EXPLORING DATA AND STATISTICS

8.4

Scientific Notation

GOAL 1 USING SCIENTIFIC NOTATION

A number is written in **scientific notation** if it is of the form $c \times 10^n$, where $1 \leq c < 10$ and n is an integer.

What you should learn

GOAL 1 Use scientific notation to represent numbers.

GOAL 2 Use scientific notation to describe **real-life** situations, such as the price per acre of the Alaska purchase in **Example 6**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the amount of water discharged by the Amazon River each year in **Example 5**.

ACTIVITY
Developing Concepts

Investigating Scientific Notation

1. Rewrite each number in decimal form.
   a. 6.43×10^4 **b.** 3.072×10^6 **c.** 4.2×10^{-2} **d.** 1.52×10^{-3}
2. Describe a general rule for writing the decimal form of a number given in scientific notation. How many places do you move the decimal point? Do you move the decimal point left or right?

EXAMPLE 1 Rewriting in Decimal Form

Rewrite in decimal form.

a. 2.834×10^2 **b.** 4.9×10^5 **c.** 7.8×10^{-1} **d.** 1.23×10^{-6}

SOLUTION

a. $2.834 \times 10^2 = 283.4$ — Move decimal point right 2 places.

b. $4.9 \times 10^5 = 490{,}000$ — Move decimal point right 5 places.

c. $7.8 \times 10^{-1} = 0.78$ — Move decimal point left 1 place.

d. $1.23 \times 10^{-6} = 0.00000123$ — Move decimal point left 6 places.

EXAMPLE 2 Rewriting in Scientific Notation

a. $34{,}690 = 3.469 \times 10^4$ — Move decimal point left 4 places.

b. $1.78 = 1.78 \times 10^0$ — Move decimal point 0 places.

c. $0.039 = 3.9 \times 10^{-2}$ — Move decimal point right 2 places.

d. $0.000722 = 7.22 \times 10^{-4}$ — Move decimal point right 4 places.

e. $5{,}600{,}000{,}000 = 5.6 \times 10^9$ — Move decimal point left 9 places.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

To multiply, divide, or find powers of numbers in scientific notation, use the properties of exponents.

EXAMPLE 3 Computing with Scientific Notation

Evaluate the expression. Write the result in scientific notation.

a. $(1.4 \times 10^4)(7.6 \times 10^3)$

b. $(1.2 \times 10^{-1}) \div (4.8 \times 10^{-4})$

c. $(4.0 \times 10^{-2})^3$

SOLUTION

a. $(1.4 \times 10^4)(7.6 \times 10^3)$

$= (1.4 \cdot 7.6) \times (10^4 \cdot 10^3)$ Associative property of multiplication

$= 10.64 \times 10^7$ Simplify.

$= 1.064 \times 10^8$ Write in scientific notation.

b. $\dfrac{1.2 \times 10^{-1}}{4.8 \times 10^{-4}} = \dfrac{1.2}{4.8} \times \dfrac{10^{-1}}{10^{-4}}$ Rewrite as a product.

$= 0.25 \times 10^3$ Simplify.

$= 2.5 \times 10^2$ Write in scientific notation.

c. $(4.0 \times 10^{-2})^3 = 4^3 \times (10^{-2})^3$ Power of a product

$= 64 \times 10^{-6}$ Power of a power

$= 6.4 \times 10^{-5}$ Write in scientific notation.

. . . . . .

Many calculators automatically switch to scientific notation to display large or small numbers. Try multiplying 98,900,000 by 500. If your calculator follows standard conventions, it will display the product using scientific notation.

4.945 E10 ← Calculator display for 4.945×10^{10}

EXAMPLE 4 Using a Calculator

Use a calculator to multiply 0.000000748 by 2,400,000,000.

STUDENT HELP

KEYSTROKE HELP
If your calculator does not have an EE function, you can enter the number in scientific notation as a product.
7.48 × 10 y^x 7 +/−

SOLUTION

Enter 0.000000748 as 7.48×10^{-7} and 2,400,000,000 as 2.4×10^9.

