8.6

Exponential Decay Functions

What you should learn

GOAL 1 Write and use models for exponential decay.

GOAL 2 Graph models for exponential decay.

Why you should learn it

▼ To solve **real-life** problems, such as comparing the buying power of a dollar in different years as in **Example 1**.

GOAL 1 WRITING EXPONENTIAL DECAY MODELS

In Lesson 8.5, you learned that a quantity is *growing exponentially* if it *increases* by the same percent in each unit of time. A quantity is *decreasing exponentially* if it *decreases* by the same percent in each unit of time. This is called **exponential decay**.

EXPONENTIAL DECAY MODEL

$$y = C(1 - r)^t$$

C is the **initial amount**.

t is the **time period**.

$(1 - r)$ is the **decay factor**, r is the **decay rate**. To assure $1 > (1 - r) > 0$, it is necessary that $0 < r < 1$.

The **percent of decrease** is $100r$.

EXAMPLE 1 *Writing an Exponential Decay Model*

PURCHASING POWER From 1982 through 1997, the purchasing power of a dollar decreased by about 3.5% per year. Using 1982 as the base for comparison, what was the purchasing power of a dollar in 1997?

▶ Source: *Statistical Abstract of the United States:* 1998.

SOLUTION Let y represent the purchasing power and let $t = 0$ represent the year 1982. The initial amount is \$1. Use an exponential decay model.

$y = C(1 - r)^t$ **Exponential decay model**

$= (1)(1 - 0.035)^t$ **Substitute 1 for C and 0.035 for r.**

$= 0.965^t$ **Simplify.**

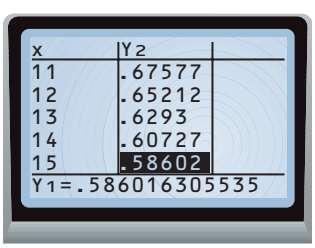
Because 1997 is 15 years after 1982, substitute 15 for t.

$y = 0.965^{15}$ **Substitute 15 for t.**

≈ 0.59 **Use a calculator.**

▶ The purchasing power of a dollar in 1997 compared to 1982 was \$.59.

✓ **CHECK** You can check your result by using a graphing calculator to make a table of values. Enter the exponential decay model and use the *Table* feature. Notice that as the prices *inflate*, the purchasing power of a dollar *deflates*.



EXAMPLE 2 Writing an Exponential Decay Model

You bought a used car for $18,000. The value of the car will be less each year because of depreciation. The car depreciates (loses value) at the rate of 12% per year.

a. Write an exponential decay model to represent this situation.

b. Estimate the value of your car in 8 years.

SOLUTION

STUDENT HELP

Skills Review
For help with writing a percent as a decimal, see page 784.

a. The initial value C is $18,000. The decay rate r is 0.12. Let y be the value and let t be the age of the car in years.

$y = C(1 - r)^t$ — Exponential decay model

$= 18{,}000(1 - 0.12)^t$ — Substitute 18,000 for C and 0.12 for r.

$= 18{,}000(0.88)^t$ — Simplify.

▶ The exponential decay model is $y = 18{,}000(0.88)^t$.

b. To find the value in 8 years, substitute 8 for t.

$y = 18{,}000(0.88)^t$ — Exponential decay model

$= 18{,}000(0.88)^8$ — Substitute 8 for t.

≈ 6473 — Use a calculator.

▶ According to this model, the value of your car in 8 years will be about $6473.

EXAMPLE 3 Making a List to Verify a Model

Verify the model you found in Example 2. Find the value of the car for each year by multiplying the value in the previous year by the decay factor.

SOLUTION The decay rate is 0.12. Decay factor $= 1 - 0.12 = 0.88$.

Year	Value
0	18,000
1	0.88(18,000) = 15,840
2	0.88(15,840) ≈ 13,939
3	0.88(13,939) ≈ 12,266
4	0.88(12,266) ≈ 10,794
5	0.88(10,794) ≈ 9499
6	0.88(9499) ≈ 8359
7	0.88(8359) ≈ 7356
8	0.88(7356) ≈ 6473

← Initial value of the car

From the list you can see that the value of the car in 8 years will be about $6473, which is consistent with your model.

