CHAPTER 8

Chapter Summary

WHAT did you learn?

WHY did you learn it?

Evaluate exponential expressions

- using multiplication properties of exponents. **(8.1)** → Find the power generated by a windmill. **(p. 454)**
- that have negative and zero exponents. **(8.2)** → Predict a basketball player's average score per game. **(p. 458)**
- using division properties of exponents. **(8.3)** → Estimate the speed of an Olympic rowing team. **(p. 468)**

Convert numbers from scientific notation to decimal form. **(8.4)** → Find the amount of water discharged by the Amazon River each year. **(p. 472)**

Convert numbers from decimal form to scientific notation. **(8.4)** → Find the price per acre of the Alaska purchase. **(p. 472)**

Perform operations with numbers in scientific notation. **(8.4)** → Find how long it takes light to travel from the Sun to Pluto. **(p. 473)**

Use scientific notation in problem solving. **(8.4)** → Estimate the number of heartbeats in a person's life. **(p. 474)**

Use exponential growth models. **(8.5)** → Find the weight of a channel catfish. **(p. 478)**

Use exponential decay models. **(8.6)** → Find the buying power of a dollar. **(p. 484)**

Sketch the graphs of exponential growth and decay models. **(8.5 and 8.6)** → Find the balance on a savings account. **(p. 480)**

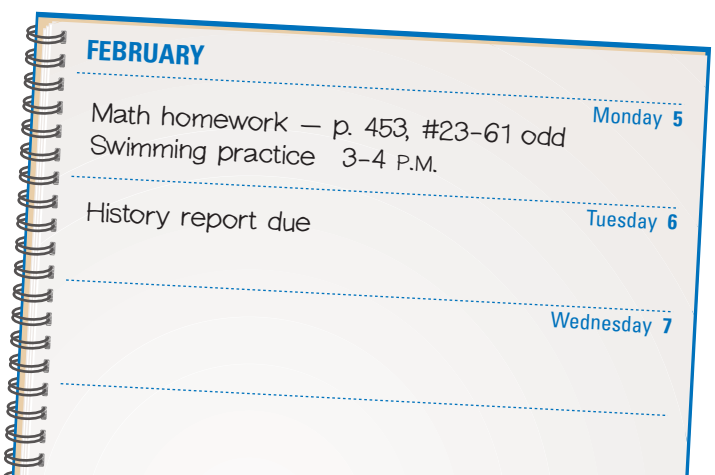
How does Chapter 8 fit into the BIGGER PICTURE of algebra?

In Chapters 3–7, you learned how to solve linear equations such as $4(x - 3y) = 24$. In this chapter you learned how to use the properties of exponents and scientific notation to solve exponential functions such as $y = 42(1.2)^x$. You will need to know how to use the properties of exponents when you solve quadratic equations in Chapter 9 and polynomial equations in Chapter 10.

STUDY STRATEGY

How did you use your schedule?

A schedule that you made using the **Study Strategy** on p. 448 may resemble this one.



CHAPTER 8

Chapter Review

VOCABULARY

- exponential function, p. 458
- scientific notation, p. 470
- exponential growth, p. 477
- growth factor, p. 477
- initial amount, pp. 477, 484
- time period, pp. 474, 484
- percent of increase, p. 477
- growth rate, p. 477
- exponential decay, p. 484
- decay factor, p. 484
- decay rate, p. 484
- percent of decrease, p. 484

8.1 Multiplication Properties of Exponents

Examples on pp. 450–452

EXAMPLES Use the multiplication properties of exponents.

a. $4^2 \cdot 4^7 = 4^{2+7} = 4^9$ — Product of powers property

b. $(5^2)^3 = 5^{2 \cdot 3} = 5^6$ — Power of a power property

c. $(6 \cdot x)^3 = 6^3 \cdot x^3$ — Power of a product property

Simplify the expression.

1. $2^2 \cdot 2^7$ **2.** $(4^3)^2$ **3.** $(3a)^3 \cdot (2a)^2$ **4.** $(w^3x^4y)^2 \cdot (wx^2y^3)^4$

Simplify. Then evaluate the expression when $s = 2$ and $t = 3$.