KEYSTROKES	DISPLAY
7.48 EE 7 +/− × 2.4 EE 9 ENTER	1.7952 E3

▶ The product is 1.7952×10^3, or 1795.2.

FOCUS ON APPLICATIONS

AMAZON RIVER The Amazon River in Brazil contributes more water to Earth's oceans than any other river. Each second, it discharges 4.2×10^6 cubic feet of water into the Atlantic Ocean.

GOAL 2 SOLVING REAL-LIFE PROBLEMS

EXAMPLE 5 *Multiplying with Scientific Notation*

AMAZON RIVER How much water does the Amazon River discharge into the Atlantic Ocean each year?

SOLUTION

First find the number of seconds in a year. Then multiply the amount of water discharged per second by the number of seconds per year to find the total amount of water discharged into the Atlantic Ocean.

Use unit analysis to find the number of seconds in a year.

$$\frac{\text{days}}{\text{year}} \cdot \frac{\text{hours}}{\text{day}} \cdot \frac{\text{minutes}}{\text{hour}} \cdot \frac{\text{seconds}}{\text{minute}} = \frac{\text{seconds}}{\text{year}}$$

$$\frac{365}{1} \cdot \frac{24}{1} \cdot \frac{60}{1} \cdot \frac{60}{1} = 31{,}536{,}000 \approx 3.2 \times 10^7$$

There are about 3.2×10^7 seconds in a year.

Multiply to find the total amount of water discharged by the Amazon River into the Atlantic Ocean in one year.

$$\begin{aligned}\text{Total amount of water} &= \frac{4.2 \times 10^6 \text{ cubic feet}}{1 \text{ second}} \cdot \frac{3.2 \times 10^7 \text{ seconds}}{1 \text{ year}} \\ &= (4.2 \times 10^6) \cdot (3.2 \times 10^7) \\ &= 13.44 \times 10^{13} \\ &= 1.344 \times 10^{14}\end{aligned}$$

▶ One trillion is 1×10^{12}, so the total amount of water discharged into the Atlantic Ocean by the Amazon River is about 134 trillion cubic feet per year.

EXAMPLE 6 *Dividing with Scientific Notation*

In 1867, the United States purchased Alaska from Russia for $7.2 million. The total area of Alaska is about 3.78×10^8 acres. What was the price per acre?

STUDENT HELP

Look Back
For help with unit rates, see p. 180.

SOLUTION The price per acre is a unit rate.

$$\begin{aligned}\text{Price per acre} &= \frac{\text{Total price}}{\text{Number of acres}} \\ &= \frac{7.2 \times 10^6}{3.78 \times 10^8} \quad \longleftarrow \text{7.2 million} = 7.2 \times 10^6 \\ &= \frac{7.2}{3.78} \times 10^{-2} \\ &\approx 0.019\end{aligned}$$

▶ The price was about 2¢ per acre.

GUIDED PRACTICE

Vocabulary Check ✓

1. Is the number 12.38×10^2 in scientific notation? Explain.

Concept Check ✓

2. Given the number 6.39×10^7, would you move the decimal point to the *left* or to the *right* to rewrite the number in decimal form?

Skill Check ✓

Rewrite in decimal form.

3. 4.3×10^2 **4.** 8.11×10^3 **5.** 2.45×10^{-1} **6.** 9.38×10^5

Rewrite in scientific notation.

