9.5 Solving Quadratic Equations by the Quadratic Formula

What you should learn

GOAL 1 Use the quadratic formula to solve a quadratic equation.

GOAL 2 Use quadratic models for **real-life** situations, such as the hot-air balloon competition in **Example 4**.

Why you should learn it

▼ To model **real-life** situations, such as the maximum height of a baseball in **Ex. 84**.

REAL LIFE

GOAL 1 USING THE QUADRATIC FORMULA

In Lesson 9.1 you learned how to solve quadratic equations of the form $ax^2 + c = 0$ by finding square roots. In this lesson you will learn how to use the **quadratic formula** to solve *any* quadratic equation.

THE QUADRATIC FORMULA

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{when } a \neq 0 \text{ and } b^2 - 4ac \geq 0.$$

You can read this formula as "x equals the opposite of b, plus or minus the square root of b squared minus $4ac$, all divided by $2a$."

EXAMPLE 1 *Using the Quadratic Formula*

Solve $x^2 + 9x + 14 = 0$.

SOLUTION

Use the quadratic formula.

$$1x^2 + 9x + 14 = 0 \qquad \text{Identify } a = 1, b = 9, \text{ and } c = 14.$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(14)}}{2(1)} \qquad \text{Substitute values in the quadratic formula.}$$

$$x = \frac{-9 \pm \sqrt{81 - 56}}{2} \qquad \text{Simplify.}$$

$$x = \frac{-9 \pm \sqrt{25}}{2} \qquad \text{Simplify.}$$

$$x = \frac{-9 \pm 5}{2} \qquad \text{Solutions}$$

▶ The equation has two solutions:

$$x = \frac{-9 + 5}{2} = -2 \text{ and } x = \frac{-9 - 5}{2} = -7.$$

✓Check these solutions in the original equation.

Check $x = -2$:	Check $x = -7$:
$x^2 + 9x + 14 = 0$	$x^2 + 9x + 14 = 0$
$(-2)^2 + 9(-2) + 14 \stackrel{?}{=} 0$	$(-7)^2 + 9(-7) + 14 \stackrel{?}{=} 0$
$0 = 0$	$0 = 0$

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 *Writing in Standard Form*

Solve $2x^2 - 3x = 8$.

SOLUTION

$2x^2 - 3x = 8$ — **Write original equation.**

$2x^2 - 3x - 8 = 0$ — **Rewrite equation in standard form.**

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-8)}}{2(2)}$ — **Substitute values into the quadratic formula: $a = 2$, $b = -3$, $c = -8$.**

$x = \dfrac{3 \pm \sqrt{9 + 64}}{4}$ — **Simplify.**

$x = \dfrac{3 \pm \sqrt{73}}{4}$ — **Solutions**

▶ The equation has two solutions:

$x = \dfrac{3 + \sqrt{73}}{4} \approx 2.89$ and $x = \dfrac{3 - \sqrt{73}}{4} \approx -1.39$.

EXAMPLE 3 *Finding the x-Intercepts of a Graph*

Find the x-intercepts of the graph of $y = -x^2 - 2x + 5$.

SOLUTION

The x-intercepts occur when $y = 0$.

$y = -x^2 - 2x + 5$ — **Write original equation.**

$0 = -x^2 - 2x + 5$ — **Substitute 0 for y.**

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(5)}}{2(-1)}$ — **Substitute values into the quadratic formula: $a = -1$, $b = -2$, and $c = 5$.**

$x = \dfrac{2 \pm \sqrt{4 + 20}}{-2}$ — **Simplify.**

$x = \dfrac{2 \pm \sqrt{24}}{-2}$ — **Solutions**

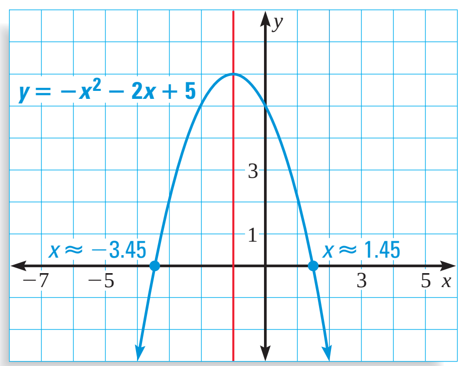
▶ The two solutions,

$x = \dfrac{2 + \sqrt{24}}{-2} \approx -3.45$ and

$x = \dfrac{2 - \sqrt{24}}{-2} \approx 1.45$,

are the x-intercepts of the graph of $y = -x^2 - 2x + 5$.