5. $s^3 \cdot s^4$ **6.** $s^4 \cdot (-t)^3$ **7.** $(s^3 \cdot t)^2$ **8.** $-(st^2)^2$

8.2 Zero and Negative Exponents

Examples on pp. 456–458

EXAMPLES Use zero and negative exponents.

a. $6^0 = 1$ — A nonzero number to the zero power is 1.

b. $7(x^{-3}) = 7\left(\frac{1}{x^3}\right) = \frac{7}{x^3}$ — a^{-n} is the reciprocal of a^n.

Evaluate the expression. Write fractions in simplest form.

9. 5^{-3} **10.** $7^{-4} \cdot 7^6$ **11.** $16\left(\frac{1}{2}\right)^{-1}$ **12.** $2^0 \cdot \left(\frac{1}{4^{-2}}\right)$

Rewrite the expression with positive exponents.

13. x^6y^{-6} **14.** $\frac{1}{5p^8q^{-3}}$ **15.** $(a^2b)^0$ **16.** $(-2y)^{-4}$

Sketch the graph of the exponential function.

17. $y = 4^x$ **18.** $y = \left(\frac{1}{2}\right)^x$ **19.** $y = 3^{-x}$ **20.** $y = \left(\frac{2}{3}\right)^{-x}$

8.3 Division Properties of Exponents

Examples on pp. 463–465

EXAMPLES Using the division properties of exponents.

a. $\frac{6^4}{6^2} = 6^{4-2} = 6^2 = 36$ Quotient of powers property

b. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$ Power of a quotient property

Evaluate the expression. Write fractions in simplest form.

21. $\frac{3^2}{3^5}$ **22.** $\frac{5^2}{5^{-2}}$ **23.** $\left(-\frac{4}{9}\right)^2$ **24.** $\left(\frac{10}{7}\right)^{-1}$

Simplify the expression. The simplified expression should have no negative exponents.

25. $\left(\frac{9}{b}\right)^6$ **26.** $\frac{x^{12}}{x^6}$ **27.** $\left(\frac{m^7}{m^4}\right)^2$ **28.** $\frac{(p^2)^3}{(p^2)^5}$

29. $\left(\frac{-9a^2b^2}{3ab}\right)^3$ **30.** $\left(\frac{25a^4b^5}{-5a^2b}\right)^3$ **31.** $\frac{32a^4b^{-2}}{2a^3b^3} \cdot \frac{3a^2b^7}{-2a}$ **32.** $\frac{9x^{-3}y^6}{x^4y^{-5}} \cdot \frac{(3x^2y)^{-2}}{xy^3}$

33. **SALES** From 1994 through 1999, the sales for a national book store increased by about the same percent each year. The sales s (in millions of dollars) for year t can be modeled by $s = 1686(1.17)^t$ where $t = 0$ represents 1994. Find the ratio of 1994 sales to 1999 sales.

8.4 Scientific Notation

Examples on pp. 470–472

EXAMPLES Rewriting numbers in decimal form and scientific notation.

a. $1.247 \times 10^2 = 124.7$ Move decimal point right 2 places.

b. $1.045 \times 10^{-3} = 0.001045$ Move decimal point left 3 places.

c. $79{,}500 = 7.95 \times 10^4$ Move decimal point left 4 places.

d. $0.0588 = 5.88 \times 10^{-2}$ Move decimal point right 2 places.

Rewrite the number in decimal form.

34. 6.667×10^{-3} **35.** 7.68×10^5 **36.** 3.75×10^{-1} **37.** 2×10^{-4}

Rewrite the number in scientific notation.

38. 523,000,000 **39.** 0.000679 **40.** 0.0000000233

41. **SPACE TRAVEL** Astronaut Shannon W. Lucid holds the United States single-mission, space-flight endurance record. Upon completion of her 1996 mission aboard the Russian Space Station *Mir*, Dr. Lucid had traveled 75,200,000 miles. Write 75,200,000 miles in scientific notation.

42. **ASTRONOMY** The distance from Earth to the star Alpha Centauri is about 4.07×10^{13} kilometers (km). Light travels at a speed of about 3.0×10^5 km per second. How long does it take light to travel from this star to Earth?

8.5 Exponential Growth Functions

Examples on pp. 477–479

EXAMPLE

You deposited $1200 in a savings account that pays 9% annual interest compounded yearly. What is the balance after 8 years?

$y = C(1 + r)^t$ Exponential growth model

$= 1200(1 + 0.09)^t$ Substitute 1200 for C and 0.09 for r.

$= 1200(1.09)^t$ Simplify.

