8.3 Division Properties of Exponents

What you should learn

GOAL 1 Use the division properties of exponents to evaluate powers and simplify expressions.

GOAL 2 Use the division properties of exponents to find a probability as in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems such as comparing the top speeds of boats in an Olympic rowing competition in **Ex. 58**.

GOAL 1 DIVIDING WITH EXPONENTS

In Lesson 8.1 you learned that you multiply powers with the same base by adding exponents. To divide powers with the same base, you subtract exponents. Here is an example.

$$\frac{4^5}{4^3} = \frac{\overbrace{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}^{\textbf{5 factors}}}{\underbrace{4 \cdot 4 \cdot 4}_{\textbf{3 factors}}} = \underbrace{4 \cdot 4}_{\textbf{2 factors}} = 4^{5-3} = 4^2$$

CONCEPT SUMMARY — **DIVISION PROPERTIES OF EXPONENTS**

Let a and b be numbers and let m and n be integers.

QUOTIENT OF POWERS PROPERTY

To divide powers having the same base, subtract exponents.

$$\frac{a^m}{a^n} = a^{m-n}, \; a \neq 0 \qquad \textbf{Example: } \frac{3^7}{3^5} = 3^{7-5} = 3^2$$

POWER OF A QUOTIENT PROPERTY

To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \; b \neq 0 \qquad \textbf{Example: } \left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$$

EXAMPLE 1 *Using the Quotient of Powers Property*

a. $\dfrac{6^5}{6^4} = 6^{5-4}$

$= 6^1$

$= 6$

b. $\dfrac{(-5)^2}{(-5)^2} = (-5)^{2-2}$

$= (-5)^0$

$= 1$

c. $\dfrac{9^4 \cdot 9^2}{9^7} = \dfrac{9^6}{9^7}$

$= 9^{6-7}$

$= 9^{-1}$

$= \dfrac{1}{9}$

d. $\dfrac{1}{y^5} \cdot y^3 = \dfrac{y^3}{y^5}$

$= y^{3-5}$

$= y^{-2}$

$= \dfrac{1}{y^2}$

EXAMPLE 2 *Using the Power of a Quotient Property*

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Simplify the expression.

a. $\left(\frac{2}{3}\right)^2$ **b.** $\left(-\frac{3}{y}\right)^3$ **c.** $\left(\frac{7}{4}\right)^{-3}$

SOLUTION

a. $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$ Square numerator and denominator and simplify.

b. $\left(-\frac{3}{y}\right)^3 = \left(\frac{-3}{y}\right)^3$ Rewrite fraction.

$= \frac{(-3)^3}{y^3}$ Power of a quotient

$= \frac{-27}{y^3}$ Simplify.

c. $\left(\frac{7}{4}\right)^{-3} = \frac{7^{-3}}{4^{-3}}$ Power of a quotient

$= \frac{4^3}{7^3}$ Definition of negative exponents

$= \frac{64}{343}$ Simplify.

EXAMPLE 3 *Simplifying Expressions*

Simplify the expression.

a. $\frac{2x^2y}{3x} \cdot \frac{9xy^2}{y^4}$ **b.** $\left(\frac{2x}{y^2}\right)^4$

SOLUTION

a. $\frac{2x^2y}{3x} \cdot \frac{9xy^2}{y^4} = \frac{(2x^2y)(9xy^2)}{(3x)(y^4)}$ Multiply fractions.

$= \frac{18x^3y^3}{3xy^4}$ Product of powers

$= 6x^2y^{-1}$ Quotient of powers

$= \frac{6x^2}{y}$ Definition of negative exponents

b. $\left(\frac{2x}{y^2}\right)^4 = \frac{(2x)^4}{(y^2)^4}$ Power of a quotient

$= \frac{2^4 \cdot x^4}{y^{2 \cdot 4}}$ Product of a product and power of a power

$= \frac{16x^4}{y^8}$ Simplify.

FOCUS ON CAREERS

FLOOR TRADER Floor traders at the New York Stock Exchange use hand signals that date back to the 1880s to relay information about their trades.

CAREER LINK
www.mcdougallittell.com

GOAL 2 USING POWERS AS REAL-LIFE MODELS

EXAMPLE 4 *Using the Quotient of Powers Property*

STOCK EXCHANGE The number of shares N (in billions) listed on the New York Stock Exchange from 1977 through 1997 can be modeled by

$$N = 92.56 \cdot (1.112)^t$$

where $t = 0$ represents 1990. Find the ratio of shares listed in 1997 to the shares listed in 1977. ▶ Source: Federal Reserve Bank of New York

SOLUTION

Use $t = -13$ for 1977 and $t = 7$ for 1997.

$$\frac{\text{Number listed in 1997}}{\text{Number listed in 1977}} = \frac{92.56 \cdot (1.112)^7}{92.56 \cdot (1.112)^{-13}}$$

$$= 1.112^{7-(-13)}$$

$$= 1.112^{20}$$

$$\approx 8.4$$

▶ The ratio of shares listed in 1997 to the shares listed in 1977 is 8.4 to 1. There were about 8.4 times as many listed in 1997 as in 1977.