✓ Check your solution graphically. You can see that the graph shows the x-intercepts between -3 and -4 and between 1 and 2.



STUDENT HELP

Study Tip
Note that the x-intercepts are x-coordinates of points that are the same distance from the axis of symmetry.

GOAL 2 USING QUADRATIC MODELS IN REAL LIFE

In Lesson 9.1 you studied the model for the height of a falling object that is *dropped*. For an object that is *thrown* down or up, the model has an extra term to take into account, the initial velocity. Problems involving these two models are called *vertical motion* problems.

VERTICAL MOTION MODELS

OBJECT IS DROPPED: $h = -16t^2 + s$

OBJECT IS THROWN: $h = -16t^2 + vt + s$

h = height (feet) $\quad$ t = time in motion (seconds)

s = initial height (feet) $\quad$ v = initial velocity (feet per second)

In these models the coefficient of t^2 is one half the acceleration due to gravity. On the surface of Earth, this acceleration is approximately 32 feet per second per second.

Remember that velocity v can be positive (for an object moving up), negative (for an object moving down), or zero (for an object that is not moving). Speed is the absolute value of velocity.

EXAMPLE 4 *Modeling Vertical Motion*

BALLOON COMPETITION You are competing in the Field Target Event at a hot-air balloon festival. You throw a marker down from an altitude of 200 feet toward a target. When the marker leaves your hand, its speed is 30 feet per second. How long will it take the marker to hit the target?

SOLUTION

Because the marker is thrown down, the initial velocity is $v = -30$ feet per second. The initial height is $s = 200$ feet. The marker will hit the target when the height is 0.

$h = -16t^2 + vt + s$ — Choose the vertical motion model for a thrown object.

$h = -16t^2 + (-30)t + 200$ — Substitute values for v and s into the vertical motion model.

$0 = -16t^2 - 30t + 200$ — Substitute 0 for h. Write in standard form.

$t = \dfrac{-(-30) \pm \sqrt{(-30)^2 - 4(-16)(200)}}{2(-16)}$ — Substitute values for a, b, and c into the quadratic formula.

$t = \dfrac{30 \pm \sqrt{13,700}}{-32}$ — Simplify.

$t \approx 2.72$ or -4.60 — Solutions

▶ The weighted marker will hit the target about 2.72 seconds after it was thrown. As a solution, -4.60 doesn't make sense in the problem.

FOCUS ON APPLICATIONS

REAL LIFE **HOT-AIR BALLOONS**
The annual Albuquerque International Balloon Fiesta has several competitive events including the Field Target Event and Prize Grab. There is also a Special Shapes Rodeo and a night time Balloon Glow event.

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

1. What formula can you use to solve any quadratic equation?

Concept Check ✓

2. Explain how to use the quadratic formula to solve $-2x^2 + 5x = -7$.
3. What is the difference between the two models for vertical motion?

Skill Check ✓

Use the quadratic formula to solve the equation.

4. $x^2 + 6x - 7 = 0$
5. $x^2 - 2x - 15 = 0$
6. $x^2 + 12x + 36 = 0$
7. $4x^2 - 8x + 3 = 0$
8. $3x^2 + x - 1 = 0$
9. $x^2 + 6x - 3 = 0$

Write in standard form. Use the quadratic formula to solve the equation.

10. $2x^2 = -x + 6$
11. $6x = -8x^2 + 2$
12. $3 = 3x^2 + 8x$
13. $-14x = -2x^2 + 36$
14. $-x^2 + 4x = 3$
15. $4x^2 + 4x = -1$

Find the *x*-intercepts of the graph of the equation.

