5 \cdot 7$.

A **common factor** of two natural numbers is a number that is a factor of both numbers. For example, **7** is a common factor of 35 and 56, because 35 = 5 • **7** and 56 = 8 • **7**. The **greatest common factor** (GCF) of two natural numbers is the largest number that is a factor of both.

EXAMPLE Find the greatest common factor of 180 and 84.

SOLUTION First write the prime factorization of each number. Multiply the common prime factors to find the greatest common factor.

$180 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot 3 \cdot 5$

$84 = \mathbf{2} \cdot \mathbf{2} \cdot \mathbf{3} \cdot 7$

▶ The greatest common factor is **2** • **2** • **3** = 12.

A **common multiple** of two natural numbers is a number that is a multiple of both numbers. For example, 42 is a common multiple of 6 and 14, because $42 = 6 \cdot 7$ and $42 = 14 \cdot 3$. The **least common multiple** (LCM) of two natural numbers is the smallest number that is a multiple of both.

EXAMPLE Find the least common multiple of 24 and 30.

SOLUTION First write the prime factorization of each number.

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$30 = 2 \cdot 3 \cdot 5$

▶ The least common multiple is the product of the common prime factors and all the prime factors that are not common. The least common multiple of 24 and 30 is $2 \cdot 3 \cdot 2 \cdot 2 \cdot 5 = 120$.

PRACTICE

List all the factors of the number.

1. 18	**2.** 10	**3.** 77	**4.** 35
5. 27	**6.** 100	**7.** 42	**8.** 49
9. 52	**10.** 81	**11.** 121	**12.** 150

Write the prime factorization of the number if it is not a prime number. If a number is prime, write *prime*.

13. 27	**14.** 24	**15.** 32	**16.** 61
17. 55	**18.** 68	**19.** 48	**20.** 225
21. 90	**22.** 75	**23.** 39	**24.** 1000
25. 728	**26.** 101	**27.** 512	**28.** 210

List all the common factors of the pair of numbers.

29. 15, 30	**30.** 36, 54	**31.** 5, 20	**32.** 14, 21
33. 9, 36	**34.** 24, 28	**35.** 20, 55	**36.** 12, 30

Find the greatest common factor of the pair of numbers.

37. 25, 30	**38.** 32, 40	**39.** 17, 24	**40.** 35, 150
41. 14, 28	**42.** 65, 39	**43.** 102, 51	**44.** 128, 104
45. 36, 50	**46.** 45, 135	**47.** 29, 87	**48.** 35, 48
49. 56, 70	**50.** 88, 231	**51.** 47, 48	**52.** 93, 124

Find the least common multiple of the pair of numbers.

53. 5, 7	**54.** 3, 4	**55.** 3, 16	**56.** 7, 12	**57.** 4, 12
58. 9, 15	**59.** 12, 35	**60.** 6, 14	**61.** 20, 25	**62.** 10, 24
63. 3, 17	**64.** 15, 40	**65.** 70, 14	**66.** 36, 50	**67.** 22, 30

Comparing and Ordering Numbers

When you compare two numbers a and b, there are exactly three possibilities. The possibilities are described at the right in words and in symbols. To compare two whole numbers or decimals, compare the digits of the two numbers from left to right. Find the first place in which the digits are different.

a is *less than* b.	$a < b$
a is *equal to* b.	$a = b$
a is *greater than* b.	$a > b$

EXAMPLE Compare the two numbers. Write the answer using <, =, or >.

a. 4723 and 4732 **b.** 27.52 and 27.39

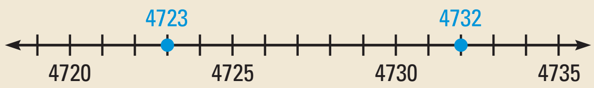
Solution

a. 4 7 **2** 3

4 7 **3** 2

▶ **2** < **3**, so 4723 < 4732.

You can picture this on a number line. The numbers on a number line increase from left to right.



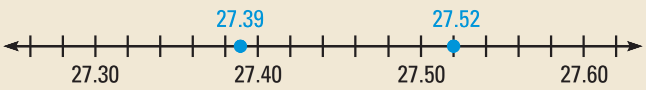
4723 is *less* than 4732.

4723 is to the *left* of 4732.

b. 2 7 . **5** 2

2 7 . **3** 9

▶ **5** > **3**, so 27.52 > 27.39.



27.52 is *greater* than 27.39.

27.52 is to the *right* of 27.39.

To compare two fractions that have the same denominator, compare the numerators. If the fractions have different denominators, first rewrite one or both fractions to produce equivalent fractions with a common denominator. The **least common denominator** (LCD) is the least common multiple of the denominators.

EXAMPLE Write the numbers $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{5}{12}$ in order from least to greatest.

Solution The LCD of the fractions is 24.

$$\frac{3}{4} = \frac{3 \cdot 6}{4 \cdot 6} = \frac{18}{24} \qquad \frac{7}{8} = \frac{7 \cdot 3}{8 \cdot 3} = \frac{21}{24} \qquad \frac{5}{12} = \frac{5 \cdot 2}{12 \cdot 2} = \frac{10}{24}$$

Compare the numerators: $10 < 18 < 21$, so $\frac{5}{12} < \frac{3}{4} < \frac{7}{8}$.

▶ In order from least to greatest, the fractions are $\frac{5}{12}$, $\frac{3}{4}$, and $\frac{7}{8}$.

EXAMPLE Compare $4\frac{3}{4}$ and $4\frac{2}{3}$. Write the answer using <, =, or >.

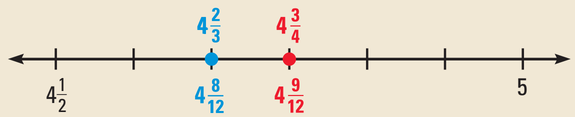
SOLUTION The whole number parts of the mixed numbers are the same, so compare the fraction parts.

