9.2 Simplifying Radicals

What you should learn

GOAL 1 Use properties of radicals to simplify radicals.

GOAL 2 Use quadratic equations to model **real-life** problems, such as the speed of a tsunami in **Ex. 55**.

Why you should learn it

▼ To compare the speeds of two different-sized sailboats in a **real-life** boat race in **Example 3**.

REAL LIFE

GOAL 1 SIMPLIFYING RADICALS

In this lesson you will learn how to simplify radical expressions and how to recognize when a radical is written in simplest form.

ACTIVITY
Developing Concepts

Investigating Properties of Radicals

1. Evaluate the radical expressions $\sqrt{ab}$ and $\sqrt{a} \cdot \sqrt{b}$ for the given values of a and b.

   a. $a = 4, b = 9$ **b.** $a = 64, b = 100$ **c.** $a = 25, b = 4$

   d. $a = 36, b = 16$ **e.** $a = 100, b = 625$ **f.** $a = 121, b = 49$

2. What can you conclude?

3. Evaluate the radical expressions $\sqrt{\frac{a}{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}}$ for the given values of a and b.

   a. $a = 4, b = 49$ **b.** $a = 16, b = 64$ **c.** $a = 25, b = 36$

   d. $a = 225, b = 4$ **e.** $a = 144, b = 100$ **f.** $a = 9, b = 81$

4. What can you conclude?

In the activity you may have discovered the following two properties that can be used to simplify expressions that contain radicals.

PROPERTIES OF RADICALS

PRODUCT PROPERTY The square root of a product equals the product of the square roots of the factors.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \text{ where } a \geq 0 \text{ and } b \geq 0$$

Example: $\sqrt{4 \cdot 100} = \sqrt{4} \cdot \sqrt{100}$

QUOTIENT PROPERTY The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } a \geq 0 \text{ and } b > 0$$

Example: $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}}$

An expression with radicals is in **simplest form** if the following are true:

- No perfect square factors other than 1 are in the radicand. $\sqrt{8} \rightarrow \sqrt{4 \cdot 2} \rightarrow 2\sqrt{2}$
- No fractions are in the radicand. $\sqrt{\frac{5}{16}} \rightarrow \frac{\sqrt{5}}{\sqrt{16}} \rightarrow \frac{\sqrt{5}}{4}$
- No radicals appear in the denominator of a fraction. $\frac{1}{\sqrt{4}} \rightarrow \frac{1}{2}$

EXAMPLE 1 *Simplifying with the Product Property*

Simplify the expression $\sqrt{50}$.

SOLUTION

You can use the product property to simplify a radical by removing perfect square factors from the radicand.

$$\sqrt{50} = \sqrt{25 \cdot 2}$$ Factor using perfect square factor.

$$= \sqrt{25} \cdot \sqrt{2}$$ Use product property.

$$= 5\sqrt{2}$$ Simplify.

STUDENT HELP

Skills Review
For help with factoring, see pp. 777–778.

EXAMPLE 2 *Simplifying with the Quotient Property*

Simplify the expression.

a. $\sqrt{\frac{3}{4}}$ **b.** $\frac{\sqrt{20}}{4}$ **c.** $\sqrt{\frac{32}{50}}$

SOLUTION

a. $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}}$ Use quotient property.

$= \frac{\sqrt{3}}{2}$ Simplify.

b. $\frac{\sqrt{20}}{4} = \frac{\sqrt{4 \cdot 5}}{4}$ Factor using perfect square factor.

$= \frac{2\sqrt{5}}{4}$ Remove perfect square factors.

$= \frac{\sqrt{5}}{2}$ Divide out common factors.

c. $\sqrt{\frac{32}{50}} = \sqrt{\frac{2 \cdot 16}{2 \cdot 25}}$ Factor using perfect square factor.

$= \sqrt{\frac{16}{25}}$ Divide out common factors.

$= \frac{\sqrt{16}}{\sqrt{25}}$ Use quotient property.

$= \frac{4}{5}$ Simplify.

STUDENT HELP

INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for extra examples.

FOCUS ON APPLICATIONS

CHASE RACES allow boats with different maximum speeds to race against each other. In Example 3, the smaller boat would start a 10-mile race 40 minutes before the bigger boat to allow for the bigger boat's speed advantage.

