8.5

Exponential Growth Functions

What you should learn

GOAL 1 Write and use models for exponential growth.

GOAL 2 Graph models for exponential growth.

Why you should learn it

▼ To solve **real-life** problems such as finding the weight of a channel catfish in **Example 2**.

GOAL 1 WRITING EXPONENTIAL GROWTH MODELS

A quantity is *growing exponentially* if it increases by the same percent in each unit of time. This is called **exponential growth**.

EXPONENTIAL GROWTH MODEL

C is the **initial amount**.

t is the **time period**.

$$y = C(1 + r)^t$$

$(1 + r)$ is the **growth factor**,

r is the **growth rate**.

The **percent of increase** is $100r$.

EXAMPLE 1 *Finding the Balance in an Account*

COMPOUND INTEREST You deposit $500 in an account that pays 8% annual interest compounded yearly. What is the account balance after 6 years?

SOLUTION

Method 1 **SOLVE A SIMPLER PROBLEM**

Find the account balance A_1 after 1 year and multiply by the growth factor to find the balance for each of the following years. The growth rate is 0.08, so the growth factor is $1 + 0.08 = 1.08$.

$A_1 = 500(1.08) = 540$ — Balance after 1 year

$A_2 = 500(1.08)(1.08) = 583.20$ — Balance after 2 years

$A_3 = 500(1.08)(1.08)(1.08) = 629.856$ — Balance after 3 years

⋮ ⋮

$A_6 = 500(1.08)^6 \approx 793.437$ — Balance after 6 years

▶ The account balance after 6 years will be about $793.44.

Method 2 **USE A FORMULA**

Use the exponential growth model to find the account balance A. The growth rate is 0.08. The initial value is 500.

$A = C(1 + r)^t$ — Exponential growth model

$= 500(1 + 0.08)^6$ — Substitute 500 for C, 0.08 for r, and 6 for t.

$= 500(1.08)^6$ — Simplify.

≈ 793.437 — Evaluate.

▶ The balance after 6 years will be about $793.44.

Catfish Growth

EXAMPLE 2 Writing an Exponential Growth Model

A newly hatched channel catfish typically weighs about 0.3 gram. During the first six weeks of life, its growth is approximately exponential, increasing by about 10% each day.

a. Write a model for the weight during the first six weeks.

b. Find the weight at the end of six weeks.

SOLUTION

a. Let W represent the weight in grams, and let t represent the time in days. The initial weight is $C = 0.3$ and the growth rate is 0.10.

$W = C(1 + r)^t$ — Exponential growth model

$= 0.3(1 + 0.10)^t$ — Substitute 0.3 for C and 0.10 for r.

$= 0.3(1.1)^t$ — Simplify.

b. To find the weight at the end of 6 weeks (or 42 days), substitute 42 for t.

$W = 0.3(1.1)^{42}$ — Substitute.

≈ 16.42910977 — Use a calculator.

≈ 16.4 — Round to the nearest tenth.

▶ The weight is about 16.4 grams.

Population Growth

EXAMPLE 3 Writing an Exponential Growth Model

A population of 20 rabbits is released into a wild-life region. The population triples each year for 5 years.

a. What is the percent of increase each year?

b. What is the population after 5 years?

SOLUTION

a. The population triples each year, so the growth factor is 3.

$1 + r = 3$

▶ So, the growth rate r is 2 and the percent of increase each year is 200%.

b. After 5 years, the population is

$P = C(1 + r)^t$ — Exponential growth model

$= 20(1 + 2)^5$ — Substitute for C, r, and t.

$= 20 \cdot 3^5$ — Simplify.

$= 4860$ — Evaluate.

▶ There will be about 4860 rabbits after 5 years.

STUDENT HELP

Study Tip

Notice that the growth factor and the percent of increase are not the same. In Example 3, the growth factor is 3 but the percent of increase is 200%.

GOAL 2 GRAPHING EXPONENTIAL GROWTH MODELS

You can graph exponential growth models in the same way you graphed exponential functions in Lesson 8.2.