7. 39.6 **8.** 0.72 **9.** 1200 **10.** 0.0003

11. 6,900,000 **12.** 0.0000205 **13.** 72,000,000 **14.** 0.000000006

15. **ASTRONOMY** The distance between the ninth planet Pluto and the Sun is 5.9×10^9 kilometers. Light travels at a speed of about 3.0×10^5 kilometers per second. How long does it take light to travel from the Sun to Pluto?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

DECIMAL FORM **Rewrite in decimal form.**

16. 2.14×10^4 **17.** 98×10^{-2} **18.** 7.75×10^0

19. 8.6521×10^3 **20.** 4.65×10^{-4} **21.** 6.002×10^{-6}

22. 4.332×10^8 **23.** 1.00012×10^8 **24.** 1.1098×10^{10}

SCIENTIFIC NOTATION **Rewrite in scientific notation.**

25. 0.05 **26.** 95.2 **27.** 0.0422 **28.** 370.207

29. 700,000,000 **30.** 19.314 **31.** 0.008551 **32.** 2,730,000,000

33. 0.000459 **34.** 0.00032954 **35.** 88,000,000 **36.** 0.0000288

EVALUATING EXPRESSIONS **Evaluate the expression without using a calculator. Write the result in scientific notation and in decimal form.**

37. $(4 \times 10^{-2}) \cdot (3 \times 10^6)$ **38.** $(7 \times 10^{-3}) \cdot (8 \times 10^{-4})$

39. $(6 \times 10^5) \cdot (2.5 \times 10^{-1})$ **40.** $(1.2 \times 10^{-6}) \cdot (2.3 \times 10^4)$

41. $\dfrac{8 \times 10^{-3}}{5 \times 10^{-5}}$ **42.** $\dfrac{1.4 \times 10^{-1}}{3.5 \times 10^{-4}}$ **43.** $\dfrac{6.6 \times 10^{-1}}{1.1 \times 10^{-1}}$

44. $(3.0 \times 10^{-3})^2$ **45.** $(9 \times 10^3)^2$ **46.** $(3 \times 10^{-2})^4$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–24
Example 2: Exs. 25–36
Example 3: Exs. 37–46
Example 4: Exs. 47–52
Example 5: Exs. 61, 62
Example 6: Exs. 58–60

CALCULATOR **Use a calculator to evaluate the expression. Write the result in scientific notation and in decimal form.**

47. 2,000,000 • 12,000 **48.** 6,000,000 • 324,000

49. 0.000279 • 3,940,000,000 **50.** 654,000 • 0.000042

51. $(2.4 \times 10^{-4})^2$ **52.** 0.000094^3

SCIENTIFIC NOTATION IN REAL LIFE **In Exercises 53–57, write the number in scientific notation.**

53. LIGHTNING The speed of a lightning bolt is 120,000,000 feet per second.

54. WORLD POPULATION In 1997, the population of the world was estimated at 5,852,000,000. INTERNET DATA UPDATE of U.S. Bureau of the Census data at www.mcdougallittell.com

55. ASTRONOMY The star Sirius in the constellation Canis Major is about 50,819,000,000,000 miles from Earth.

56. CHEMISTRY The mass of a carbon atom is 0.000000000000000000000002 gram.

57. SIZE OF JUPITER Jupiter, the largest planet in our solar system, has a radius of about 4.4×10^4 miles. Use the equation $V = \frac{4}{3}\pi r^3$ to find Jupiter's volume.

STUDENT HELP

INTERNET APPLICATION LINK Visit our Web site www.mcdougallittell.com for more information about the purchase of Alaska, the Louisiana Purchase, and the Gadsden Purchase in Example 6 and Exs. 58–60.

HISTORY CONNECTION **In Exercises 58–60, use the following information.**

In 1803, the Louisiana Purchase added 8.28×10^5 square miles to the United States. The cost of this land was \$15 million. In 1853, the Gadsden Purchase added 2.94×10^4 square miles, and the cost was \$10 million.

58. Find the average cost of a square mile for each of the purchases.

59. Find the average cost of an acre for each of the purchases. (*Hint:* There are 640 acres in a square mile.)

60. *Writing* Describe a factor that you think might explain the difference in the price per acre for these two purchases.

61. WATERFALL Stanley Falls in Congo, Africa, has an average flow of about 1.7×10^4 cubic meters per second. How much water goes over Stanley Falls in a typical 30-day month?

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Ex. 62.

