EXPLORING DATA AND STATISTICS

9.8 Comparing Linear, Exponential, and Quadratic Models

What *you should learn*

GOAL 1 Choose a model that best fits a collection of data.

GOAL 2 Use models in **real-life** settings, such as the stretch of a spring in **Ex. 20**.

Why *you should learn it*

▼ To select the best model for **real-life** data, such as the increase in volume of the chambers of a nautilus in **Example 2**.

REAL LIFE

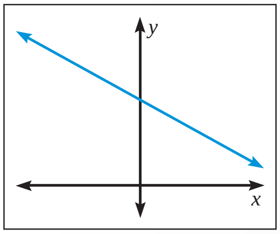
Chambered Nautilus

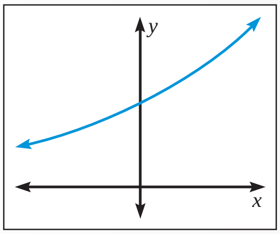
GOAL 1 CHOOSING A MODEL

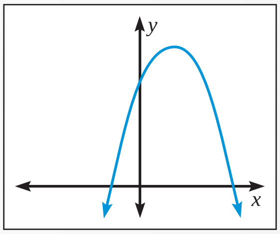
This lesson will help you choose the type of model that best fits a collection of data.

CONCEPT SUMMARY — THREE BASIC MODELS

LINEAR MODEL	EXPONENTIAL MODEL	QUADRATIC MODEL
$y = mx + b$	$y = C(1 \pm r)^t$	$y = ax^2 + bx + c$







EXAMPLE 1 *Choosing a Model*

Name the type of model that best fits each data collection.

a. $(-3, 4), \left(-2, \frac{7}{2}\right), (-1, 3), \left(0, \frac{5}{2}\right), (1, 2), \left(2, \frac{3}{2}\right), (3, 1)$

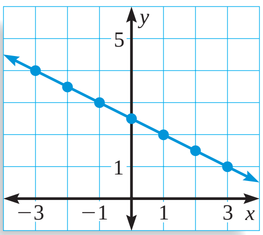
b. $(-3, 4), (-2, 2), (-1, 1), \left(0, \frac{1}{2}\right), \left(1, \frac{1}{4}\right), \left(2, \frac{1}{8}\right), \left(3, \frac{1}{16}\right)$

c. $(-3, 4), \left(-2, \frac{7}{3}\right), \left(-1, \frac{4}{3}\right), (0, 1), \left(1, \frac{4}{3}\right), \left(2, \frac{7}{3}\right), (3, 4)$

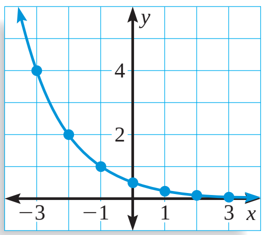
SOLUTION

Make scatter plots of the data. Then decide whether the points appear to lie on a line (linear model), an exponential curve (exponential model), or a parabola (quadratic model).

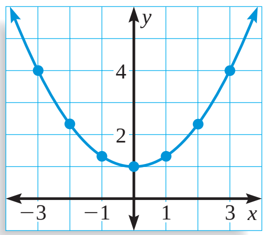
a. Linear Model



b. Exponential Model



c. Quadratic Model



FOCUS ON APPLICATIONS

CHAMBERED NAUTILUS is a marine animal whose soft body is covered with a shell. The nautilus lives in the outermost chamber. Shown here is a cross section of the shell.

GOAL 2 USING MODELS IN REAL-LIFE SETTINGS

EXAMPLE 2 *Writing a Model*

NAUTILUS As a chambered nautilus grows, its chambers get larger and larger. The relationship between the volume and consecutive chambers usually follows the same pattern.

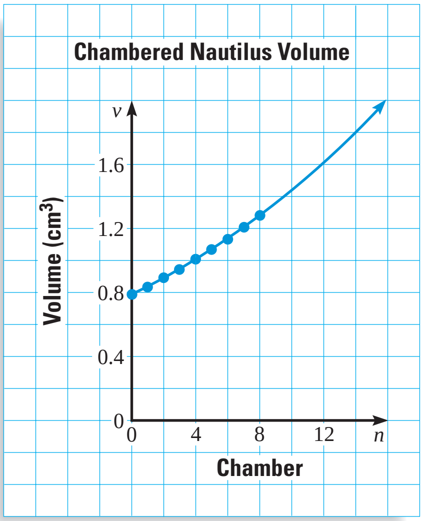
The volumes v (in cubic centimeters) of nine consecutive chambers of a chambered nautilus are given in the table. The chambers are numbered from 0 to 8, with 0 being the first and smallest chamber.