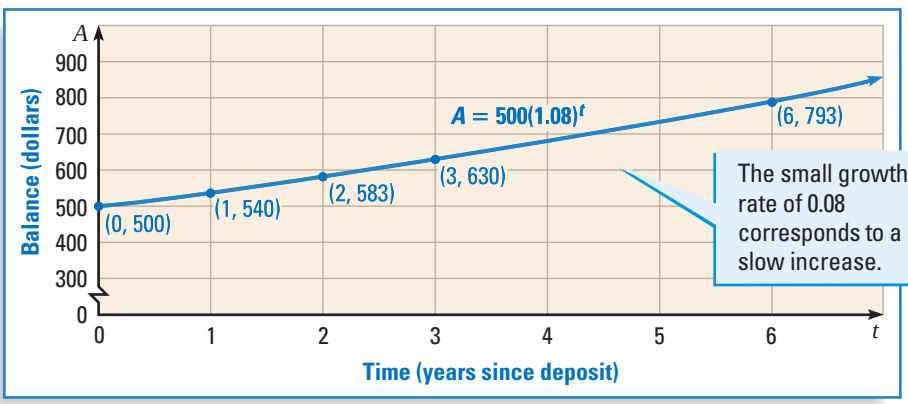
REAL LIFE *Compound Interest*

EXAMPLE 4 *A Model with a Small Growth Factor*

Graph the exponential growth model in Example 1.

SOLUTION

Use the values found in Method 1 of Example 1 to plot points in a coordinate plane. Then, draw a smooth curve through the points.



REAL LIFE *Population Growth*

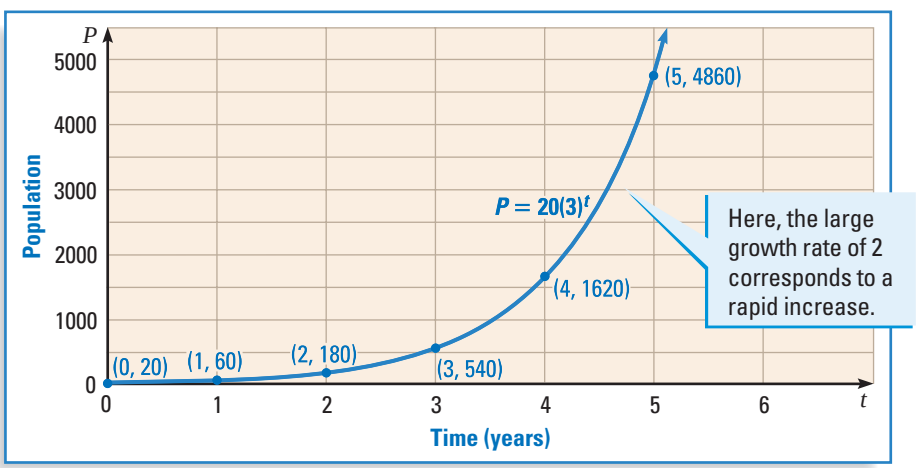
EXAMPLE 5 *A Model with a Large Growth Factor*

Graph the exponential growth model in Example 3.

SOLUTION

Make a table of values, plot the points in a coordinate plane, and draw a smooth curve through the points.

t	0	1	2	3	4	5
P	20	60	180	540	1620	4860



GUIDED PRACTICE

Vocabulary Check ✓

1. In the exponential growth model, $y = C(1 + r)^t$, C is the ? and $(1 + r)$ is the ?.

Concept Check ✓

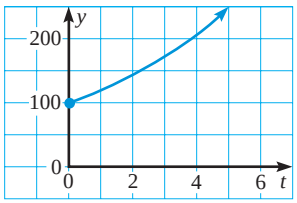
2. Look back at Example 1. Suppose that you got a 12% annual interest rate. Write a new exponential growth model for the balance in the account.

3. Look back at Example 3. Suppose that the rabbit population doubled every year for 5 years. What was the percent of increase? the growth factor?

Skill Check ✓

4. **ACCOUNT BALANCE** You deposit \$500 in an account that pays 4% interest compounded yearly. What is the balance after 5 years? after 10 years?

5. **CHOOSE A MODEL** Which model best represents the growth curve shown in the graph at the right?