GOAL 2 GRAPHING EXPONENTIAL DECAY MODELS

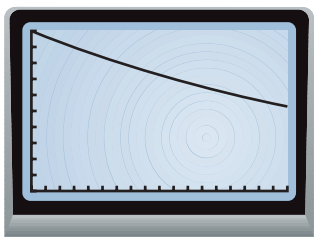
REAL LIFE
Purchasing Power

EXAMPLE 4 *Graphing an Exponential Decay Model*

Graph the exponential decay model in Example 1.

SOLUTION

Use your graphing calculator to graph the model. Set the viewing rectangle so that $0 \leq x \leq 20$ and $0 \leq y \leq 1$.



REAL LIFE
Depreciation

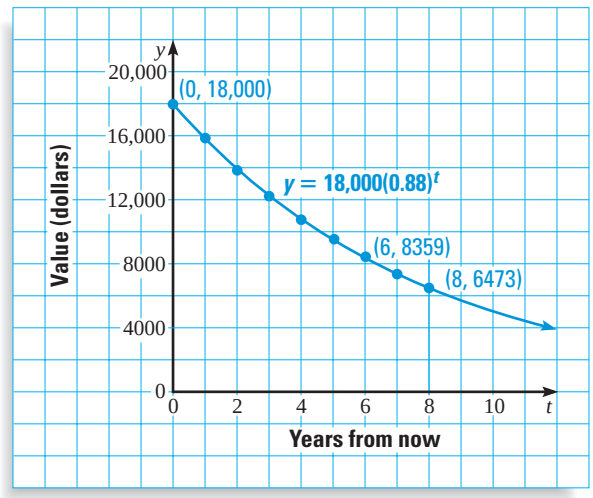
EXAMPLE 5 *Graphing an Exponential Decay Model*

a. Graph the exponential decay model in Example 2.

b. Use the graph to estimate the value of your car in 10 years.

SOLUTION

a. Use the table of values in Example 3. Plot the points in a coordinate plane, and draw a smooth curve through the points.



b. From the graph, the value of your car in 10 years will be about $5000.

An exponential model $y = a \cdot b^t$ represents exponential growth if $b > 1$ and exponential decay if $0 < b < 1$.

EXAMPLE 6 Comparing Growth and Decay Models

Classify each model as *exponential growth* or *exponential decay*. In each case identify the growth or decay factor and the percent of increase or percent of decrease per time period. Then graph each model.

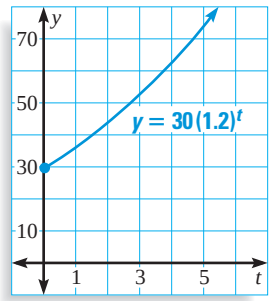
a. $y = 30(1.2)^t$ **b.** $y = 30\left(\frac{3}{5}\right)^t$

SOLUTION

a. Because $1.2 > 1$, the model $y = 30(1.2)^t$ is an exponential growth model.

The growth factor $(1 + r)$ is 1.20.

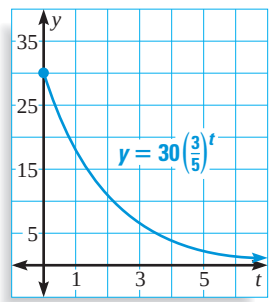
The growth rate is 0.2, so the percent of increase is 20%.



b. Because $0 < \frac{3}{5} < 1$, the model $y = 30\left(\frac{3}{5}\right)^t$ is an exponential decay model.

The decay factor $(1 - r) = \frac{3}{5}$.

The decay rate is $\frac{2}{5}$ or 0.4, so the percent of decrease is 40%.