The LCD of $\frac{3}{4}$ and $\frac{2}{3}$ is 12.

$\frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$ $\frac{2}{3} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$

Compare the numerators: $9 > 8$, so $\frac{9}{12} > \frac{8}{12}$.

▶ Since $\frac{3}{4} > \frac{2}{3}$, it follows that $4\frac{3}{4} > 4\frac{2}{3}$.



$4\frac{3}{4}$ is to the right of $4\frac{2}{3}$.

PRACTICE

Compare the two numbers. Write the answer using <, =, or >.

1. 12,428 and 15,116
2. 905 and 961
3. 142,109 and 140,999
4. 16.82 and 14.09
5. 0.00456 and 0.40506
6. 23.03 and 23.3
7. 1005.2 and 1050.7
8. 932,778 and 934,112
9. 0.058 and 0.102
10. $\frac{7}{13}$ and $\frac{3}{13}$
11. $17\frac{1}{4}$ and $15\frac{11}{12}$
12. $\frac{7}{10}$ and $\frac{3}{4}$
13. $\frac{5}{9}$ and $\frac{15}{27}$
14. $\frac{1}{2}$ and $\frac{3}{8}$
15. $\frac{1}{8}$ and $\frac{1}{9}$
16. $\frac{4}{5}$ and $\frac{2}{3}$
17. $42\frac{1}{5}$ and $41\frac{7}{8}$
18. 508.881 and 508.793
19. 32,227 and 32,226.5
20. $\frac{5}{8}$ and $\frac{2}{3}$
21. $17\frac{5}{6}$ and $17\frac{5}{7}$

Write the numbers in order from least to greatest.

22. 1207, 1702, 1220, 1772
23. 45,617, 45,242, 40,099, 40,071
24. 23.12, 23.5, 24.0, 23.08, 24.01
25. 9.027, 9.10, 9.003, 9.3, 9.27
26. 4.07, 4.5, 4.01, 4.22
27. $\frac{1}{3}, \frac{5}{6}, \frac{3}{8}, \frac{5}{4}$
28. $\frac{3}{5}, \frac{3}{2}, \frac{3}{4}, \frac{3}{10}, \frac{3}{7}$
29. $1\frac{2}{5}, \frac{7}{4}, \frac{5}{3}, 1\frac{1}{8}, \frac{15}{16}$
30. $14\frac{7}{9}, 15\frac{1}{3}, 14\frac{5}{6}, 15\frac{1}{4}$
31. $\frac{7}{8}, \frac{5}{4}, 1\frac{1}{3}, \frac{5}{12}$

32. You need a piece of trim $6\frac{5}{8}$ yards long to complete a craft project. You have a piece $6\frac{3}{4}$ yards long left over from another project. Is the trim long enough?

33. Goran finished a ski race in 53.56 seconds. Markus finished in 53.78 seconds. Which skier finished first?

FRACTION OPERATIONS

To add or subtract two fractions with the same denominator, add or subtract the numerators.

EXAMPLE

$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5}$ **Add numerators.**

$= \frac{7}{5}$, or $1\frac{2}{5}$ **Simplify.**

EXAMPLE

$\frac{7}{10} - \frac{2}{10} = \frac{7-2}{10}$ **Subtract numerators.**

$= \frac{5}{10}$ **Simplify.**

$= \frac{\cancel{5}}{2 \cdot \cancel{5}}$ **Factor numerator and denominator.**

$= \frac{1}{2}$ **Simplify fraction to lowest terms.**

To add or subtract two fractions with different denominators, write equivalent fractions with a common denominator.

EXAMPLE

$\frac{3}{5} + \frac{5}{6} = \frac{18}{30} + \frac{25}{30}$ **Use the LCD, 30.**

$= \frac{18+25}{30}$ **Add numerators.**

$= \frac{43}{30}$, or $1\frac{13}{30}$ **Simplify.**

To add or subtract mixed numbers, you can first rewrite them as fractions.

EXAMPLE

$3\frac{2}{3} - 2\frac{1}{4} = \frac{11}{3} - \frac{9}{4}$ **Rewrite mixed numbers as fractions.**

$= \frac{44}{12} - \frac{27}{12}$ **The LCD is 12.**

$= \frac{44-27}{12}$ **Subtract numerators.**

$= \frac{17}{12}$, or $1\frac{5}{12}$ **Simplify.**

To multiply two fractions, multiply the numerators and multiply the denominators.

EXAMPLE

$\frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6}$ **Multiply numerators and multiply denominators.**

$= \frac{15}{24}$ **Simplify.**

$= \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 8}$ **Factor numerator and denominator.**

$= \frac{5}{8}$ **Simplify fraction to lowest terms.**

Two numbers are **reciprocals** of each other if their product is 1. Every number except 0 has a reciprocal.

$5 \times \frac{1}{5} = 1$, so 5 and $\frac{1}{5}$ are reciprocals.

$\frac{2}{3} \times \frac{3}{2} = 1$, so $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals.

$1\frac{1}{4} \times \frac{4}{5} = \frac{5}{4} \cdot \frac{4}{5} = 1$, so $1\frac{1}{4}$ and $\frac{4}{5}$ are reciprocals.

To find the reciprocal of a number, write the number as a fraction. Then interchange the numerator and the denominator.

EXAMPLE Find the reciprocal of $3\frac{1}{4}$.

SOLUTION $3\frac{1}{4} = \frac{13}{4}$ Write $3\frac{1}{4}$ as a fraction.

$\frac{13}{4} \rightarrow \frac{4}{13}$ Interchange numerator and denominator.

▶ The reciprocal of $3\frac{1}{4}$ is $\frac{4}{13}$.