Decide which type of model best fits the data. Write a model.

Chamber, n	Volume (cm^3), v
0	0.787
1	0.837
2	0.889
3	0.945
4	1.005
5	1.068
6	1.135
7	1.207
8	1.283

SOLUTION

Draw a scatter plot of the data. You can see that the graph has a slight curve.



Test whether an exponential model fits the data by finding the ratios of consecutive volumes of the chambers.

$$\frac{\text{Volume of Chamber 1}}{\text{Volume of Chamber 0}} = \frac{0.837}{0.787} \approx 1.064$$

$$\frac{\text{Volume of Chamber 2}}{\text{Volume of Chamber 1}} = \frac{0.889}{0.837} \approx 1.062$$

$$\frac{\text{Volume of Chamber 3}}{\text{Volume of Chamber 2}} = \frac{0.945}{0.889} \approx 1.063$$

STUDENT HELP

Look Back
For help writing an exponential growth model, see p. 477.

The ratios show that each chamber's volume is about 0.063, or 6.3%, larger than the volume of the chamber just before it. Assuming this pattern applies to all data points, it is appropriate to use an exponential growth model $v = C(1 + r)^n$.

The rate of growth r is 0.063 and the initial volume in chamber 0 is 0.787.

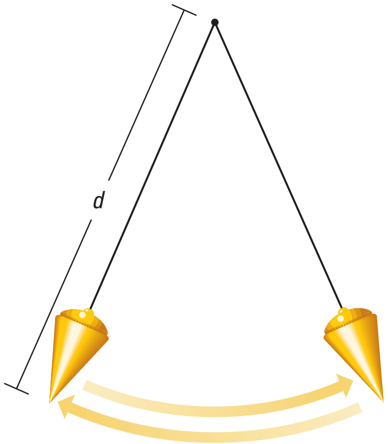
✓**CHECK** Check values of n in the exponential model $v = 0.787(1.063)^n$.

$v = 0.787(1.063)^n$	$v = 0.787(1.063)^n$
$v = 0.787(1.063)^3$	$v = 0.787(1.063)^8$
$v = 0.945$	$v = 1.283$

The exponential model $v = 0.787(1.063)^n$ fits the data.

EXAMPLE 3 *Writing a Model*

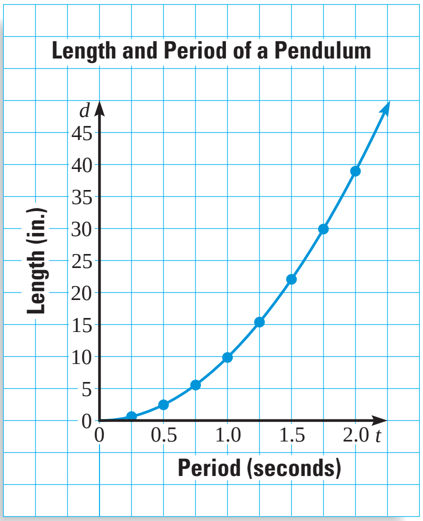
PENDULUMS A simple pendulum can be constructed with a string hung from a fixed point and a weight attached at the other end. The time it takes for the pendulum to swing from one side to the other and all the way back again is called its *period*. The period t (in seconds) of the pendulum is related to its length d (in inches). The table gives values of t and d for several different pendulums. Decide which type of model best fits the data. Write a model.



Period (sec), t	Length (in.), d
0.25	0.61
0.50	2.45
0.75	5.50
1.00	9.78
1.25	15.28
1.50	22.00
1.75	29.95
2.00	39.12

SOLUTION

Draw a scatter plot for the data. From the scatter plot, you can see that the pattern is not linear.



You can test whether an exponential model fits by finding the ratios of consecutive pendulum lengths. Since the lengths do not increase by the same percent, an exponential model does not fit the data.

Test whether a quadratic model fits. Write the simple quadratic model $d = at^2$. To find a, substitute any known values of d and t.

