9.1 Solving Quadratic Equations by Finding Square Roots

What you should learn

GOAL 1 Evaluate and approximate square roots.

GOAL 2 Solve a quadratic equation by finding square roots.

Why you should learn it

▼ To solve **real-life** problems such as finding the time it takes an egg to hit the ground in an engineering contest in **Example 7**.

REAL LIFE

GOAL 1 EVALUATING SQUARE ROOTS

You know how to find the square of a number. For instance, the square of 3 is $3^2 = 9$. The square of -3 is also 9.

In this lesson you will study the reverse problem: finding a *square root* of a number.

SQUARE ROOT OF A NUMBER

If $b^2 = a$, then b is a **square root** of a.

Example: If $3^2 = 9$, then 3 is a square root of 9.

All positive real numbers have two square roots: a **positive square root** (or principal square root) and a **negative square root**. Square roots are written with a radical symbol $\sqrt{}$. The number or expression inside a radical symbol is the **radicand**. In the following examples, 9 is the radicand.

Meaning	Positive square root	Negative square root	The positive and negative square roots
Symbol	$\sqrt{}$	$-\sqrt{}$	$\pm\sqrt{}$
Example	$\sqrt{9} = 3$	$-\sqrt{9} = -3$	$\pm\sqrt{9} = \pm 3 = +3$ or -3

The symbol $\pm$ is read as "plus or minus" and refers to both the positive square root and the negative square root. Zero has only one square root: zero. Negative numbers have no real square roots, because the square of every real number is either positive or zero.

EXAMPLE 1 *Finding Square Roots of Numbers*

Evaluate the expression.

a. $\sqrt{64}$ **b.** $-\sqrt{64}$ **c.** $\sqrt{0}$ **d.** $\pm\sqrt{0.25}$ **e.** $\sqrt{-4}$

SOLUTION

a. $\sqrt{64} = 8$ — Positive square root

b. $-\sqrt{64} = -8$ — Negative square root

c. $\sqrt{0} = 0$ — Square root of zero

d. $\pm\sqrt{0.25} = \pm 0.5$ — Two square roots

e. $\sqrt{-4}$ (undefined) — No real square root

Numbers whose square roots are integers or quotients of integers are called **perfect squares**. The square roots of numbers that are not perfect squares must be written using the radical symbol or approximated. These numbers are part of the set of *irrational numbers*. An **irrational number** is a number that cannot be written as the quotient of two integers.

EXAMPLE 2 *Evaluating Square Roots*

STUDENT HELP

Square Root Table
For a table of square roots, see p. 811.

Evaluate the expression. Use a calculator if necessary.

a. $-\sqrt{121}$ **b.** $-\sqrt{1.44}$ **c.** $\sqrt{0.09}$ **d.** $\sqrt{7}$

SOLUTION

a. $-\sqrt{121} = -11$ 121 is a perfect square: $11^2 = 121$.

b. $-\sqrt{1.44} = -1.2$ 1.44 is a perfect square: $1.2^2 = 1.44$.

c. $\sqrt{0.09} = 0.3$ 0.09 is a perfect square: $0.3^2 = 0.09$.

d. $\sqrt{7} \approx 2.65$ Round to the nearest hundredth.

....

A **radical expression** involves square roots (or *radicals*). If the symbol $\pm$ precedes the radical, the expression represents two different numbers, as in Example 4. The square root symbol is a grouping symbol. Operations inside a radical symbol must be performed before the square root is evaluated.

EXAMPLE 3 *Evaluating a Radical Expression*

Evaluate $\sqrt{b^2 - 4ac}$ when $a = 1$, $b = -2$, and $c = -3$.

SOLUTION

$\sqrt{b^2 - 4ac} = \sqrt{(-2)^2 - 4(1)(-3)}$ Substitute values.

$= \sqrt{4 + 12}$ Simplify.

$= \sqrt{16}$ Simplify.

$= 4$ Positive square root

EXAMPLE 4 *Evaluating an Expression with a Calculator*

STUDENT HELP

KEYSTROKE HELP
To find the square root of 3 on your calculator you may need to press [√] [3] or [3] [√]. Test your calculator to find out which order is correct.