APPLICATION LINK
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GOAL 2 USING QUADRATIC MODELS IN REAL LIFE

Quadratic equations can be used to model real-life situations, such as comparing the maximum speeds of two sailboats with different water line lengths.

EXAMPLE 3 *Simplifying Radical Expressions*

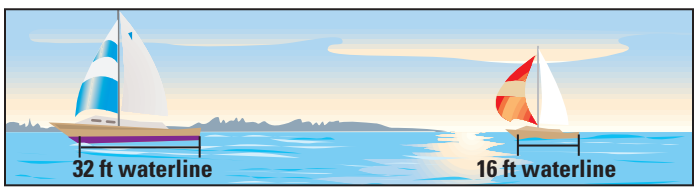
BOAT RACING The maximum speed s (in knots or nautical miles per hour) that some kinds of boats can travel can be modeled by

$$s^2 = \frac{16}{9}x, \text{ where } x \text{ is the length of the water line in feet.}$$

A sailboat with a 16-foot water line is racing against a sailboat with a 32-foot water line. The smaller sailboat is traveling at a maximum speed of 5.3 knots.

a. Find the maximum speed of the sailboat with the 32-foot water line. Find the speed to the nearest tenth.

b. Is the maximum speed of the sailboat with the 32-foot water line twice that of the sailboat with the 16-foot water line? Explain.



SOLUTION

a. Write the model for maximum speed of a sailboat and let $x = 32$ feet.

$$s^2 = \frac{16}{9}x$$ Write quadratic model.

$$s^2 = \frac{16}{9} \cdot 32$$ Substitute 32 for x.

$$\sqrt{s^2} = \sqrt{\frac{16}{9} \cdot 32}$$ Find square root of both sides.

$$s = \frac{\sqrt{16}}{\sqrt{9}} \cdot \sqrt{32}$$ Use quotient and product properties.

$$= \frac{\sqrt{16}}{\sqrt{9}} \cdot \sqrt{16} \cdot \sqrt{2}$$ Identify perfect square factors.

$$= \frac{4}{3} \cdot 4\sqrt{2}$$ Remove perfect square factors.

$$= \frac{16\sqrt{2}}{3}$$ Simplify.

$$\approx 7.5$$ Use a calculator or square root table.

b. No, because 7.5 knots is not twice 5.3 knots. The maximum speed of the smaller sailboat is $s = \sqrt{\frac{16^2}{9}}$, or $\frac{16}{3}$, so the longer sailboat's maximum speed is only $\sqrt{2}$ times the smaller sailboat's maximum speed.

GUIDED PRACTICE

Vocabulary Check ✓

1. Is the radical expression in simplest form? Explain.

a. $\frac{3}{5}\sqrt{2}$ **b.** $\sqrt{\frac{3}{16}}$ **c.** $5\sqrt{40}$

Concept Check ✓

2. Explain how to use the product property of radicals to simplify $\sqrt{3} \cdot \sqrt{15}$.

3. Explain how to use the quotient property of radicals to simplify $\sqrt{\frac{4}{25}}$.

Skill Check ✓

Match the radical expression with its simplified form.

A. $3\sqrt{6}$ **B.** $9\sqrt{6}$ **C.** $2\sqrt{2}$ **D.** $4\sqrt{2}$

4. $\sqrt{32}$ **5.** $\sqrt{54}$ **6.** $\sqrt{486}$ **7.** $\sqrt{8}$

8. **SAILING** Find the maximum speed of a sailboat with a 25-foot water line using the formula in Example 3.

9. **ERROR ANALYSIS** Describe the error. Simplify correctly.

$$\sqrt{50} = \sqrt{5 \cdot 10}$$
$$= 5\sqrt{10}$$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

PRODUCT PROPERTY **Simplify the expression.**

10. $\sqrt{44}$ **11.** $\sqrt{27}$ **12.** $\sqrt{48}$ **13.** $\sqrt{75}$

14. $\sqrt{90}$ **15.** $\sqrt{125}$ **16.** $\sqrt{200}$ **17.** $\sqrt{80}$

18. $\frac{1}{2}\sqrt{112}$ **19.** $\frac{1}{3}\sqrt{54}$ **20.** $\sqrt{2} \cdot \sqrt{8}$ **21.** $\sqrt{6} \cdot \sqrt{8}$