62. HEARTBEATS Consider a person whose heart beats 70 times per minute and who lives to be 85 years old. Estimate the number of times the person's heart beats during his or her life. Do not acknowledge leap years. Write your answer in decimal form and in scientific notation.

63. TELEPHONE SURVEY Use the table which shows the population and the number of local telephone calls made in five states in 1994 to find the number of local calls made per person in each state.

State	Local Calls	Population
Texas	3.9×10^{10}	1.8×10^7
Minnesota	7.0×10^9	4.6×10^6
Pennsylvania	1.9×10^{10}	1.2×10^7
Vermont	4.7×10^8	5.8×10^5
California	5.6×10^{10}	3.1×10^7

▶ Source: U.S. Bureau of the Census

Test Preparation

64. MULTIPLE CHOICE Which number is not in scientific notation?

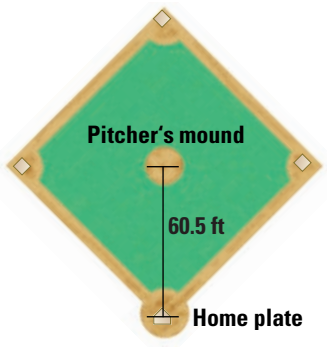
Ⓐ 1×10^4 Ⓑ 3.4×10^{-3} Ⓒ 9.02×10^2 Ⓓ 12.25×10^{-5}

65. MULTIPLE CHOICE Evaluate $\dfrac{1.1 \times 10^{-1}}{5.5 \times 10^{-5}}$ using scientific notation.

Ⓐ 0.2×10^{-4} Ⓑ 2.0×10^4 Ⓒ 2.0×10^3 Ⓓ 0.2×10^4

Challenge

66. BASEBALL A baseball pitcher can throw a ball to home plate in about 0.5 second. The distance between the pitcher's mound and home plate is 60.5 feet.



a. Fill in the missing numbers and simplify the following expression to find the rate, in scientific notation, at which the ball is traveling in millimeters per second.

$$\frac{?\text{ feet}}{?\text{ second}} \cdot \frac{1\text{ meter}}{3.3\text{ feet}} \cdot \frac{?\text{ millimeters}}{1\text{ meter}} \approx \underline{?} \times 10^{?} \text{ millimeters per second}$$

b. To hit a home run, a batter has a leeway of about 200 millimeters in the point of contact between the bat and the ball. What is the most the batter's timing could be off and still hit a home run? Explain your calculations. (*Hint:* Find the time it takes the ball to travel 200 millimeters.)

c. CRITICAL THINKING You are at bat. The pitcher throws you a pitch. Your timing is off by 0.006 second. Could you hit a home run on this pitch? Explain your answer.

EXTRA CHALLENGE
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MIXED REVIEW

PERCENTS AS DECIMALS **Write the percent as a decimal.** (Skills Review page 784 for 8.5)

67. 22% **68.** 87.5% **69.** 0.07% **70.** 8.42%

71. $\frac{1}{2}\%$ **72.** $\frac{3}{4}\%$ **73.** 255% **74.** $1\frac{1}{4}\%$

GRAPHING LINEAR SYSTEMS **Use the graphing method to solve the linear system and describe its solution(s).** (Review 7.5)

75. $4x + 2y = 12$
$-6x + 3y = 6$

76. $3x - 2y = 0$
$3x - 2y = -4$

GRAPHING **Graph the system of linear inequalities.** (Review 7.6)

77. $2x + y \leq 1$
$-2x + y \leq 1$

78. $x + 2y < 3$
$x - 3y > 1$

79. $2x + y \geq 2$
$x \leq 2$

SIMPLIFYING EXPRESSIONS **Simplify the expression.** (Review 8.2 and 8.3)

80. 2^{-4} **81.** $\left(\frac{1}{10}\right)^{-3}$ **82.** $\dfrac{1}{(2x)^{-2}}$ **83.** $\dfrac{7^4 \cdot 7}{7^7}$