Evaluate $\dfrac{1 \pm 2\sqrt{3}}{4}$ to the nearest hundredth.

SOLUTION This expression represents two numbers.

[(] 1 [+] 2 [×] 3 [√] [)] [÷] 4 [ENTER] 1.116025404

[(] 1 [−] 2 [×] 3 [√] [)] [÷] 4 [ENTER] −0.61602540

▶ Rounded to the nearest hundredth, the expression represents 1.12 and −0.62.

GOAL 2 SOLVING A QUADRATIC EQUATION

A **quadratic equation** is an equation that can be written in the following **standard form**.

$$ax^2 + bx + c = 0, \text{ where } a \neq 0$$

In standard form, a is the **leading coefficient**.

When $b = 0$, this equation becomes $ax^2 + c = 0$. To solve quadratic equations of this form, isolate x^2 on one side. Then find the square root(s) of each side.

CONCEPT SUMMARY — SOLVING $x^2 = d$ BY FINDING SQUARE ROOTS

- If $d > 0$, then $x^2 = d$ has two solutions: $x = \pm\sqrt{d}$.
- If $d = 0$, then $x^2 = d$ has one solution: $x = 0$.
- If $d < 0$, then $x^2 = d$ has no real solution.

EXAMPLE 5 *Solving Quadratic Equations*

Solve each equation.

a. $x^2 = 4$ **b.** $x^2 = 5$ **c.** $x^2 = 0$ **d.** $x^2 = -1$

SOLUTION

a. $x^2 = 4$ has two solutions: $x = +2$ and $x = -2$.

b. $x^2 = 5$ has two solutions: $x = \sqrt{5}$ and $x = -\sqrt{5}$.

c. $x^2 = 0$ has one solution: $x = 0$.

d. $x^2 = -1$ has no real solution.

EXAMPLE 6 *Rewriting Before Finding Square Roots*

Solve $3x^2 - 48 = 0$.

SOLUTION

$3x^2 - 48 = 0$ — Write original equation.

$3x^2 = 48$ — Add 48 to each side.

$x^2 = 16$ — Divide each side by 3.

$x = \pm\sqrt{16}$ — Find square roots.

$x = \pm 4$ — 16 is a perfect square: $4^2 = 16$, $(-4)^2 = 16$.

▶ The solutions are 4 and -4. Check both solutions in the original equation. Both 4 and -4 make the equation true, so $3x^2 - 48 = 0$ has two solutions.

STUDENT HELP

Study Tip
When checking solutions of quadratic equations, remember to check *all* solutions in the original equation. In Example 6, check both 4 and -4.

$3(4)^2 - 48 \stackrel{?}{=} 0$

$3(-4)^2 - 48 \stackrel{?}{=} 0$

FALLING OBJECT MODEL When an object is dropped, the speed with which it falls continues to increase. Ignoring air resistance, its height h can be approximated by the falling object model.

$$h = -16t^2 + s \longleftarrow \text{Falling object model}$$

Here h is measured in feet, t is the number of seconds the object has fallen, and s is the initial height from which the object was dropped.

The object's increase in speed is due to Earth's gravitational pull. On a planet whose gravitational pull is different from that of Earth, the factor -16 would be replaced by another constant.

REAL LIFE
Engineering

EXAMPLE 7 *Using a Falling Object Model*

An engineering student is in an "egg dropping contest." The goal is to create a container for an egg so it can be dropped from a height of 32 feet without breaking the egg. To the nearest tenth of a second, about how long will it take for the egg's container to hit the ground? Assume there is no air resistance.

SOLUTION Write an equation to model the egg container's height h as function of time t, where the initial height $s = 32$.

$h = -16t^2 + s$ Write falling object model.

$h = -16t^2 + 32$ Substitute 32 for s.

The falling object model for the egg container is $h = -16t^2 + 32$.

PROBLEM SOLVING STRATEGY

Method 1 **MAKE A TABLE** One way to solve the problem is to find the height h for different values of time t in the function $h = -16t^2 + 32$. Organize the data in a table. The egg container will hit the ground when $h = 0$.