A. $y = 100(1.08)^t$

B. $y = 100(1.2)^t$

C. $y = 200(1.08)^t$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

FIND THE BALANCE **You deposit \$1400 in an account that pays 6% interest compounded yearly. Find the balance for the given time period.**

6. 5 years **7.** 8 years **8.** 12 years **9.** 20 years

FIND THE BALANCE **Find the balance after 5 years of an account that pays 4.8% interest compounded yearly given the following investment amounts.**

10. \$250 **11.** \$300 **12.** \$350 **13.** \$400

14. **ACCOUNT BALANCE** A principal of \$200 is deposited in an account that pays 4.2% interest compounded yearly. Find the balance after 5 years.

15. **INVESTING** How much must you deposit in an account that pays 3.5% interest compounded yearly to have a balance of \$400 after 6 years?

16. **INVESTING** How much must you deposit in an account that pays 6% interest compounded yearly to have a balance of \$1000 after 8 years?

17. **BICYCLE RACING** In the Chapter Opener you learned that there is a relationship between the breathing rate of a cyclist and the bicycle speed.

Bicycle speed, x	0	5	10	15	20
Breathing rate, y	6.4	10.7	18.1	30.5	51.4

Let x represent the speed of the bike in miles per hour, and let y represent the cyclist's breathing rate in liters of air taken into the lungs per minute. The breathing rate of a cyclist can be modeled by $y = 6.37(1.11)^x$. What is the cyclist's breathing rate if the bike is traveling 19 miles per hour? 25 miles per hour?

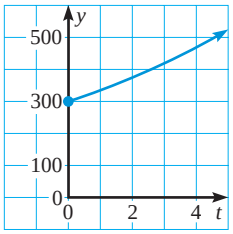
STUDENT HELP

HOMEWORK HELP

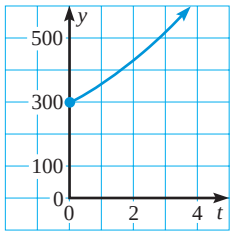
Example 1: Exs. 6–17
Example 2: Exs. 21, 22
Example 3: Ex. 24
Example 4: Exs. 18–20
Example 5: Exs. 25–28

MATCHING THE GRAPH **Match the description with its graph.**

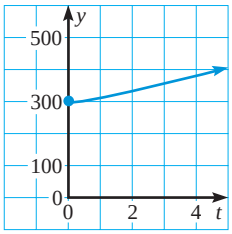
A.



B.



C.



18. Deposit: \$300
Annual rate: 6%

19. Deposit: \$300
Annual rate: 12%

20. Deposit: \$300
Annual rate: 20%

BUSINESS **In Exercises 21 and 22, write an exponential growth model.**

21. A business had a \$10,000 profit in 1990. Then the profit increased by 25% per year for the next 10 years.

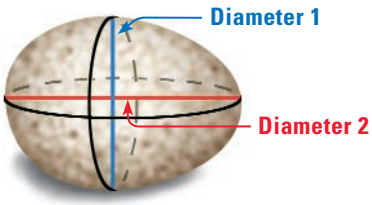
22. A business had a \$20,000 profit in 1990. Then the profit increased by 20% per year for the next 10 years.

23. **VISUAL THINKING** Graph the exponential growth models in Exercises 21 and 22 in the same coordinate plane. Which business would you rather own? Explain.

24. **POPULATION GROWTH** A population of 30 mice is released in a wildlife region. The population doubles each year for 4 years. What is the population after 4 years?

BIOLOGY CONNECTION **In Exercises 25–28, use the following information.**

The average of the diameters of the two cross-sections shown can be related to the length of the adult bird. This average for a robin's egg is 25 millimeters (mm); for a bluebird, 19 mm; and for a chickadee, 13 mm. The graph shows the relationship between the average A of the two diameters and the length L (in centimeters) of an adult bird.

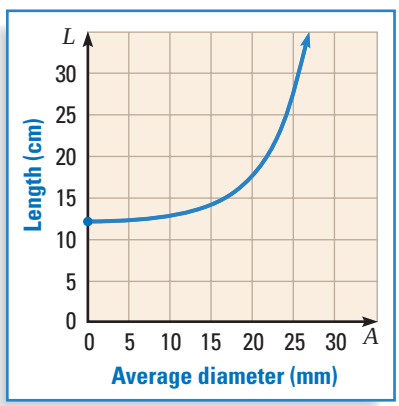


25. **CHOOSING A MODEL** Which model best represents the growth curve shown in the graph?