CONCEPT SUMMARY EXPONENTIAL GROWTH AND DECAY MODELS

EXPONENTIAL GROWTH MODEL

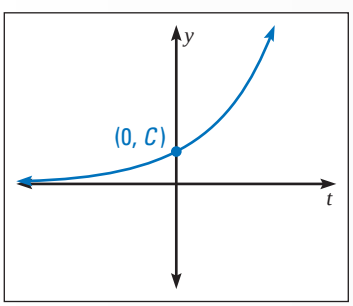
$y = C(1 + r)^t$ ← time period

initial amount; growth factor

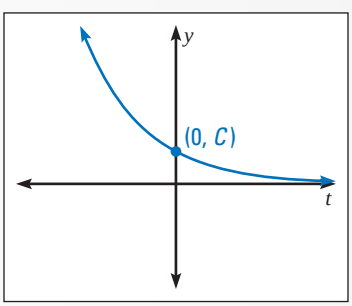
EXPONENTIAL DECAY MODEL

$y = C(1 - r)^t$ ← time period

initial amount; decay factor



$1 + r > 1$



$0 < 1 - r < 1$

GUIDED PRACTICE

Vocabulary Check ✓

1. In the exponential decay model, $y = C(1 - r)^t$, what is the decay factor?

Concept Check ✓

2. Is $y = 1.02^t$ an exponential decay model? Explain.

3. Look back at Example 2. Suppose that the car was depreciating at the rate of 20% per year. Write a new exponential decay model.

Skill Check ✓

CAR VALUE **You buy a used car for $7000. The car depreciates at the rate of 6% per year. Find the value of the car in the given years.**

4. 2 years **5.** 5 years **6.** 8 years **7.** 10 years

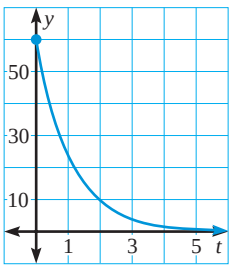
8. **BUSINESS DECLINE** A business earned $85,000 in 1990. Then its earnings decreased by 2% each year for 10 years. Write an exponential decay model for the earnings E in year t. Let $t = 0$ represent 1990.

9. **CHOOSE A MODEL** Which model best represents the decay curve shown in the graph at the right?

A. $y = 60(0.80)^t$

B. $y = 60(1.20)^t$

C. $y = 60(0.40)^t$



PRACTICE AND APPLICATIONS

STUDENT HELP

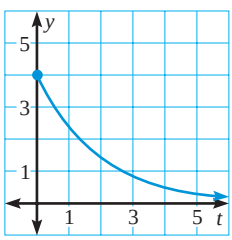
Extra Practice to help you master skills is on p. 804.

TRUCK VALUE **You buy a used truck for $20,000. It depreciates at the rate of 15% per year. Find the value of the truck in the given years.**

10. 3 years **11.** 8 years **12.** 10 years **13.** 12 years

MATCHING THE GRAPH **Match the equation with its graph.**

A.



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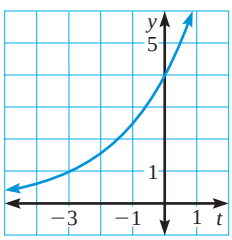
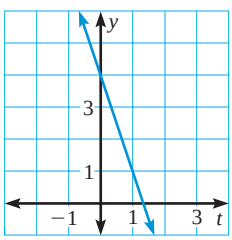


C.



14. $y = 4 - 3t$ **15.** $y = 4(1.6)^t$ **16.** $y = 4(0.6)^t$

RECOGNIZING MODELS **Classify the model as *exponential growth* or *exponential decay*. Identify the growth or decay factor and the percent of increase or decrease per time period.**

17. $y = 24(1.18)^t$ **18.** $y = 14(0.98)^t$ **19.** $y = 35\left(\frac{5}{4}\right)^t$

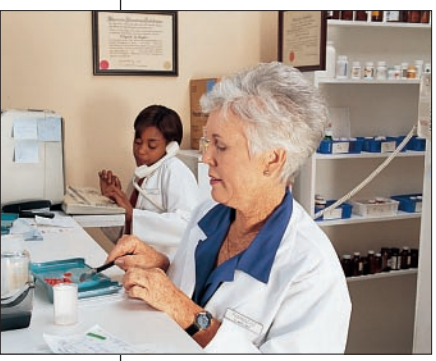
20. $y = 112(0.4)^t$ **21.** $y = 9\left(\frac{2}{5}\right)^t$ **22.** $y = 97(1.01)^t$

STUDENT HELP

HOMEWORK HELP

Examples 1, 2: Exs. 10–13, 23–29
Example 3: Exs. 30–33
Examples 4, 5: Exs. 14–16, 30, 33
Example 6: Exs. 14–22

FOCUS ON CAREERS



PHARMACISTS Pharmacists must understand the use, composition, and effects of drugs.