✓**CHECK** $3\frac{1}{4} \times \frac{4}{13} = \frac{13}{4} \times \frac{4}{13} = \frac{13 \times 4}{4 \times 13} = 1$

To divide by a fraction, multiply by its reciprocal.

EXAMPLE

$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5}$ The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

$= \frac{3 \times 6}{4 \times 5}$ Multiply numerators and denominators.

$= \frac{18}{20}$ Simplify.

$= \frac{\cancel{2} \cdot 9}{\cancel{2} \cdot 10}$ Factor numerator and denominator.

$= \frac{9}{10}$ Simplify fraction to lowest terms.

EXAMPLE

$2\frac{1}{2} \div 4\frac{1}{6} = \frac{5}{2} \div \frac{25}{6}$ Write mixed numbers as fractions.

$= \frac{5}{2} \times \frac{6}{25}$ The reciprocal of $\frac{25}{6}$ is $\frac{6}{25}$.

$= \frac{5 \times 6}{2 \times 25}$ Multiply numerators and denominators.

$= \frac{30}{50}$ Simplify.

$= \frac{3 \times \cancel{10}}{5 \times \cancel{10}}$ Factor numerators and denominators.

$= \frac{3}{5}$ Simplify fraction to lowest terms.

PRACTICE

Add or subtract. Write the answer as a fraction or a mixed number in lowest terms.

1. $\frac{1}{6}+\frac{4}{6}$ **2.** $\frac{5}{8}-\frac{3}{8}$ **3.** $\frac{4}{9}-\frac{1}{9}$ **4.** $\frac{5}{12}+\frac{3}{12}$

5. $\frac{1}{2}+\frac{1}{8}$ **6.** $\frac{3}{5}-\frac{1}{10}$ **7.** $\frac{7}{10}+\frac{1}{3}$ **8.** $\frac{15}{24}-\frac{7}{12}$

9. $5\frac{1}{8}-2\frac{3}{4}$ **10.** $1\frac{3}{7}+\frac{1}{2}$ **11.** $4\frac{3}{8}-2\frac{5}{6}$ **12.** $\frac{3}{7}+\frac{3}{4}$

13. $7\frac{1}{2}+\frac{7}{10}$ **14.** $5\frac{5}{9}-2\frac{1}{3}$ **15.** $4\frac{5}{8}-1\frac{3}{16}$ **16.** $9\frac{2}{5}+3\frac{1}{3}$

Find the reciprocal of each number.

17. 7 **18.** $\frac{1}{14}$ **19.** $\frac{7}{12}$ **20.** $\frac{5}{8}$

21. $\frac{1}{20}$ **22.** 100 **23.** $\frac{5}{13}$ **24.** $\frac{6}{7}$

25. $1\frac{1}{5}$ **26.** $2\frac{3}{5}$ **27.** $\frac{3}{9}$ **28.** $\frac{12}{17}$

29. $6\frac{2}{5}$ **30.** $10\frac{1}{3}$ **31.** $\frac{2}{7}$ **32.** $4\frac{3}{4}$

Multiply or divide. Write the answer as a fraction or a mixed number in lowest terms.

33. $\frac{1}{2}\times\frac{1}{2}$ **34.** $\frac{2}{3}\times\frac{4}{5}$ **35.** $\frac{5}{8}\times\frac{4}{15}$ **36.** $\frac{3}{7}\times\frac{7}{9}$

37. $\frac{3}{4}\times\frac{8}{9}$ **38.** $1\frac{2}{3}\times\frac{3}{5}$ **39.** $3\times 2\frac{5}{9}$ **40.** $5\frac{1}{4}\times 1\frac{1}{7}$

41. $\frac{7}{8}\div\frac{3}{4}$ **42.** $\frac{5}{12}\div\frac{1}{2}$ **43.** $\frac{4}{5}\div\frac{2}{3}$ **44.** $\frac{11}{16}\div 1\frac{1}{2}$

45. $4\frac{1}{2}\div\frac{3}{4}$ **46.** $2\frac{1}{4}\div 1\frac{1}{3}$ **47.** $3\frac{2}{5}\div 4$ **48.** $7\frac{1}{5}\div 2\frac{1}{4}$

Add, subtract, multiply, or divide. Write the answer as a fraction or a mixed number in lowest terms.

49. $\frac{15}{16}-\frac{1}{8}$ **50.** $\frac{5}{9}\times 1\frac{1}{2}$ **51.** $\frac{12}{13}\div\frac{12}{13}$ **52.** $\frac{24}{25}+\frac{1}{5}$

53. $5\frac{1}{2}-\frac{1}{8}$ **54.** $\frac{3}{10}\div\frac{1}{5}$ **55.** $\frac{7}{8}\times\frac{4}{9}$ **56.** $\frac{1}{3}+\frac{1}{6}$

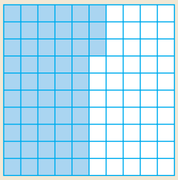
57. $4\frac{1}{4}\times\frac{2}{3}$ **58.** $9\frac{2}{5}+3\frac{1}{2}$ **59.** $\frac{4}{5}\div\frac{1}{2}$ **60.** $6\frac{5}{7}-2\frac{1}{5}$

61. $\frac{9}{10}+\frac{3}{8}$ **62.** $8\frac{1}{2}\times\frac{1}{4}$ **63.** $\frac{11}{15}\times\frac{3}{8}$ **64.** $\frac{4}{7}\div\frac{4}{5}$

65. You went trout fishing. You caught trout with the lengths (in inches) of $8\frac{5}{8}$, $10\frac{1}{2}$, $9\frac{1}{4}$, $8\frac{3}{4}$, $10\frac{2}{3}$, and $9\frac{5}{6}$. Find the average length of the trout you caught.