EXAMPLE 5 *Using the Power of a Quotient Property*

PROBABILITY CONNECTION You toss a fair coin ten times. Show that the probability that the coin lands heads up each time is about 0.001.

SOLUTION

Probability that the first toss is heads: $\frac{1}{2}$

Probability that the first two tosses are heads: $\left(\frac{1}{2}\right)^2$

Probability that the first three tosses are heads: $\left(\frac{1}{2}\right)^3$

$\vdots$ $\quad$ $\vdots$

Probability that the first nine tosses are heads: $\left(\frac{1}{2}\right)^9$

Probability that the first ten tosses are heads: $\left(\frac{1}{2}\right)^{10}$

Use the power of a quotient property to evaluate.

$$\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}} = \frac{1}{1024} \approx 0.001$$

▶ The probability is $\left(\frac{1}{2}\right)^{10}$ or about 0.001.

STUDENT HELP

Look Back
For help with probability, see page 114.

GUIDED PRACTICE

Vocabulary Check ✓

1. The expression $\frac{a^4}{a^6}$ can be simplified by using the ? property.

Concept Check ✓

2. Can $\frac{x^8}{y^3}$ be simplified? Explain your answer.

Skill Check ✓

Use the quotient of powers property to simplify the expression.

3. $\frac{5^4}{5^1}$
4. $\frac{7^6}{7^9}$
5. $\frac{a^{12}}{a^9}$
6. $\frac{m^5}{m^{11}}$
7. $\frac{a^5}{a^2}$
8. $\frac{(-2)^8}{(-2)^3}$
9. $\frac{5^3 \cdot 5^5}{5^9}$
10. $\frac{x^7 \cdot x}{x^{-2}}$

Use the power of a quotient property to simplify the expression.

11. $\left(\frac{1}{2}\right)^5$
12. $\left(\frac{3}{5}\right)^3$
13. $\left(\frac{5}{m}\right)^2$
14. $\left(\frac{2}{b}\right)^4$
15. $\left(\frac{5}{4}\right)^{-3}$
16. $\left(\frac{x^4}{2^3}\right)^2$
17. $\left(\frac{x^3}{y^5}\right)^6$
18. $\left(\frac{a^6}{b^9}\right)^5$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

EVALUATING EXPRESSIONS **Evaluate the expression. Write fractions in simplest form.**

19. $\frac{5^6}{5^3}$
20. $\frac{8^3}{8^1}$
21. $\frac{(-3)^6}{-3^6}$
22. $\frac{(-3)^9}{(-3)^9}$
23. $\frac{3^3}{3^{-4}}$
24. $\frac{8^3 \cdot 8^2}{8^5}$
25. $\frac{5 \cdot 5^4}{5^8}$
26. $\left(\frac{3}{4}\right)^2$
27. $\left(\frac{6}{2}\right)^3$
28. $\left(-\frac{2}{3}\right)^3$
29. $\left(-\frac{3}{5}\right)^2$
30. $\left(\frac{9}{6}\right)^{-1}$

SIMPLIFYING EXPRESSIONS **Simplify the expression. The simplified expression should have no negative exponents.**

31. $\left(\frac{3}{x}\right)^4$
32. $\frac{x^4}{x^5}$
33. $\left(\frac{1}{x}\right)^5$
34. $x^3 \cdot \frac{1}{x^2}$
35. $x^5 \cdot \frac{1}{x^8}$
36. $\left(\frac{a^9}{a^5}\right)^{-1}$
37. $\left(\frac{y^2}{y^3}\right)^{-2}$
38. $\frac{m^3 \cdot m^5}{m^2}$
39. $\frac{(r^3)^4}{(r^3)^8}$
40. $\left(\frac{-6x^2y}{2xy^3}\right)^3$
41. $\left(\frac{2x^3y^4}{3xy}\right)^3$
42. $\frac{16x^3y}{-4xy^3} \cdot -\frac{2xy}{-x^{-1}}$
43. $\frac{4x^3y^3}{2xy} \cdot \frac{5xy^2}{2y}$
44. $\frac{36a^8b^2}{ab} \cdot \left(\frac{6}{ab^2}\right)^{-1}$
45. $\frac{16x^5y^{-8}}{x^7y^4} \cdot \left(\frac{x^3y^2}{8xy}\right)^4$
46. $\frac{6x^{-2}y^2}{xy^{-3}} \cdot \frac{(4x^2y)^{-2}}{xy^2}$
47. $\frac{5x^{-3}y^2}{x^5y^{-1}} \cdot \frac{(2xy^3)^{-2}}{xy}$
48. $\left(\frac{2xy^{-2}y^4}{3x^{-1}y}\right)^{-2} \cdot \left(\frac{4xy}{2x^{-1}y^{-3}}\right)^2$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 19–50
Example 2: Exs. 19–48
Example 3: Exs. 31–50
Example 4: Exs. 51–59
Example 5: Exs. 60, 61