INTERNET CAREER LINK
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SCIENCE CONNECTION **In Exercises 23–25, use the following information.** The concentration of aspirin in a person's bloodstream can be modeled by the equation $y = A(0.8)^t$, where y represents the concentration of aspirin in a person's bloodstream in milligrams (mg), A represents the amount of aspirin taken, and t represents the number of hours since the medication was taken. Find the amount of aspirin remaining in a person's bloodstream at the given dosage.

23. Dosage: 250 mg
Time: after 2 hours

24. Dosage: 500 mg
Time: after 3.5 hours

25. Dosage: 750 mg
Time: after 5 hours

26. **BUYING A TRUCK** You buy a used truck in 1996 for $10,500. Each year the truck depreciates by 10%. Write an exponential decay model to represent this situation. Then estimate the value of the truck in 10 years.

BASKETBALL **In Exercises 27–29, use the following information.** Each year in the month of March, the NCAA basketball tournament is held to determine the national champion. At the start of the tournament there are 64 teams, and after each round, one half of the remaining teams are eliminated.

27. Write an exponential decay model showing the number of teams N left in the tournament after round t.

28. How many teams remain after 3 rounds? after 4 rounds?

29. **CRITICAL THINKING** If a team won 6 games in a row in the tournament, does it mean that it won the national championship? Explain your reasoning.

30. **SUMMER CAMP** A summer youth camp had a declining enrollment from 1995 to 2000. The enrollment in 1995 was 320 people. Each year for the next five years, the enrollment decreased by 2%. Copy and complete the table showing the enrollment for each year. Sketch a graph of the results.

Year	1995	1996	1997	1998	1999	2000
Enrollment	?	?	?	?	?	?

CABLE CARS **In Exercises 31–33, use the following information. From 1894 to 1903 the number of miles of cable car track decreased by about 10% per year. There were 302 miles of track in 1894.**

31. Write an exponential decay model showing the number of miles M of cable car track left in year t.

32. Copy and complete the table. You may want to use a calculator.

Year	1894	1896	1898	1899	1900	1901	1903
Miles of track	?	?	?	?	?	?	?

33. Sketch a graph of the results.

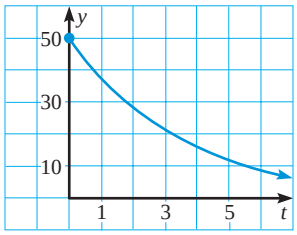
34. **CRITICAL THINKING** A store is having a sale on sweaters. On the first day the price of a sweater is reduced by 20%. The price will be reduced another 20% each day until the sweater is sold. Denise thinks that on the fifth day of the sale the sweater will be free. Is she right? Explain.

Test Preparation

35. MULTIPLE CHOICE In 1995, you purchase a parcel of land for $8000. The value of the land depreciates by 4% every year. What will the approximate value of the land be in 2002?

(A) $5760 (B) $5771 (C) $6012 (D) $6262

36. MULTIPLE CHOICE Which model best represents the decay curve shown in the graph at the right?



(A) $y = 20(1.16)^t$

(B) $y = 50(0.75)^t$

(C) $y = 20(0.75)^t$

(D) $y = 50 + 20(1.16)^t$

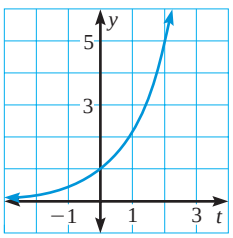
Challenge

UNDERSTANDING GRAPHS **In Exercises 37–39, use a graphing calculator.**

37. Make an input-output table for the equations $y = 4^t$ and $y = \left(\frac{1}{4}\right)^t$. Use -3, -2, -1, 0, 1, 2, and 3 as the input. Then sketch the graph of each equation.