Fractions, Decimals, and Percents

Percent (%) means "divided by 100."



$53\% = 53 \text{ divided by } 100 = \frac{53}{100}$

The grid contains 100 squares.

53 of the 100 squares are shaded.

53% of the squares are shaded.

To write a percent as a decimal, move the decimal point two places to the *left* and remove the percent symbol.

EXAMPLES

a. $85\% = 85\% = 0.85$

b. $3\% = 03\% = 0.03$

c. $427\% = 427\% = 4.27$

d. $12.5\% = 12.5\% = 0.125$

To write a percent as a fraction in lowest terms, first write the percent as a fraction with a denominator of 100. Then simplify if possible.

EXAMPLES

a. $71\% = \frac{71}{100}$

b. $10\% = \frac{10}{100} = \frac{1}{10}$

c. $4\% = \frac{4}{100} = \frac{1}{25}$

d. $350\% = \frac{350}{100} = \frac{7}{2} = 3\frac{1}{2}$

To write a decimal as a percent, move the decimal point two places to the *right* and add a percent symbol.

EXAMPLES

a. $0.93 = 0.93 = 93\%$

b. $1.47 = 1.47 = 147\%$

c. $0.025 = 0.025 = 2.5\%$

d. $0.005 = 0.005 = 0.5\%$

To write a decimal as a fraction in lowest terms, first write the decimal as a fraction with a denominator of 100. Then simplify if possible.

EXAMPLES

a. $0.73 = \frac{73}{100}$

b. $0.25 = \frac{25}{100} = \frac{1}{4}$

c. $0.05 = \frac{5}{100} = \frac{1}{20}$

d. $2.75 = 2\frac{75}{100} = 2\frac{3}{4}$

It is simple to write a fraction as a percent if the denominator of the fraction is a factor of 100. If not, divide the numerator by the denominator.

EXAMPLES

a. $\frac{17}{25}$ → 25 is a factor of 100, so write $\frac{17}{25} = \frac{17 \cdot 4}{25 \cdot 4} = \frac{68}{100} = 68\%$.

b. $\frac{1}{8}$ → 8 is not a factor of 100, so divide: $1 \div 8 = 0.125 = 12.5\%$.

c. $\frac{1}{6}$ → 6 is not a factor of 100, so divide: $1 \div 6 = 0.1666\ldots \approx 0.167 = 16.7\%$.

You should memorize the percent, decimal, and fraction relationships in this chart.

Equivalent percents, decimals, and fractions		
$1\% = 0.01 = \frac{1}{100}$	$33\frac{1}{3}\% = 0.\overline{3} = \frac{1}{3}$	$66\frac{2}{3}\% = 0.\overline{6} = \frac{2}{3}$
$10\% = 0.1 = \frac{1}{10}$	$40\% = 0.4 = \frac{2}{5}$	$75\% = 0.75 = \frac{3}{4}$
$20\% = 0.2 = \frac{1}{5}$	$50\% = 0.5 = \frac{1}{2}$	$80\% = 0.8 = \frac{4}{5}$
$25\% = 0.25 = \frac{1}{4}$	$60\% = 0.6 = \frac{3}{5}$	$100\% = 1$

PRACTICE

Write the percent as a decimal and as a fraction or a mixed number in lowest terms.

1. 63% **2.** 7% **3.** 24% **4.** 35% **5.** 17%

6. 125% **7.** 45% **8.** 250% **9.** $33\frac{1}{3}\%$ **10.** 96%

11. 62.5% **12.** 725% **13.** 5.2% **14.** 0.8% **15.** 0.12%

Write the decimal as a percent and as a fraction or a mixed number in lowest terms.

16. 0.39 **17.** 0.08 **18.** 0.12 **19.** 1.5 **20.** 0.72

21. 0.05 **22.** 2.08 **23.** 4.8 **24.** 0.02 **25.** 3.75

26. 0.85 **27.** 0.52 **28.** 0.9 **29.** 0.005 **30.** 2.01

Write the fraction or mixed number as a decimal and as a percent. Round decimals to the nearest thousandth. Round percents to the nearest tenth of a percent.

31. $\frac{7}{10}$ **32.** $\frac{13}{20}$ **33.** $\frac{11}{25}$ **34.** $\frac{3}{10}$ **35.** $\frac{3}{8}$

36. $2\frac{3}{4}$ **37.** $5\frac{1}{8}$ **38.** $\frac{19}{20}$ **39.** $\frac{7}{8}$ **40.** $3\frac{7}{25}$

41. $\frac{5}{6}$ **42.** $3\frac{3}{5}$ **43.** $\frac{8}{15}$ **44.** $8\frac{1}{5}$ **45.** $1\frac{5}{12}$

Using Percent

A fraction shows what part one number is of another. You can first write a fraction to determine what percent one number is of another.

EXAMPLE What percent of 120 is 48?

SOLUTION First write a fraction that compares 48 to 120: $\frac{48}{120}$.

$\frac{48}{120} = 0.4$ **Write the fraction as a decimal.**

$= 40\%$ **Write the decimal as a percent.**

▶ 48 is 40% of 120.

To find a percent of a given number, first write the percent as a decimal or as a fraction. Then multiply.

EXAMPLE What is 75% of 160?

SOLUTION $75\% = \frac{3}{4}$ **Write 75% as a fraction.**

$\frac{3}{4} \times 160 = 120$ **Multiply.**

▶ 75% of 160 is 120.

EXAMPLE What is 6% of \$29.95?

SOLUTION $6\% = 0.06$ **Write 6% as a decimal.**

$0.06 \times \$29.95 = \$1.797$ **Multiply.**

$= \$1.80$ **Round to the nearest cent.**

▶ To the nearest cent, 6% of \$29.95 is \$1.80.