$d = at^2$	**Write quadratic model.**
$9.78 = a(1^2)$	**Substitute 9.78 for *d* and 1 for *t*.**
$9.78 = a$	**Solve for *a*.**
$d = 9.78t^2$	**Substitute *a* in the equation.**

✓**CHECK** Check values of t in the quadratic model $d = 9.78t^2$

$d = 9.78t^2$	$d = 9.78t^2$
$d = 9.78(0.25)^2$	$d = 9.78(2.0)^2$
$d = 0.61$	$d = 39.12$

▶ The quadratic model $d = 9.78t^2$ fits the data.

FOCUS ON APPLICATIONS

REAL LIFE **PENDULUM CLOCKS** One of the first clocks to use a pendulum was invented by the Dutch physicist, astronomer, and mathematician, Christiaan Huygens in the 1650s.

GUIDED PRACTICE

Vocabulary Check ✓

1. Name, sketch, and write an equation for each of the three types of algebraic models you have studied.

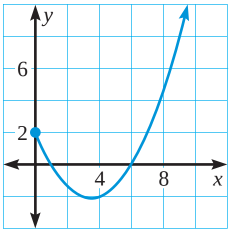
Concept Check ✓

2. Describe the steps you would follow to decide which of the three basic models best fits a collection of data.

Skill Check ✓

Name the type of model suggested by the graph.

3.



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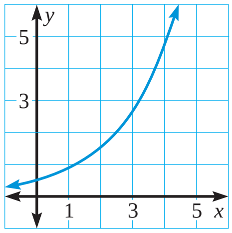

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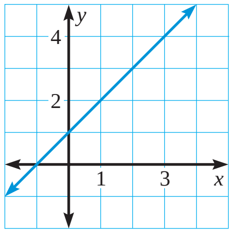


Use the data: (0, 1), (1, 1.25), (2, 2), (3, 3.25), (4, 5), (5, 7.25).

6. Draw a scatter plot of the data.

7. Decide which type of model best fits the data. Explain your reasoning.

8. Write a model that fits the data.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

CHOOSING MODELS **Make a scatter plot of the data. Then name the type of model that best fits the data.**

9. $(-1, -6), (-3, 4), (2, 9), (-2, -3), (0, -5), (1, 0)$

10. $(0, 3), (8, 3), (-4, -1), (4, 4), (-6, -3), (10, 1)$

11. $(1, 3), (2.5, 16.5), (0.5, 1.5), (-2, 0.1), (0, 1), (1.5, 5)$

12. $(-2, -1), (-1, -2.5), (0, -3), (1, -2.5), (2, -1), (3, 1.5)$

13. $(-3, 2), \left(-2, \frac{5}{2}\right), \left(-1, \frac{7}{2}\right), (0, 5), (1, 7), \left(2, \frac{19}{2}\right)$

14. $(-2, 2), \left(-1, \frac{5}{2}\right), (0, 3), \left(1, \frac{7}{2}\right), (2, 4), \left(3, \frac{9}{2}\right)$

15.

x	y
5	4
0	−6
7	−6
−1	−14
6	0
3	6

16.

x	y
−1	8
1	2
−2	16
3	0.5
0	4
2	1

17.

x	y
0	5
2	−3
−2	13
1	1
4	−11
3	−7

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 9–17
Example 2: Exs. 18–20
Example 3: Exs. 18–20

18. **GEOMETRY CONNECTION** The surface area S of a sphere with radius r is shown in the table. Write a model that best fits the data.

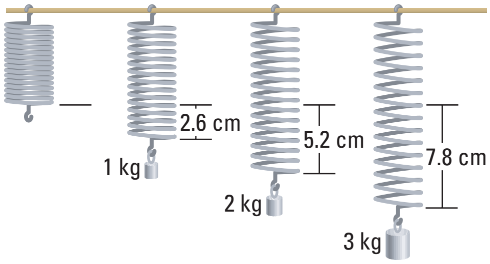
Radius, r	1	2	3	4	5
Surface area, S	4π	16π	36π	64π	100π

19. **BODY MASS INDEX** Body mass index is a measure of weight in relation to height. The table shows the body mass index B of a person who is 152 centimeters tall and weighs w kilograms. Which type of model best fits the data? Write a model.