A. $L = (0.09)(1.23)^A$

B. $L = 12 + (0.09)(1.23)^A$

C. $L = 12 + [(0.09)(1.23)]^A$

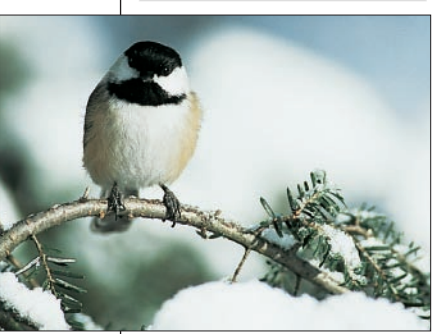


26. Estimate the length of an adult bluebird from the graph.

27. Estimate the length of an adult bluebird using the exponential growth model. How does it compare to the length you found in Exercise 26?

28. Use the exponential growth model to find the ratio of the length of a chickadee to the length of a robin.

FOCUS ON APPLICATIONS



REAL LIFE **CHICKADEES**
Black-capped Chickadees survive freezing weather by lowering their body temperatures at night, entering a state of controlled hypothermia.

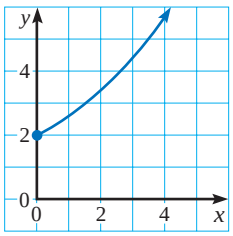
INTERNET **APPLICATION LINK**
www.mcdougallittell.com

Test Preparation

29. MULTIPLE CHOICE The hourly rate of your new job is \$5.00 per hour. You expect a raise of 9% each year. At the end of your first year, you receive your first raise. What will your hourly rate be at the end of your fifth year?

(A) \$5.45 (B) \$7.25 (C) \$7.69 (D) \$7.76

30. MULTIPLE CHOICE What is the equation of the graph?



(A) $y = 2(0.88)^x$ (B) $y = 4(0.88)^x$

(C) $y = 2(1.3)^x$ (D) $y = 4(1.3)^x$

Challenge

31. EXTENSION: COMPOUND INTEREST What is the value of an \$8000 investment after 5 years if it earns 8% annual interest compounded quarterly? To solve, use the compound interest formula, $A = P(1 + i)^n$, where P is the original value of the investment, i is the interest rate per compounding period, n is the total number of compounding periods, and A is the value of the investment after n periods.

a. What is the interest rate per quarter?

b. How many compounding periods (quarters) are there in 5 years?

c. Use the formula $A = P(1 + i)^n$ to find the value of the investment after 5 years.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

PERCENT OF A NUMBER **Find the percent of a number.** (Skills Review page 786)

32. 12% of 56 **33.** 75% of 235 **34.** 1.25% of 90

35. 200% of 130 **36.** 2% of 105 **37.** 0.8% of 120

EVALUATING VARIABLE EXPRESSIONS **Evaluate the expression.** (Review 1.3)

38. $24 + m^3$ when $m = 5$ **39.** $\frac{a^2 - b^2}{ab}$ when $a = 3$ and $b = 5$

40. $x^6 - 1$ when $x = 1.2$ **41.** $3y^4 + 15y$ when $y = -0.02$

42. $(1 - x)^t$ when $x = 0.5$ and $t = 3$ **43.** $\frac{(1 - x)^t}{2}$ when $x = 0.09$ and $t = 2$

44. BREAKFAST You are in charge of bringing breakfast for your scout troop. You buy 6 bagels and 8 donuts for a total of \$4.10. Then you decide to buy 3 extra of each for a total of \$1.80. How much did each bagel and donut cost? (Review 7.4)

SOLVING EQUATIONS **Solve the equation.** (Review 3.3, 3.4, 3.6)

45. $-2(7 - 5x) = 10$ **46.** $25 - (6x + 5) = 4(3x - 5) + 4$

47. $\frac{3}{2}(8m - 30) = -3m$ **48.** $1.4(6.4y - 3.5) = -9.54y + 22.85$