QUOTIENT PROPERTY **Simplify the expression.**

22. $\sqrt{\frac{7}{9}}$ **23.** $\sqrt{\frac{11}{16}}$ **24.** $2\sqrt{\frac{5}{4}}$ **25.** $18\sqrt{\frac{5}{81}}$

26. $2\sqrt{\frac{10}{2}}$ **27.** $3\sqrt{\frac{9}{3}}$ **28.** $8\sqrt{\frac{13}{9}}$ **29.** $3\sqrt{\frac{8}{64}}$

30. $4\sqrt{\frac{16}{4}}$ **31.** $3\sqrt{\frac{3}{16}}$ **32.** $5\sqrt{\frac{6}{2}}$ **33.** $8\sqrt{\frac{20}{4}}$

SIMPLIFYING **Simplify the expression.**

34. $\frac{\sqrt{32}}{\sqrt{25}}$ **35.** $\sqrt{\frac{27}{36}}$ **36.** $\frac{\sqrt{49}}{\sqrt{4}}$ **37.** $\frac{\sqrt{36}}{\sqrt{9}}$

38. $\frac{\sqrt{9}}{\sqrt{49}}$ **39.** $\frac{\sqrt{48}}{\sqrt{81}}$ **40.** $\frac{\sqrt{64}}{\sqrt{16}}$ **41.** $\frac{\sqrt{120}}{\sqrt{4}}$

42. $\frac{1}{2}\sqrt{32} \cdot \sqrt{2}$ **43.** $3\sqrt{63} \cdot \sqrt{4}$ **44.** $\sqrt{9} \cdot 4\sqrt{25}$ **45.** $-2\sqrt{27} \cdot \sqrt{3}$

46. $\sqrt{7} \cdot \frac{\sqrt{18}}{\sqrt{2}}$ **47.** $-\sqrt{4} \cdot \frac{\sqrt{81}}{\sqrt{36}}$ **48.** $\frac{\sqrt{10} \cdot \sqrt{16}}{\sqrt{5}}$ **49.** $\frac{-2\sqrt{20}}{\sqrt{100}}$

STUDENT HELP

HOMEWORK HELP

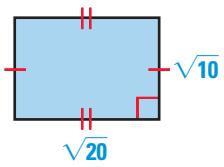
Example 1: Exs. 10–21
Example 2: Exs. 22–41
Example 3: Exs. 42–57

STUDENT HELP

Skills Review
For help with area of geometric figures, see pp. 790–791.

GEOMETRY CONNECTION **Find the area of the figure. Give both the exact answer in simplified form and the decimal approximation rounded to the nearest hundredth.**

50.



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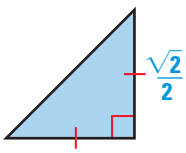

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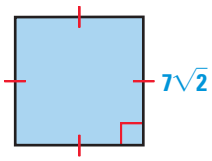


BODY SURFACE AREA **In Exercises 53 and 54, use the following information.**

Physicians can approximate the Body Surface Area of an adult (in square meters) using an index called *BSA* where H is height in centimeters and W is weight in kilograms.

Body Surface Area: $\sqrt{\frac{HW}{3600}}$

53. Find the *BSA* of a person who is 180 centimeters tall and weighs 75 kilograms.

54. Find the *BSA* of a person who is 160 centimeters tall and weighs 50 kilograms.

TSUNAMI **In Exercises 55–57, use the following information.**

A *tsunami* is a destructive, fast-moving ocean wave that is caused by an undersea earthquake, landslide, or volcano. The Pacific Tsunami Warning Center is responsible for monitoring earthquakes that could potentially cause tsunamis in the Pacific Ocean. Through measuring the water level and calculating the speed of a tsunami, scientists can predict arrival times of tsunamis.