ERROR ANALYSIS **In Exercises 49 and 50, find and correct the errors.**

49. $6^3 \div 6 = \frac{6^3}{6}$

$= 1^3$

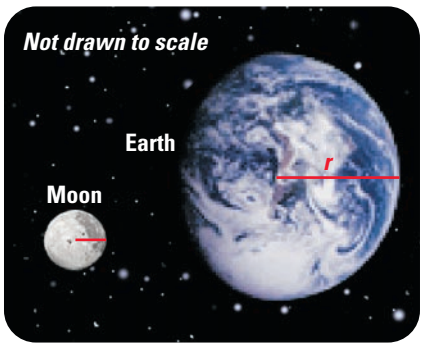
$= 1$

50. $\frac{x^{-9}}{x^{-3}} = x^{-9-3}$

$= x^{-12}$

$= \frac{1}{x^{-12}}$

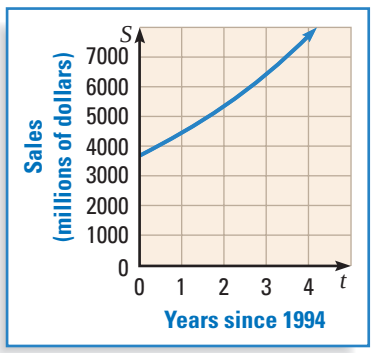
51. **EARTH AND MOON** The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Assuming that the radius of the Moon is $\frac{1}{4}$ the radius of Earth, find the ratio of the volume of Earth to the volume of the Moon. Let r represent the radius of Earth.



52. **RETAIL SALES** From 1994 to 1998, the sales for a national clothing store increased by about the same percent each year. The sales S (in millions of dollars) for year t can be modeled by

$$S = 3723\left(\frac{6}{5}\right)^t$$

where $t = 0$ corresponds to 1994. Find the ratio of 1998 sales to 1995 sales.



53. **BASEBALL SALARIES** The average salary for a professional baseball player in the United States can be approximated by $y = 283(1.2)^t$, where $t = 0$ represents the year 1984. Using this approximation, find the ratio of an average salary in 1988 to an average salary in 1994.

FOCUS ON APPLICATIONS

REAL LIFE **ATLANTIC COD** Adult Atlantic cod average about 3 feet in length and weigh from 10 to 25 pounds, though some may grow much larger.

ATLANTIC COD **In Exercises 54–56, use the following information. The average weight w (in pounds) of an Atlantic cod t years old can be modeled by the equation $w = 1.16(1.44)^t$.** ▶ Source: National Marine Fisheries Service, Northeast Science Center

54. Find the ratio of the weight of a 5-year-old cod to the weight of a 2-year-old cod. Express this ratio as a power of 1.44.

55. A 5-year-old cod weighs how many times as much as a 2-year-old cod?

56. According to the model, what is the average weight of an Atlantic cod when it is hatched? How did you get your answer?

57. **SALES** From 1995 through 1999, the sales for a national furniture store increased by about the same percent each year. The sales s (in millions of dollars) for year t can be modeled by $s = 476(1.13)^t$, where $t = 0$ represents 1995. Find the ratio of 1997 sales to 1999 sales.

FOCUS ON APPLICATIONS

REAL LIFE **ROWING** Shells with 4 or 8 rowers usually have an additional nonrowing member of the team to direct the rest of the crew. This person is called the coxswain.

58. **OLYMPIC ROWING** The racing shells (boats) used in rowing competition usually have 1, 2, 4, or 8 rowers. Top speeds for racing shells in the Olympic 2000-meter races can be modeled by $s = 16.3(1.0285)^n$, where s is the speed in kilometers per hour and n is the number of rowers. Use the model to estimate the ratio of the speed of an 8-rower shell to the speed of a 2-rower shell.

59. **LEARNING SPANISH** You memorized a list of 200 Spanish vocabulary words. Unfortunately, each week you forget one fifth of the words you knew the previous week. The number of words S you remember after n weeks can be approximated by the following equation.

Vocabulary words remembered: $S = 200\left(\frac{4}{5}\right)^n$

a. Copy and complete the table showing the number of words you remember after n weeks.

Weeks, n	0	1	2	3	4	5	6
Words, S	?	?	?	?	?	?	?

b. **CRITICAL THINKING** How many weeks does it take to forget all but three words? Explain your answer.