Practice

Find the answer.

1. What percent of 90 is 15?
2. 12 is what percent of 60?
3. What percent of 80 is 30?
4. What percent of 60 is 90?
5. What percent of 90 is 60?
6. 6 is what percent of 120?
7. 15 is what percent of 90?
8. What percent of 18 is 4.5?
9. 18 is what percent of 96?

Find the number. Round sums of money to the nearest cent.

10. 38% of 250
11. 12% of 75
12. 25% of 84
13. 50% of 96
14. 42% of 115
15. $33\frac{1}{3}\%$ of 114
16. 5.5% of \$102.95
17. 70% of 60
18. $66\frac{2}{3}\%$ of 48
19. 8% of \$12.99
20. 150% of 90
21. 4.5% of \$75
22. You are charged 6.5% tax on a \$42 purchase. Find the amount of the tax.

Ratio and Rate

A ratio compares two numbers using division. If a and b are two quantities measured in the same units, then the **ratio of a to b** is $\frac{a}{b}$. The ratio of a to b can be written in three ways: a to b, $a : b$, and $\frac{a}{b}$.

EXAMPLE Write the ratio 7 to 15 in three ways.

SOLUTION 7 to 15 $\quad$ 7 : 15 $\quad$ $\frac{7}{15}$

EXAMPLE Write the ratio 12 to 60 in lowest terms.

SOLUTION

$$12 \text{ to } 60 = \frac{12}{60} \qquad \text{First write the ratio as a fraction.}$$

$$= \frac{1}{5} \qquad \text{Then write the fraction in lowest terms.}$$

If a and b are two quantities measured in different units, then the **rate of a per b** is $\frac{a}{b}$. A **unit rate** is a rate per one unit of a given quantity. To determine a unit rate, write the rate with a denominator of 1.

EXAMPLE A car traveled 648 miles using 18 gallons of gas. Find the unit rate in miles per gallon.

SOLUTION $\frac{\text{miles}}{\text{gallons}} \rightarrow \frac{648}{18} = \frac{36}{1}$

▶ The unit rate is 36 miles per gallon.

Practice

Write each ratio in lowest terms.

1. 75 to 20
2. 162 : 36
3. $\frac{36}{60}$
4. 21 : 15
5. $\frac{60}{85}$
6. 42 to 72
7. 136 : 24
8. $\frac{216}{120}$
9. 24 cars to 36 cars
10. 39 students : 104 students
11. 75 hours to 35 hours
12. 8 hours : 2 hours
13. 9 inches to 36 inches
14. 48 hours to 10 hours

Find each unit rate.

15. \$90 for 4 tickets
16. \$51 for 6 hours
17. 208 miles in 4 hours
18. \$1050 in 6 weeks
19. 476 miles for 17 gallons
20. 128 ounces for 16 people
21. \$2.70 for 27 minutes
22. \$8.67 for 3 notebooks
23. 65 meters in 3 seconds
24. 18 hours in 4 days
25. \$476 for 425 miles
26. \$50 for 5 hours

Counting Methods

Suppose your family's favorite pizza toppings are hamburger, sausage, pepperoni, mushroom, onion, and pepper. You have to choose one meat topping and one vegetable topping. How many choices are possible?

There are many ways to answer that question, including **making a list**, drawing a **tree diagram**, and using the **counting principle**.

EXAMPLE Make a list to determine the number of topping choices.

SOLUTION Pair each meat topping with each vegetable topping.

hamburger, mushroom	**hamburger**, onion	**hamburger**, pepper
sausage, mushroom	**sausage**, onion	**sausage**, pepper
pepperoni, mushroom	**pepperoni**, onion	**pepperoni**, pepper

▶ There are nine possible choices.

EXAMPLE Use a tree diagram to determine the number of topping choices.

SOLUTION Draw a tree diagram and count the number of branches.

MEAT	VEGETABLE
hamburger	mushroom
	onion
	pepper
sausage	mushroom
	onion
	pepper
pepperoni	mushroom
	onion
	pepper

▶ The tree has nine branches. There are nine possible choices.

THE COUNTING PRINCIPLE When one item is selected from each of two or more lists, the total number of possible combinations is the product of the number of items in each list.

EXAMPLE Use the counting principle to determine the number of topping choices.

SOLUTION Multiply the number of meat choices by the number of vegetable choices.

3 meat choices $\times$ **3** vegetable choices $=$ **9** possible topping choices

▶ There are nine possible choices.

In situations involving a greater number of items, it may not be practical to make a list or to draw a tree diagram. Use the counting principle instead.

EXAMPLE One state plans to issue a series of license plates using codes consisting of three letters followed by two digits. Letters and digits may both be repeated. How many different plates are possible?

SOLUTION Use the Counting Principle.

26 choices for first letter	×	26 choices for second letter	×	26 choices for third letter	×	10 choices for first digit	×	10 choices for second digit

$26 \times 26 \times 26 \times 10 \times 10 = 1{,}757{,}600$

▶ There are 1,757,600 different license plates possible.

PRACTICE

Use the indicated method to answer each question.

1. Jacob is choosing an outfit from three pairs of jeans and three shirts. The jeans are blue, black, and gray. The shirts are white, blue, and yellow. How many such outfits are possible? (Make a list.)
2. The student council is selling sweatshirts with the school name and emblem. The shirts come in the colors, styles, and sizes shown at the right. How many different types of sweatshirt are available? (Make a tree diagram.)
3. There are 270 girls and 210 boys enrolled in the Jefferson School. The cover of the yearbook will show a photograph of a pair of students, one girl and one boy. How many different pairs of students are possible? (Use the counting principle.)