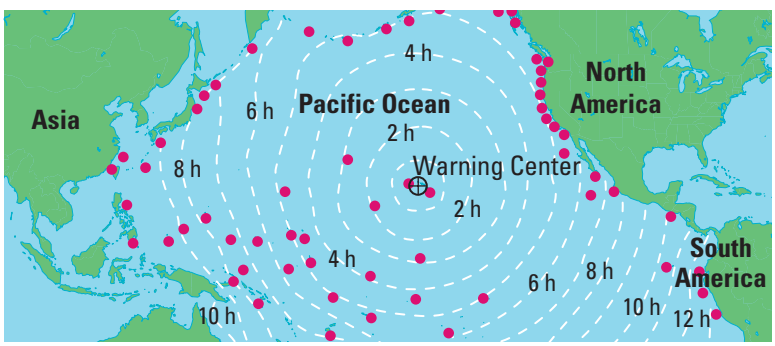
The speed s (in meters per second) at which a tsunami moves is determined by the depth d (in meters) of the ocean.

$s = \sqrt{gd}$, where g is 9.8 meters per second per second

55. Find the speed of a tsunami in a region of the ocean that is 1000 meters deep. Write the result in simplified form.

56. Find the speed of a tsunami in a region of the ocean that is 4000 meters deep. Write the result in simplified form.

57. CRITICAL THINKING Is the speed of a tsunami at a depth of 4000 meters four times the speed of a tsunami at 1000 meters? Explain why or why not.



Test Preparation

58. MULTIPLE CHOICE Which is the simplified form of $4\frac{\sqrt{125}}{\sqrt{25}}$?

Ⓐ $2\sqrt{5}$ Ⓑ $4\sqrt{5}$ Ⓒ $20\sqrt{5}$ Ⓓ $\frac{4\sqrt{5}}{5}$

59. MULTIPLE CHOICE Which is the simplified form of $33\sqrt{\frac{2}{121}}$?

Ⓐ $\frac{2\sqrt{33}}{\sqrt{11}}$ Ⓑ $33\sqrt{2}$ Ⓒ $3\sqrt{2}$ Ⓓ $\frac{3\sqrt{2}}{\sqrt{11}}$

60. MULTIPLE CHOICE Which is the simplified form of $\frac{6\sqrt{52}}{\sqrt{2}\cdot\sqrt{8}}$?

Ⓐ $\frac{3\sqrt{13}}{\sqrt{2}}$ Ⓑ $3\sqrt{13}$ Ⓒ $3\sqrt{26}$ Ⓓ $\frac{6\sqrt{13}}{\sqrt{2}}$

Challenge

EXTENSION: FRACTIONAL EXPONENTS **Sometimes $\sqrt{3}$ is written as $3^{1/2}$.**

61. You can obtain a graphical representation of the relationship $2^{1/2} = \sqrt{2}$ by investigating the graph of $f(x) = 2^x$.

a. Graph $f(x) = 2^x$.

b. Use the *Trace* feature to find values of f when $x = \frac{1}{2}$.

c. Compare the value from part (b) with the value of $\sqrt{2}$.

EXTRA CHALLENGE
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Using the fact that $x^{1/2} = \sqrt{x}$, rewrite in simplest radical form.

62. $6x^{1/2}$ **63.** $x^{1/2} \cdot 4\sqrt{2}$ **64.** $18^{1/2}x \cdot 9x^{1/2}x$

MIXED REVIEW

GRAPHING EQUATIONS **Use a table of values to graph the equation.** (Review 4.2 for 9.3)

65. $y = -x + 5$ **66.** $y = x - 7$ **67.** $y = 3x - 1$

EVALUATING EXPRESSIONS **Simplify the expression. Then evaluate the expression when $a = 1$ and $b = 2$.** (Review 8.1)

68. $(a^3)^2$ **69.** $b^6 \cdot b^2$ **70.** $(a^3b)^4$

REWRITING EXPRESSIONS **Rewrite the expression using positive exponents.** (Review 8.2)

71. $\frac{1}{2x^{-5}}$ **72.** $\frac{1}{4x^{-7}}$ **73.** $x^{-4}y^3$ **74.** $6x^{-2}y^{-6}$

75. BUSINESS From 1994 to 1999, the sales for a chain of home furnishing stores increased by about the same annual rate. The sales S (in millions of dollars) in year t can be modeled by $S = 455\left(\frac{13}{10}\right)^t$ where t represents years since 1994. Find the ratio of 1999 sales to 1995 sales. (Review 8.3)