Time (sec)	0.0	0.5	1.0	1.1	1.2	1.3	1.4	1.5
Height (ft)	32	28	16	12.64	8.96	4.96	0.64	−4

▶ From the table, you can see that $h = 0$ between 1.4 and 1.5 seconds. The egg container will take between 1.4 and 1.5 seconds to hit the ground.

Method 2 **USE AN EQUATION** Another way to approach the problem is to solve the quadratic equation for the time t that gives a height of $h = 0$ feet.

$h = -16t^2 + 32$ Write falling egg model.

$0 = -16t^2 + 32$ Substitute 0 for h.

$-32 = -16t^2$ Subtract 32 from each side.

$2 = t^2$ Divide each side by -16.

$\sqrt{2} = t$ Find positive square root.

$1.4 \approx t$ Use a calculator.

▶ The egg container will hit the ground in about 1.4 seconds. You can ignore the negative square root, because -1.4 seconds is not a reasonable solution.

STUDENT HELP

Look Back
For help with using a table of values to graph an equation, see pp. 211–212.

Guided Practice

Vocabulary Check ✓

1. State the meanings of the symbols $\sqrt{}$, $-\sqrt{}$, and $\pm\sqrt{}$.

2. Give an example of a perfect square and an example of an irrational number.

Concept Check ✓

3. Explain how to find solutions of an equation of the form $ax^2 + c = 0$.

If the statement is *true*, give an example. If false, give a counterexample.

4. Some numbers have no real square root.

5. No number has only one square root.

6. All positive numbers have two different square roots.

7. The square root of the sum of two numbers is equal to the sum of the square roots of the numbers.

Skill Check ✓

Evaluate the expression.

8. $\sqrt{36}$ **9.** $\sqrt{0.81}$ **10.** $-\sqrt{0.04}$ **11.** $\pm\sqrt{9}$

Evaluate the radical expression when *a* = 2 and *b* = 4.

12. $\sqrt{b^2 + 10a}$ **13.** $\dfrac{10 \pm 2\sqrt{b}}{a}$ **14.** $\sqrt{b^2 - 8a}$

Solve the equation. If there is no solution, state the reason.

15. $2x^2 - 8 = 0$ **16.** $x^2 + 25 = 0$ **17.** $x^2 - 1.44 = 0$ **18.** $5x^2 = -15$

FALLING OBJECTS **If an object is dropped from an initial height *s*, how long will it take to reach the ground? Assume there is no air resistance.**

19. $s = 48$ **20.** $s = 96$ **21.** $s = 192$

22. If you double the height from which the object falls, do you double the falling time? If not, why?

Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

FINDING SQUARE ROOTS **Find all square roots of the number or write *no square roots*. Check the results by squaring each root.**

23. 49 **24.** 144 **25.** -4 **26.** -25

27. 0 **28.** 100 **29.** 0.09 **30.** 0.16

EVALUATING SQUARE ROOTS **Evaluate the expression. Give the exact value if possible. Otherwise, approximate to the nearest hundredth.**

31. $-\sqrt{169}$ **32.** $\sqrt{64}$ **33.** $\sqrt{13}$ **34.** $-\sqrt{125}$

35. $\sqrt{0.04}$ **36.** $\pm\sqrt{0.75}$ **37.** $-\sqrt{0.1}$ **38.** $\pm\sqrt{6.25}$

EVALUATING EXPRESSIONS **Evaluate $\sqrt{b^2 - 4ac}$ for the given values.**

39. $a = 4, b = 5, c = 1$ **40.** $a = -2, b = 8, c = -8$ **41.** $a = 3, b = -7, c = 6$

42. $a = 2, b = 4, c = 0.5$ **43.** $a = -3, b = 7, c = 5$ **44.** $a = 6, b = -8, c = 4$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 23–30
Example 2: Exs. 31–38
Example 3: Exs. 39–44

continued on p. 508

STUDENT HELP

HOMEWORK HELP

continued from p. 507

Example 4: Exs. 45–53
Example 5: Exs. 54–59
Example 6: Exs. 60–77
Example 7: Exs. 79–86