Weight, w	45.40	49.94	54.48	59.02	63.56	68.10	72.64
Body mass index, B	19.55	21.50	23.46	25.41	27.37	29.32	31.28

▶ Source: Reuters Health Information

20. **SCIENCE CONNECTION** Different masses M (in kilograms) are hung from a spring. The distances d (in centimeters) that the spring stretches are shown in the table. Test different models to see which type of model best fits the data. Write a model that accurately represents the data.



Mass, M	1	2	3	4	5	6	7
Distance, d	2.6	5.2	7.8	10.4	13	15.6	18.2

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 21 and 22.

NHL ATTENDANCE **In Exercises 21 and 22, use the following information.** The regular season attendance A (in thousands) of National Hockey League games during a recent five-year period can be modeled by

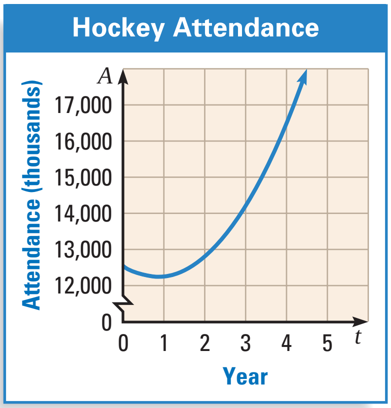
$$A = 341.93t^2 - 583.61t + 12{,}580.66$$

where t is the number of years since the beginning of the five-year period.

DATA UPDATE of National Hockey League data at www.mcdougallittell.com

21. Copy and complete the table.

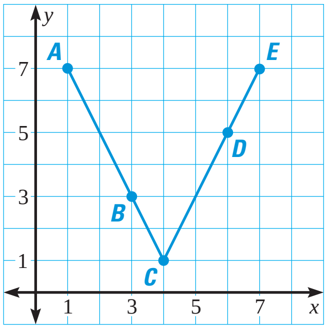
Year t	Attendance A	Change in A from previous year
0	12,580,660	———
1	?	?
2	?	?
3	?	?
4	?	?



22. Describe the pattern in the change of attendance.

EXTENSION: ABSOLUTE-VALUE EQUATIONS

In Exercises 23–26, use the V-shaped graph. It is the graph of an absolute-value equation of the form $y = a|x - b| + c$.



23. Write an equation of the line passing through the points A and B. What is its slope?

24. Write an equation of the line passing through the points D and E. What is its slope?

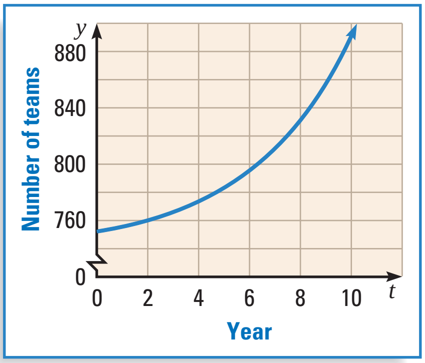
25. The intersection of the two lines in Exercises 23 and 24 is the vertex of the graph. Find the intersection of the two lines.

26. Find values of a, b, and c such that the equation of the graph is of the form $y = a|x - b| + c$, where a is the slope of the line to the right of the vertex and (b, c) is the vertex.

Test Preparation

27. MULTIPLE CHOICE The graph shows the number of NCAA men's college basketball teams during a recent ten-year period, where t is the number of years since the beginning of the period. Which type of model best fits the data?

▶ Source: National College Athletic Association



Ⓐ Exponential   Ⓑ Quadratic

Ⓒ Linear   Ⓓ None

28. MULTIPLE CHOICE In 1970 the population A of Arizona was 1,775,000. Since then, the average annual percent of increase has been about 3.41%. Which model best fits the situation where t is the number of years since 1970?

▶ Source: U.S. Bureau of the Census

Ⓐ $A = 3.41t + 1{,}775{,}000$ Ⓑ $A = 60{,}527.5t + 1{,}775{,}000$

Ⓒ $A = 1{,}835{,}527.5t^2$ Ⓓ $A = 1{,}775{,}000(1.0341)^t$

29. MULTIPLE CHOICE The area of an ellipse whose semi-minor and semi-major axes are r and $r + 1$ respectively is given below. Which model best fits the data?

Radius, r	1	2	3	4	5	6
Area A	2π	6π	12π	20π	30π	42π

Ⓐ $A = \pi r + 2$ Ⓑ $A = \pi r(r + 1)$ Ⓒ $A = \pi(1 + r)^t$ Ⓓ $A = r^2 + \pi r + 2$

★ Challenge

CHOOSING A MODEL Use a graphing calculator to graph the points. Which type of model best fits the data?