38. VISUAL THINKING Interpret the graphs. How are they related?

39. VISUAL THINKING The graph at the right is $y = 2.26^t$. Based on the relationships between the graphs in Exercise 37, predict the graph of $y = \left(\frac{1}{2.26}\right)^t$.



37. $y = 4^t$

t	y
−3	0.015625
−2	0.0625
−1	0.25
0	1
1	4
2	16
3	64

$y = \left(\frac{1}{4}\right)^t$

t	y
−3	64
−2	16
−1	4
0	1
1	0.25
2	0.0625
3	0.015625

EXTRA CHALLENGE
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MIXED REVIEW

VARIABLE EXPRESSIONS **Evaluate the expression.** (Review 1.3 for 9.1)

40. $4a^2 + 11$ when $a = 5$

41. $c^3 + 6cd$ when $c = 2$ and $d = 1$

42. $b^2 - 4ac$ when $a = 1$, $b = 3$, and $c = 5$

43. $\frac{a^2 - b^2}{2c^2} + 9$ when $a = -3$, $b = 5$, and $c = -2$

SOLVING EQUATIONS **Solve the equation. Round the result to the nearest tenth if necessary.** (Review 3.6)

44. $12m - 9 = 5m - 2$

45. $5(2x + 2.3) - 11.2 = 6x - 5$

46. $-1.3y + 3.7 = 4.2 - 5.4y$

47. $2.5(3.5p + 6.4) = 18.2p - 6.5$

48. TELEVISION A TV station's local news program has 50,000 viewers. The managers of the station plan to increase the number of viewers by 2% per month. Write an exponential growth model to represent the number of viewers in t months. (Review 8.5)

QUIZ 2

Self-Test for Lessons 8.4–8.6

SCIENTIFIC NOTATION **Rewrite in scientific notation.** (Lesson 8.4)

1. 0.011205 **2.** 140,000,000 **3.** 0.000000067 **4.** 30,720,000,000

DECIMAL FORM **Rewrite in decimal form.** (Lesson 8.4)

5. 4.82×10^{3} **6.** 5×10^{9} **7.** 7.04×10^{-6} **8.** 1.112×10^{-2}

9. **INVESTMENT** In 1995, you bought a baseball card for $50 that you expect to increase in value 5% each year for the next 10 years. Write an exponential growth model and estimate the value of the baseball card in 2002. (Lesson 8.5)

10. **BUYING A CAMPER** You buy a used camper in 1995 for $20,000. Each year the camper depreciates by 15%. Write an exponential decay model to represent this situation. Then estimate the value of the camper in 5 years. (Lesson 8.6)

MATH & History — History of Microscopes

APPLICATION LINK
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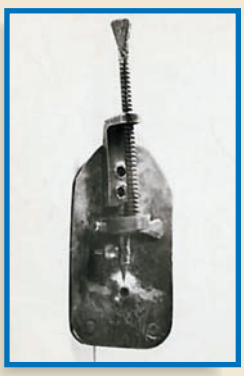
THEN

THE EARLIEST MICROSCOPES consisted of a single, strongly curved lens mounted on a metal plate. These simple microscopes used visible light to illuminate the object being viewed and could magnify objects as much as 400 times.



NOW

TODAY, scientists use microscopes that use electrons instead of light. The *transmission electron microscope* (or TEM) can resolve two objects as close together as 0.0000005 millimeter (mm) with magnifications up to 1,000,000 times. At this magnification, a sneaker would be long enough to reach from Boston to New York City.

1. Write 0.0000005 mm in scientific notation.
2. The distance from Boston to New York City is 1,003,200 feet. Write this number in scientific notation.
3. How many times stronger is the magnification of the TEM compared to the earliest microscopes? Write your answer in scientific notation.