EVALUATING EXPRESSIONS **Use a calculator to evaluate the expression. Round the results to the nearest hundredth.**

45. $\frac{3 \pm 4\sqrt{6}}{3}$ **46.** $\frac{6 \pm 4\sqrt{2}}{-1}$ **47.** $\frac{2 \pm 5\sqrt{3}}{5}$

48. $\frac{1 \pm 6\sqrt{8}}{6}$ **49.** $\frac{7 \pm 3\sqrt{2}}{-1}$ **50.** $\frac{2 \pm 5\sqrt{6}}{2}$

51. $\frac{5 \pm 6\sqrt{3}}{3}$ **52.** $\frac{3 \pm 4\sqrt{5}}{4}$ **53.** $\frac{7 \pm 0.3\sqrt{12}}{-6}$

QUADRATIC EQUATIONS **Solve the equation or write *no solution*. Write the solutions as integers if possible. Otherwise write them as radical expressions.**

54. $x^2 = 36$ **55.** $b^2 = 64$ **56.** $5x^2 = 500$

57. $x^2 = 16$ **58.** $x^2 = 0$ **59.** $x^2 = -9$

60. $3x^2 = 6$ **61.** $a^2 + 3 = 12$ **62.** $x^2 - 7 = 57$

63. $2s^2 - 5 = 27$ **64.** $5x^2 + 5 = 20$ **65.** $7x^2 + 30 = 9$

66. $x^2 + 4.0 = 0$ **67.** $6x^2 - 54 = 0$ **68.** $7x^2 - 63 = 0$

SOLVING EQUATIONS **Use a calculator to solve the equation or write *no solution*. Round the results to the nearest hundredth.**

69. $4x^2 - 3 = 57$ **70.** $6y^2 + 22 = 34$ **71.** $2x^2 - 4 = 10$

72. $\frac{2}{3}n^2 - 6 = 2$ **73.** $\frac{4}{5}x^2 + 12 = 5$ **74.** $\frac{1}{2}x^2 + 3 = 8$

75. $3x^2 + 7 = 31$ **76.** $6s^2 - 12 = 0$ **77.** $5a^2 + 10 = 20$

78. **CRITICAL THINKING** Write your own example of a quadratic equation in the form $x^2 = d$ for each of the following.

a. An equation that has no real solution.

b. An equation that has one solution.

c. An equation that has two solutions.

ROAD SAFETY **In Exercises 79–82, a boulder falls off the top of a cliff during a storm. The cliff is 60 feet high. Find how long it will take for the boulder to hit the road below.**

79. Write the falling object model for $s = 60$.

80. Try to determine the time when $h = 0$ from an input-output table for this model.

81. Solve the falling object model for $h = 0$.

82. *Writing* Which problem solving method do you prefer? Why?

FALLING OBJECT MODEL **In Exercises 83–86, an object is dropped from a height *s*. How long does it take to reach the ground? Assume there is no air resistance.**

83. $s = 144$ feet **84.** $s = 256$ feet **85.** $s = 400$ feet **86.** $s = 600$ feet

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 87–88.

87. **HOME COMPUTER SALES** The sales S (in millions of dollars) of home computers in the United States from 1988 to 1995 can be modeled by $S = 145.63t^2 + 3327.56$, where t is the number of years since 1988. Use this model to estimate the year in which sales of home computers will be \$36,000 million. ▶ Source: Electronic Industries Association

88. **COMPUTER SOFTWARE SALES** The sales S (in millions of dollars) of computer software in the United States from 1990 to 1995 can be modeled by $S = 61.98t^2 + 1001.15$, where t is the number of years since 1990. Use this model to estimate the year in which sales of computer software will be \$7200 million. ▶ Source: Electronic Industries Association

FOCUS ON CAREERS

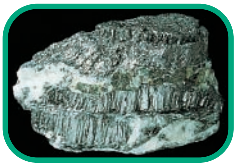
REAL LIFE MINERALOGISTS The properties and distribution of minerals are studied by mineralogists. The Vickers scale applies to thin slices of minerals that can be examined with a microscope.