16. $y = x^2 - 11x + 24$
17. $y = x^2 + 10x + 16$
18. $y = 2x^2 + 4x - 30$
19. $y = 2x^2 + 6x - 9$
20. $y = 5x^2 + 8x - 8$
21. $y = 4x^2 + 8x - 1$
22. **FIELD TARGET EVENT** From a hot-air balloon directly over a target, you throw a marker with an initial downward velocity of -40 feet per second from a height of 180 feet. How long does it take the marker to reach the target?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

FINDING VALUES **Find the value of $b^2 - 4ac$ for the equation.**

23. $x^2 - 3x - 4 = 0$
24. $4x^2 + 5x + 1 = 0$
25. $x^2 - 11x + 30 = 0$
26. $5w^2 - 3w - 2 = 0$
27. $s^2 - 13s + 42 = 0$
28. $3x^2 - 5x - 12 = 0$
29. $5x^2 + 5x + \frac{1}{5} = 0$
30. $\frac{1}{2}a^2 + 5a - 8 = 0$
31. $\frac{1}{4}v^2 - 6v - 3 = 0$

SOLVING EQUATIONS **Use the quadratic formula to solve the equation.**

32. $4x^2 - 13x + 3 = 0$
33. $y^2 + 11y + 10 = 0$
34. $7x^2 + 8x + 1 = 0$
35. $-3y^2 + 2y + 8 = 0$
36. $6n^2 - 10n + 3 = 0$
37. $9x^2 + 14x + 3 = 0$
38. $8m^2 + 6m - 1 = 0$
39. $7x^2 + 2x - 1 = 0$
40. $-6x^2 - 3x + 2 = 0$
41. $-\frac{1}{2}x^2 + 6x + 13 = 0$
42. $-10a^2 + 3a + 2 = 0$
43. $-2d^2 - 5d + 19 = 0$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 23–43
Example 2: Exs. 44–52
Example 3: Exs. 53–61
Example 4: Exs. 71–80

STANDARD FORM **Write the quadratic equation in standard form. Solve using the quadratic formula.**

44. $2x^2 = 4x + 30$
45. $2 - 3x + x^2 = 0$
46. $16 = -x^2 + 11x$
47. $5x - 1 = -6x^2$
48. $2q^2 - 6 = -4q$
49. $5z - 2z^2 + 15 = 8$
50. $-1 + 3x^2 = 2x$
51. $-5c^2 + 9c = 4$
52. $-16b = -8b^2 - 8$

FINDING INTERCEPTS **Find the x-intercepts of the graph of the equation.**

53. $y = 3x^2 - 6x - 24$ **54.** $y = 2x^2 - 6x - 8$ **55.** $y = 2x^2 - 2x - 12$

56. $y = -2x^2 + 6x + 9$ **57.** $y = x^2 + x - 4$ **58.** $y = x^2 + 7x - 2$

59. $y = -3x^2 - 2x + 1$ **60.** $y = -4x^2 + 8x - 2$ **61.** $y = -5x^2 + 5x + 5$

CHOOSING A METHOD **Solve the quadratic equation by finding square roots or by using the quadratic formula. Explain why you chose the method.**

62. $6x^2 + 20x + 5 = 0$ **63.** $m^2 = 32$ **64.** $x^2 - 625 = 0$

65. $4y^2 - 49 = 0$ **66.** $-2x^2 + 6x + 1 = 0$ **67.** $h^2 + 18h + 81 = 0$

68. $5x^2 = 25$ **69.** $9y^2 - 3y = 1$ **70.** $v^2 = 8v + 2$

FIELD TARGET EVENT **In Exercises 71–76, six balloonists compete in the Field Target Event at a hot-air balloon festival. Calculate the amount of time it takes for the marker to hit the target when thrown down from the given initial height (in feet) with the given initial velocity (in feet per second).**

71. $s = 200$; $v = -50$ **72.** $s = 150$; $v = -25$ **73.** $s = 100$; $v = -10$

74. $s = 150$; $v = -33$ **75.** $s = 80$; $v = -40$ **76.** $s = 50$; $v = 0$

VERTICAL MOTION **In Exercises 77–80, use a vertical motion model to find how long it will take for the object to reach the ground.**