After 8 years, the balance would be

$y = 1200(1.09)^8$ Substitute 8 for t.

≈ 2391.08 Use a calculator.

After 8 years, the balance would be $2391.08.

FITNESS PROGRAM In Exercises 43 and 44, use the following information. You start a walking program. The first week you walk 2 miles. Over the next 9 weeks, you increase your distance 5% per week.

43. Write an exponential growth model to represent the number of miles w you are walking after x weeks.

44. How far are you walking in the tenth week?

8.6 Exponential Decay Functions

Examples on pp. 484–487

EXAMPLE

In 1995 you bought a 32-inch television for $600. The television is depreciating at the rate of 8% per year. Write an exponential decay model and estimate the value of the television in 6 years.

$y = C(1 - r)^t$ Exponential decay model

$= 600(1 - 0.08)^t$ Substitute 600 for C and 0.08 for r.

$= 600(0.92)^t$ Simplify.

After 6 years, the balance would be

$y = 600(0.92)^6$ Substitute 6 for t.

≈ 363.81 Use a calculator.

After 6 years, the television would be worth $363.81.

TENNIS CLUB In Exercises 45 and 46, use the following information. A tennis club had a declining enrollment from 1993 to 2000. The enrollment in 1993 was 125 people. Each year for 7 years, the enrollment decreased by 3%.

45. Write an exponential decay model to represent enrollment e after x weeks.

46. Estimate the enrollment in 2000.

CHAPTER 8

Chapter Test

In Exercises 1–12, simplify the expression. The simplified expression should have no negative exponents.

1. $x^3 \cdot x^4$ **2.** $a^0 \cdot a^4$ **3.** $b^2 \cdot b^{-5}$ **4.** $5y^{-4}$

5. $(x^3)^7$ **6.** $(a^{-2})^3$ **7.** $\frac{n^3}{n^5}$ **8.** $(2b)^3(b^{-4})$

9. $(mn)^2 \cdot n^4$ **10.** $3a^5 \cdot 5a^{-2} \cdot a^3$ **11.** $\left(\frac{x^3}{xy^4}\right)\left(\frac{y}{x}\right)^5$ **12.** $\frac{a^{-1}b^2}{ab} \cdot \frac{a^2b^3}{b^{-2}}$

In Exercises 13–20, evaluate the expression.

13. $5^4 \cdot 5^{-1}$ **14.** 4^{-3} **15.** $(425^2)^0$ **16.** $\left(\frac{5}{2}\right)^{-2}$

17. $\frac{3 \cdot 3^5}{3^4}$ **18.** $\left(\frac{3}{4}\right)^3 \cdot 4^2 \cdot 3^0$ **19.** $(5 \cdot 4)^3 \cdot 5^{-2}$ **20.** $[(-2)^5]^2$

In Exercises 21–24, write the number in decimal form.

21. $4.27 \cdot 10^5$ **22.** $6.283 \cdot 10^{-9}$ **23.** 4.56×10^{10} **24.** 5×10^{-12}

In Exercises 25–28, write the number in scientific notation.

25. 9,875,000 **26.** 0.00125 **27.** 6,557,000,000 **28.** 0.0000000317

In Exercises 29–31, sketch a graph of the equation.

29. $y = 2^x$ **30.** $y = \left(\frac{1}{3}\right)^x$ **31.** $y = 10(1.4)^x$

32. **GEOMETRY CONNECTION** The volume of a cube is given by $V = s^3$, where s is the length of a side. The cube has a side of length $3a$. What is the volume of the cube if $a = 2$?

33. **SAVINGS ACCOUNT** You started a savings account in 1996. The balance A is given by $A = 400(1.1)^t$, where $t = 0$ represents the year 1996. What is the balance in 2000? in 2003?

34. **SALES** In 1996, you started your own business. In the first year, your sales totaled $88,500. Then each year for the next 4 years, your sales increased by 20%. Write an exponential growth model to represent this situation. Then estimate your sales in 2001.

35. **RADIOISOTOPES** The amount of time it takes for a radioactive substance to reduce to half of its original amount is called its half-life. The half-life of carbon 11 (^{11}C) is 20 minutes. If you start with 16 grams of ^{11}C, the number of grams remaining after h half-life periods would be $W = 16(0.5)^h$. Copy and complete the table and use the results to sketch the graph.

Half-life periods, h	0	1	2	3	4
Grams remaining, W	?	?	?	?	?