30. $(-3, 4), \left(-2, \frac{7}{2}\right), (-1, 3), \left(0, \frac{5}{2}\right), (1, 2), \left(2, \frac{3}{2}\right), (3, 1)$

31. $(-3, 4), (-2, 2), (-1, 1), \left(0, \frac{1}{2}\right), \left(1, \frac{1}{4}\right), \left(2, \frac{1}{8}\right), \left(3, \frac{1}{16}\right)$

32. $(-3, 4), \left(-2, \frac{7}{3}\right), \left(-1, \frac{4}{3}\right), (0, 1), \left(1, \frac{4}{3}\right), \left(2, \frac{7}{3}\right), (3, 4)$

EXTRA CHALLENGE
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MIXED REVIEW

FINDING TERMS **List the terms of the expression.** (Review 2.3)

33. $-3 + x$ **34.** $a - 5$ **35.** $-5 - 8x$ **36.** $-4r + s - 1$

SIMPLIFYING EXPRESSIONS **Simplify the variable expression.** (Review 2.5 for 10.1)

37. $-(-5)(y)(-y)$ **38.** $(-3)^3(x)(x)$ **39.** $(-2)(-r)(-r)(-r)$

40. $(-n)(-n)(-4)$ **41.** $-(y)^3$ **42.** $(-1)(-x)^4$

SOLVING EQUATIONS **Solve the equation. Round the result to two decimal places.** (Review 3.6)

43. $15.67x + 23.61 = 1.56 + 45.8x$ **44.** $17.87 - 2.87x = 1.87 - 4.92x$

45. $6.35x - 9.94 = 3.88 + 40.34x$ **46.** $5.6(1.2 + 1.9x) = 20.4x + 6.8$

WRITING EQUATIONS **Write an equation of the line that passes through the two points.** (Review 5.3)

47. $(3, -2), (5, 4)$ **48.** $(-3, -9), (5, 7)$ **49.** $(2, 3), (-4, 6)$

QUIZ 3

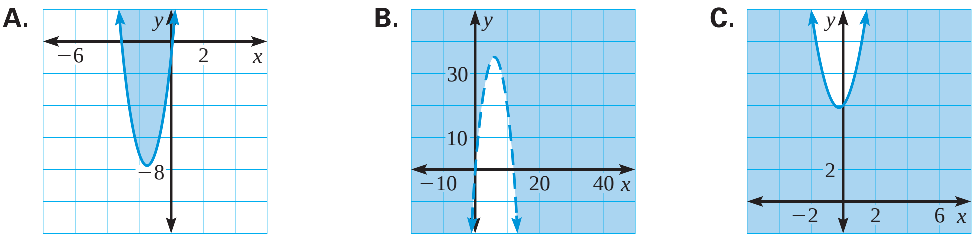
Self-Test for Lessons 9.7 and 9.8

Decide whether the ordered pair is a solution of the inequality. (Lesson 9.7)

1. $y \geq 2x^2 - x + 9, (2, 10)$ **2.** $y > 4x^2 - 64x + 115, (12, -60)$

3. $y < x^2 + 6x + 12, (-1, 6)$ **4.** $y \leq x^2 - 7x + 9, (-1, 16)$

Match the inequality with its graph. (Lesson 9.7)



5. $y > -x^2 + 12x - 1$ **6.** $y \leq 2x^2 + x + 6$ **7.** $y \geq 3x^2 + 9x - 1$

Graph the equation. Name the model it represents. (Lesson 9.8)

8. $y = 3x - 6$ **9.** $y = 0.5(1.4)^x$ **10.** $y = 2x^2 + 8x - 4$

Make a scatter plot of the data. Then name the type of model that best fits the data and write a model. (Lesson 9.8)

11. $(-8.5, 20), (-8, 19), (-2, 7), (0, 3), (1, 1), (6, -9), (7.5, -12), (10, -17)$

12. $(-3, 1.5), (-1.5, 2.7), (-0.5, 4), (0, 5), (1, 7.5), (3, 17)$

13. $(3.3, 15.0), (2.1, 11.7), (1.0, 6.8), (0, 2.9), (-1.1, -0.9)$