INTERNET CAREER LINK www.mcdougallittell.com

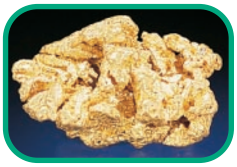
MINERALS **In Exercises 89–94, use the following information.** Mineralogists use the Vickers scale to measure the hardness of minerals. The hardness H of a mineral can be determined by hitting the mineral with a pyramid-shaped diamond and measuring the depth d of the indentation. The harder the mineral, the smaller the depth of the indentation. A model that relates mineral hardness with the indentation depth (in millimeters) is $Hd^2 = 1.89$.

Use a calculator to find the depth of the indentation for the mineral with the given value of H. Round to the nearest hundredth of a millimeter.

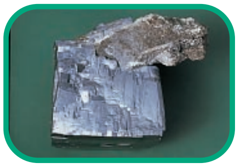
89. Graphite: $H = 12$



90. Gold: $H = 50$



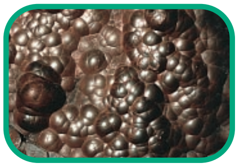
91. Galena: $H = 80$



92. Platinum: $H = 125$

93. Copper: $H = 140$

94. Hematite: $H = 755$



Test Preparation

QUANTITATIVE COMPARISON **In Exercises 95–96, choose the statement below that is true about the given numbers.**

(A) The number in column A is greater.

(B) The number in column B is greater.

(C) The two numbers are equal.

(D) The relationship cannot be determined from the given information.

	Column A	Column B
95.	$-\sqrt{121}$	$-2\sqrt{121}$
96.	The positive value of x in $2x^2 - 14 = 18$	The positive value of x in $2x^2 + 14 = 46$

★ **Challenge**

SCIENCE CONNECTION **In Exercises 97–100, use the following information.** Scientists simulate a gravity-free environment called *microgravity* in free-fall situations. A similar microgravity environment can be felt on free-fall rides at amusement parks or when stepping off a high diving platform. The distance d (in meters) that an object that is dropped falls in t seconds can be modeled by the equation $d = \frac{1}{2}g(t^2)$, where g is the acceleration due to gravity (9.8 meters per second per second).

97. The NASA Lewis Research Center has two microgravity facilities. One provides a 132-meter drop into a hole and the other provides a 24-meter drop inside a tower. How long will each free-fall period be?

98. In Japan a 490-meter-deep mine shaft has been converted into a microgravity facility. This creates the longest period of free fall currently available on Earth. How long will a period of free-fall be?

99. If you want to double the free-fall time, how much do you have to increase the height from which the object was dropped?

100. **CRITICAL THINKING** How are these formulas similar?

$d = \frac{1}{2}g(t^2)$ when d is distance, g is gravity, and t is time

$h = -16t^2 + s$ when h is height, s is initial height, and t is time

EXTRA CHALLENGE
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MIXED REVIEW

FACTORING INTEGERS **Write the prime factorization.** (Skills Review, p. 777)

101. 11 **102.** 24 **103.** 72 **104.** 108

GRAPH AND CHECK **Use a graph to solve the linear system. Check your solution algebraically.** (Review 7.1)

105. $-3x + 4y = -5$
$4x + 2y = -8$

106. $4x + 5y = 20$
$\frac{5}{4}x + y = 4$

107. $\frac{1}{2}x + 3y = 18$
$2x + 6y = -12$

SOLVING SYSTEMS **In Exercises 109–111, use linear combinations to solve the system.** (Review 7.3)

108. $12x - 4y = -32$
$x + 3y = 4$

109. $10x - 3y = 17$
$-7x + y = 9$

110. $8x - 5y = 100$
$2x + \frac{1}{2}y = 4$

111. **BASKETBALL TICKETS** You are selling tickets at a high school basketball game. Student tickets cost \$2 and general admission tickets cost \$3. You sell 2342 tickets and collect \$5801. How many of each type of ticket did you sell? (Review 7.2)

112. **FLOWERS** You are buying a combination of irises and white tulips for a flower arrangement. The irises are \$1 each and the white tulips are \$.50. You spend \$20 total to purchase an arrangement of 25 flowers. How many of each kind did you purchase? (Review 7.2)