Sweatshirt Choices

Colors: white, gray, black

Styles: hooded, plain

Sizes: small, medium, large

Ex. 2

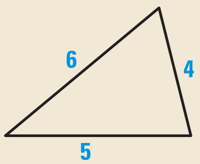
Answer each question using any method you choose.

4. A student can choose one sandwich, one salad, and one piece of fruit in the cafeteria. The choices include the following: grilled cheese sandwich, hamburger, tuna sandwich, garden salad, fruit salad, apple, banana, and orange. How many different lunches are possible?
5. In a given calling area, telephone numbers have the same three first digits. The four-digit number that follows is different for each telephone number. How many numbers are possible?
6. A radio host plans to play one song from each of three CDs by a single artist. The first CD includes 12 songs. The second includes 14 songs, and the third includes 13 songs. How many different combinations are possible?
7. Members of a video club choose a personal identification number (PIN) consisting of two letters followed by two digits. How many PINs are possible if letters and digits may be repeated? How many are possible if neither letters nor digits may be repeated?

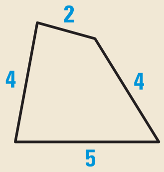
Perimeter, Area, and Volume

The **perimeter** P of a figure is the distance around it.

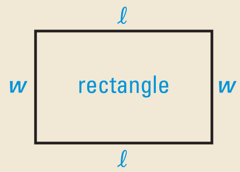
EXAMPLES



$P = 6 + 5 + 4$
$= 15$



$P = 4 + 2 + 4 + 5$
$= 15$



$P = l + w + l + w$
$= 2l + 2w$

EXAMPLE Find the perimeter of a rectangle with length 14 centimeters and width 6 centimeters.

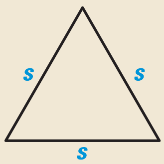
SOLUTION $P = 2l + 2w = (2 \times 14) + (2 \times 6) = 28 + 12 = 40$

▶ The perimeter is 40 centimeters.

A **regular polygon** is a polygon in which all the angles have the same measure and all the sides have the same length. The perimeter of a regular polygon can be found by multiplying the length of a side by the number of sides.

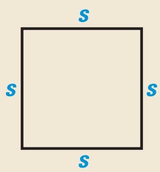
EXAMPLES

regular (equilateral) triangle



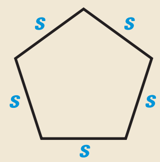
$P = 3s$

square



$P = 4s$

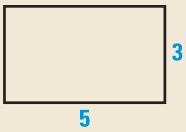
regular pentagon



$P = 5s$

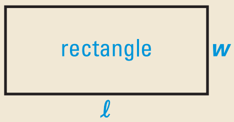
The **area** A of a figure is the number of square units enclosed by the figure.

EXAMPLES



$A = 5 \times 3$

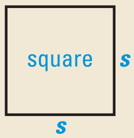
$= 15$ square units



$A = \text{length} \times \text{width}$

$= l \times w$

$= lw$



$A = \text{side} \times \text{side}$

$= s \times s$

$= s^2$

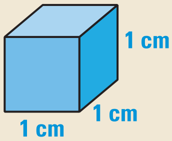
EXAMPLE Find the area of a square with sides 15 inches long.

SOLUTION $A = s^2 = 15^2 = 225$

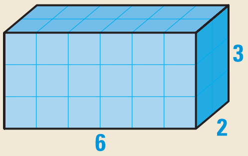
▶ The area is 225 square inches.

The **volume** of a space figure is the amount of space enclosed by the figure. Volume is measured in cubic units.

One such unit is the cubic centimeter (cm^3). It is the amount of space enclosed by a cube whose length, width, and height are each 1 cm.

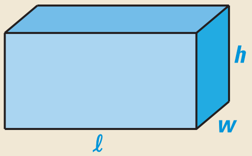


EXAMPLES



$V = 6 \times 2 \times 3$

$= 36$ cubic units



$V = l \times w \times h$

EXAMPLE Find the volume of a box with length 8 feet, width 5 feet, and height 9 feet.

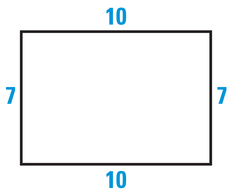
SOLUTION $V = l \times w \times h = 8 \times 5 \times 9 = 360$

▶ The volume is 360 cubic feet (ft^3).

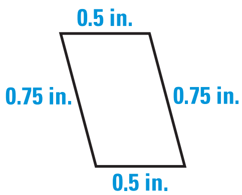
PRACTICE

Find the perimeter of each figure.

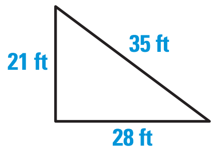
1.



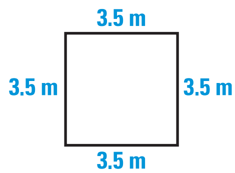
2.



3.



4.



5. a square with sides of length 18 ft

6. a rectangle with length 6 m and width 7 m

Find the area of each figure.

7. a square with sides of length 29 yd

8. a rectangle with length 7 km and width 4 km

9. a square with sides of length 3.5 in.

10. a rectangle with length 24 ft and width 6 ft

11. a square with sides of length 7.2 cm

12. a rectangle with length 7.5 in. and width 8 in.

13. a square with sides of length 45 km

14. a rectangle with length 5.3 m and width 4 m

Find the volume of each box.

15. a cube with sides of length 25 ft

16. a cube with sides of length 4.2 cm

17. a box with length 15 yd, width 7 yd, and height 4 yd

18. a box with length 7.3 cm, width 5 cm, and height 3.2 cm

19. a box with length 5.3 in., width 4 in., and height 10 in.