77. You drop keys from a window 30 feet above ground to your friend below. Your friend does not catch them.

78. An acorn falls 45 feet from the top of a tree.

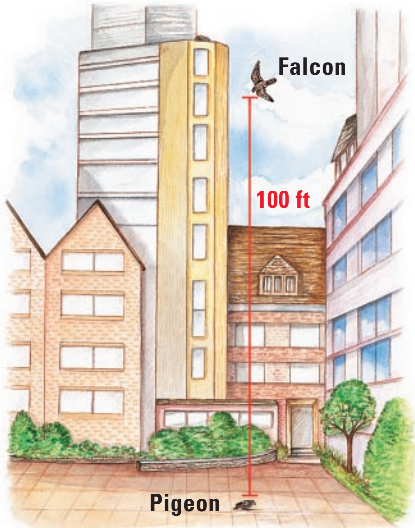
79. A lacrosse player throws a ball upward from her playing stick with an initial height of 7 feet, at an initial speed of 90 feet per second.

80. You throw a ball downward with an initial speed of 10 feet per second out of a window to a friend 20 feet below. Your friend does not catch the ball.

FOCUS ON APPLICATIONS

URBAN BIRDS Cities provide a habitat for many species of wildlife including birds of prey such as Peregrine falcons and red-tailed hawks.

81. **PEREGRINE FALCON** A falcon dives toward a pigeon on the ground. When the falcon is at a height of 100 feet the pigeon sees the falcon, which is diving at 220 feet per second. Estimate the time the pigeon has to escape.



82. **RED-TAILED HAWK** A hawk dives toward a snake. When the hawk is at a height of 200 feet the snake sees the hawk, which is diving at 105 feet per second. Estimate the time the snake has to escape.

83. **BALD EAGLE** An eagle dives toward a mouse. When the eagle is at a height of 125 feet the mouse sees the eagle, which is diving at 145 feet per second. Estimate the time the mouse has to escape.

Test Preparation

84. **MULTI-STEP PROBLEM** In parts (a)–(d), a batter hits a pitched baseball when it is 3 feet off the ground. After it is hit, the height h (in feet) of the ball at time t (in seconds) is modeled by

$$h = -16t^2 + 80t + 3$$

where t is the time (in seconds).

a. Find the time when the ball hits the ground in the outfield.

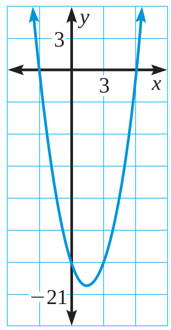
b. Write a quadratic equation that you can use to find the time when the baseball is at its maximum height of 103 feet. Solve the quadratic equation.

c. Use a graphing calculator to graph the function. Use the zoom feature to approximate the time when the baseball is at its maximum height. Compare your results with those you obtained in part (b).

d. **CRITICAL THINKING** What factors change the path of a baseball? What factors would contribute to hitting a home run?

★ Challenge

85. **VISUAL THINKING** Write an equation of the axis of symmetry of the graph and show that it lies halfway between the two x-intercepts.



Ex. 85

86. *Writing* Explain how you can use this two-part form of the quadratic formula

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

to find the distance between the axis of symmetry of a parabola and either of its x-intercepts.

EXTRA CHALLENGE
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MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 1.3 for 9.6)

87. x^2 when $x = -5$

88. $-y^2$ when $y = -1$

89. $-4xy$ when $x = -2$ and $y = -6$

90. $y^2 - y$ when $y = -2$

INEQUALITIES **Solve the inequality and graph the solution.** (Review 6.3, 6.4)

91. $2 \le x < 5$

92. $8 > 2x > -4$

93. $-12 < 2x - 6 < 4$

94. $-3 < -x < 1$

95. $|x + 5| \ge 10$

96. $|2x + 9| \le 15$

SKETCHING GRAPHS **Sketch the graph of the function. Label the vertex.** (Review 9.3)

97. $y = 6x^2 - 4x - 1$

98. $y = -3x^2 - 5x + 3$

99. $y = -2x^2 - 3x + 2$

100. $y = \frac{1}{2}x^2 + 2x - 1$

101. $y = 4x^2 - \frac{1}{4}x + 4$

102. $y = -5x^2 - 0.5x + 0.5$

103. **RECREATION** There were 1.4×10^7 people who visited Golden Gate Recreation Area in California in 1996. On average, how many people visited per day? per month? ▶ Source: National Park Service (Review 8.4)