DATA DISPLAYS

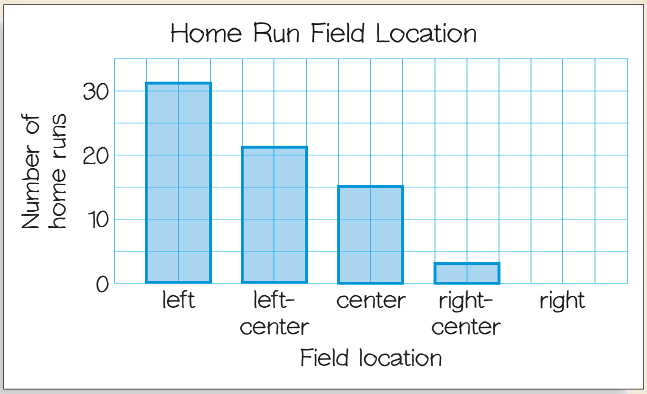
A **bar graph** can be used to display data that fall into distinct categories. The bars in a bar graph are the same width. The height or length of each bar is determined by the data it represents and by the scale you choose.

EXAMPLE In 1998, baseball player Mark McGwire hit a record 70 home runs. The table shows the locations to which the home runs were hit. Draw a bar graph to display the data. ▶ Source: *The Boston Globe*

Field location	Number of home runs
left	31
left-center	21
center	15
right-center	3
right	0

SOLUTION

1. First, choose a scale. Since the data range from 0 to 31, make the scale increase from 0 to 35 by fives.
2. Draw and label the axes. Mark intervals on the vertical axis according to the scale you chose.
3. Draw a bar for each category.
4. Give the bar graph a title.



A **histogram** is a bar graph that shows how many data items occur within given intervals. The number of data items in an interval is the **frequency** for that interval.

EXAMPLE The table shows the distances of McGwire's home runs. Draw a histogram to display them.

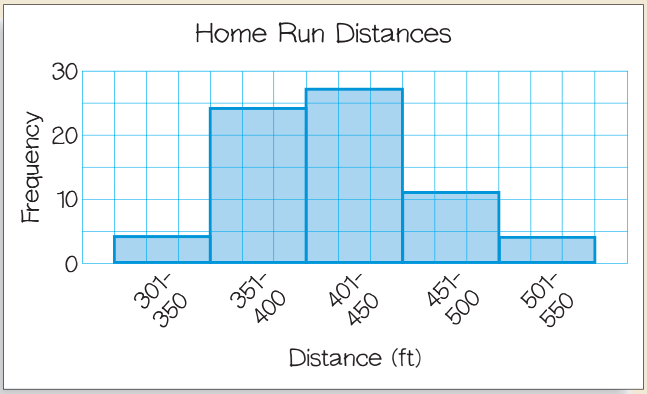
Distance (ft)	Frequency
301–350	4
351–400	24
401–450	27
451–500	11
501–550	4

SOLUTION Use the same method you used for drawing the bar graph above. However, do not leave spaces between the bars.

Since the frequencies range from 4 to 27, make the scale increase from 0 to 30 by fives.

Draw and label the axes. Mark intervals on the vertical axis and draw a bar for each category. Do not leave spaces between the bars.

Give the histogram a title.



A **line graph** can be used to show how data change over time.

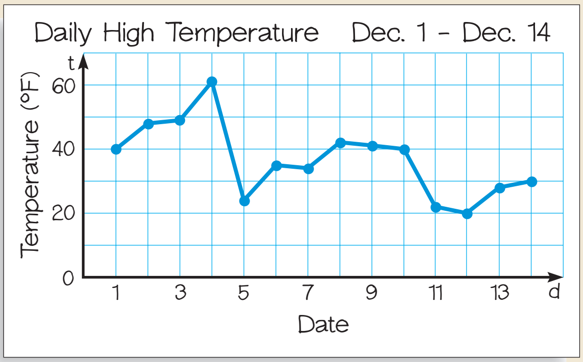
EXAMPLE A science class recorded the highest temperature each day from December 1 to December 14. The temperatures are given in the table. Draw a line graph to display the data.

Date	1	2	3	4	5	6	7
Temperature (°F)	40	48	49	61	24	35	34

Date	8	9	10	11	12	13	14
Temperature (°F)	42	41	40	22	20	28	30

SOLUTION

1. Choose a scale.
2. Draw and label the axes. Mark evenly spaced intervals on both axes.
3. Graph each data item as a point. Connect the points.
4. Give the line graph a title.



A **circle graph** can be used to show how parts relate to a whole and to each other.

EXAMPLE The table shows the number of sports-related injuries treated in a hospital emergency room in one year. Draw a circle graph to display the data.

Related sport	Number of injuries
basketball	56
football	34
skating/hockey	22
track and field	10
other	28

SOLUTION

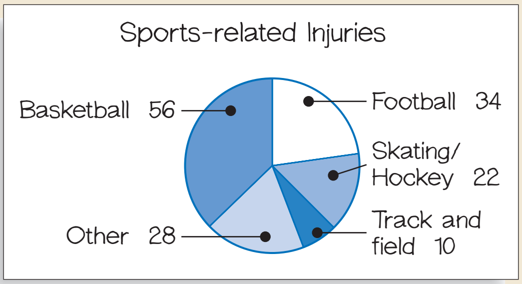
1. To begin, find the total number of injuries.

   $56 + 34 + 22 + 10 + 28 = \mathbf{150}$

   To find the degree measure for each sector of the circle, write a fraction comparing the number of injuries to the total. Then multiply the fraction by 360°. For example:

   Football: $\frac{34}{\mathbf{150}} \cdot 360^\circ = 81.6^\circ$

2. Next, draw a circle. Use a protractor to draw the angle for each sector.
3. Label each sector.
4. Give the circle graph a title.



PRACTICE

In 1998, baseball player Sammy Sosa hit 66 home runs. The tables show the field locations and distances of his home runs. ▶ Source: *The Boston Globe*

1. The location data range from 10 to 22. The scale must start at 0. Choose a reasonable scale for a bar graph.

2. Draw a bar graph to display the field locations of Sosa's home runs.

3. The distance data range from 1 to 16. The scale must start at 0. Choose a reasonable scale for a histogram.

4. Draw a histogram to display the distances of Sosa's home runs.

Field location	Number of home runs
left	12
left-center	22
center	10
right-center	11
right	11

Exs. 1 and 2

Distance (ft)	Frequency
326–350	5
351–375	12
376–400	14
401–425	16
426–450	14
451–475	1
476–500	4

Exs. 3 and 4

5. There are 150 runs at the Mountain Mania ski resort, including 51 expert runs, 60 intermediate runs, and 39 beginner runs. Draw a circle graph to display the data.

6. A nurse recorded a patient's temperature (°F) every 3 hours from 9 A.M. until noon of the following day. The temperatures were 102°, 102°, 101.5°, 101.1°, 100°, 101°, 101.5°, 100°, 99.8°, and 99°. Draw a line graph to display the data.

Choose an appropriate graph to display the data. Draw the graph.

7.

Value of one share of Company Stock						
Year	1994	1995	1996	1997	1998	1999
Value ($)	15	18	16	12	10	15

8.

Passenger Car Stopping Distance (dry road)				
Speed (mi/h)	35	45	55	65
Distance (ft)	160	225	310	410

9.

Fat in One Tablespoon of Canola Oil	
Type of fat	Number of Grams
saturated	1
polyunsaturated	4
monounsaturated	8

▶ Source: U.S. Department of Agriculture

10.

Population of Meridian City by Age					
Age	Under 5	5–19	20–44	45–64	65 and older
Population	912	2556	4812	2232	1502

PROBLEM SOLVING

One of your primary goals in mathematics should be to become a good problem solver. It will help to approach every problem with an organized plan.

STEP 1 **UNDERSTAND THE PROBLEM.**
Read the problem carefully. Organize the information you are given and decide what you need to find. Determine whether some of the information given is unnecessary, or whether enough information is given. Supply missing facts, if possible.

STEP 2 **MAKE A PLAN TO SOLVE THE PROBLEM.**
Choose a strategy. (Get ideas from the list given on page 796.) Choose the correct operations. Decide if you will use a tool such as a calculator, a graph, or a spreadsheet.

STEP 3 **CARRY OUT THE PLAN TO SOLVE THE PROBLEM.**
Use the strategy and any tools you have chosen. Estimate before you calculate, if possible. Do any calculations that are needed. Answer the question that the problem asks.

STEP 4 **CHECK TO SEE IF YOUR ANSWER IS REASONABLE.**
Reread the problem and see if your answer agrees with the given information.

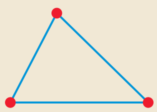
EXAMPLE How many segments can be drawn between 7 points, no three of which lie on the same line?

1. You are given a number of points, along with the information that no three points lie on the same line. You need to determine how many segments can be drawn between the points.

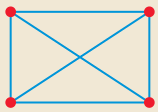
2. Some strategies to consider are: draw a diagram, solve a simpler problem and look for a pattern.

3. Consider the problem for fewer points.

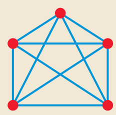
2 points
1 segment



3 points
3 segments



4 points
6 segments



5 points
10 segments

Look for a pattern. Then continue the pattern to find the number of segments for 7 points.

Number of points	2	3	4	5	6	7
Number of segments	1	3	6	10	15	21

+2 +3 +4 +5 +6

▶ Given 7 points, no three of which lie on the same line, 21 segments can be drawn between the points.

4. You can check your solution by making a sketch.

In **Step 2** of the problem solving plan, you may want to consider the following strategies.

PROBLEM SOLVING STRATEGIES

• **Guess, check, and revise.**	When you do not seem to have enough information.
• **Draw a diagram or a graph.**	When words describe a picture.
• **Make a table or an organized list.**	When you have data or choices to organize.
• **Use an equation or a formula.**	When you know a relationship between quantities.
• **Use a proportion.**	When you know that two ratios are equal.
• **Look for a pattern.**	When you can examine several cases.
• **Break the problem into simpler parts.**	When you have a multi-step problem.
• **Solve a simpler problem.**	When easier numbers help you understand a problem.
• **Work backward.**	When you are looking for a fact leading to a known result.

PRACTICE

1. Tasha bought salads at \$2.75 each and cartons of milk at \$.80 each. The total cost was \$16.15. How many of each did Tasha buy?
2. A rectangular garden is 45 feet long and has perimeter 150 feet. Rows of plants are planted 3 feet apart. Find the area of the garden.
3. If five turkey club sandwiches cost \$18.75, how much would seven sandwiches cost?
4. How many diagonals can be drawn from one vertex of a 12-sided polygon?
5. Nguyen wants to arrive at school no later than 7:25 A.M. for his first class. It takes him 25 minutes to shower and dress, 15 minutes to eat breakfast, and at least 20 minutes to get to school. What time should he plan to get out of bed?
6. There are 32 players in a single-elimination chess tournament. That is, a player who loses once is eliminated. Assuming that no ties are allowed, how many games must be played to determine a champion?
7. Andrea, Betty, Joyce, Karen, and Paula are starters on their school basketball team. How many different groups of three can be chosen for a newspaper photo?
8. Carl has \$135 in the bank and plans to save \$5 per week. Jean has \$90 in the bank and plans to save \$10 per week. How many weeks will it be before Jean has at least as much in the bank as Carl?
9. The Peznolas are planning to use square tiles to tile a kitchen floor that is 18 feet long and 15 feet wide. Each tile covers one square foot. A carton of tiles costs \$18. How much will it cost to cover the entire kitchen floor?