60. **PROBABILITY CONNECTION** You roll a die eight times. What is the probability that you will roll eight sixes in a row?

61. **PROBABILITY CONNECTION** You roll a die six times. What is the probability that you will roll six even numbers in a row?

Test Preparation

MULTI-STEP PROBLEM **In Exercises 62–65, use the following information.** You work for a real estate company that wants to build a new apartment complex. A team is formed to decide in which state to build the complex. One team member wants to build in Arizona. Another team member wants to build in Michigan. Your boss asks you to decide where to build the complex.

62. You find that the population P of Arizona (in thousands) in 1995 projected through 2025 can be modeled by $P = 4264(1.0208)^t$, where $t = 0$ represents 1995. Find the ratio of the population in 2025 to the population in 2000.

INTERNET **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

63. You find that the population P of Michigan (in thousands) in 1995 projected through 2025 can be modeled by $P = 9540(1.0026)^t$, where $t = 0$ represents 1995. Find the ratio of the population in 2025 to the population in 2000.

▶ Source: U.S. Bureau of the Census

64. Which population is projected to grow more rapidly?

65. *Writing* Use the results from Exercises 62–64 to decide where to build the complex. Write a memo to your boss explaining your decision.

★ Challenge

66. **STACKING PAPER** A piece of notebook paper is about 0.0032 inch thick. If you begin with a stack consisting of a single sheet and double the stack 25 times, how tall will the stack be in inches? How tall will it be in feet? (*Hint:* Write and solve an exponential equation to find the height of the stack in inches. Then use unit analysis to find the height in feet.)

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

POWERS OF TEN Evaluate the expression. (Review 1.2, 8.2 for 8.4)

67. 10^5 **68.** 10^3 **69.** 10^{-4} **70.** 10^{-8}

SKETCHING GRAPHS Sketch the graph of the inequality in a coordinate plane. (Review 6.5)

71. $x \geq 5$ **72.** $x + 3 < 4$ **73.** $y > -2$ **74.** $y \leq -1.5$

75. $x \geq 2.5$ **76.** $3x - y < 0$ **77.** $y \leq \frac{x}{2}$ **78.** $\frac{3}{4}x + \frac{1}{4}y \geq 1$

CHECKING FOR SOLUTIONS Decide whether the ordered pair is a solution of the system. (Review 7.1)

79. $2x + 4y = 2$
$-x + 5y = 13$ $(-3, 2)$

80. $x - 5y = 9$
$3x + y = 11$ $(1, -4)$

81. $8a + 4b = 6$
$4a + b = 3$ $\left(\frac{3}{4}, 0\right)$

82. $3c - 8d = 11$
$c + 6d = 8$ $\left(5, -\frac{1}{2}\right)$

SOLVING LINEAR SYSTEMS Use linear combinations to solve the system. (Review 7.3)

83. $x - y = 4$
$x + y = 12$

84. $-x + 2y = 12$
$x + 6y = 20$

85. $2a + 3b = 17$
$3a + 4b = 24$

QUIZ 1

Self-Test for Lessons 8.1–8.3

Evaluate the expression. (Lessons 8.1, 8.2, and 8.3)

1. $3^3 \cdot 3^4$ **2.** $(2^2)^4$ **3.** $[(8 + 2)^2]^2$ **4.** 7^{-4}

5. $4^{-3} \cdot 4^{-4}$ **6.** $\left(\frac{6}{7}\right)^{-1}$ **7.** $\frac{5^{-3}}{5^2}$ **8.** $\frac{3^4 \cdot 3^6}{3^3}$

9. $\left(\frac{5}{4}\right)^{-3}$ **10.** $\frac{(-2)^9}{(-2)^2}$ **11.** $6^0 \cdot \frac{1}{4^{-3}}$ **12.** $\frac{2^3 \cdot 2^{-4}}{2^{-3}}$

Simplify the expression. Write your answer with no negative exponents. (Lessons 8.1, 8.2, and 8.3)

13. $x^4 \cdot x^5$ **14.** $(-2x)^5$ **15.** $-\frac{3}{a^{-5}}$ **16.** 200^0c^5

17. $\frac{x^6}{x^4}$ **18.** $\frac{x^{-5}}{x^{-6}}$ **19.** $\left(\frac{-2m^2n}{3mn^2}\right)^4$ **20.** $x^4 \cdot \frac{1}{x^3}$

21. $(3a)^3 \cdot (-4a)^3$ **22.** $(8m^3)^2\left(\frac{1}{2}m^2\right)^2$ **23.** $\frac{20x^3y}{4xy^2} \cdot \frac{-6xy}{-x}$

24. **SAVINGS ACCOUNT** You started a savings account in 1994. The balance A is given by $A = 250(1.08)^t$, where $t = 0$ represents the year 2001. What is the balance in the account in 1994? in 1999? in 2001? (Lesson 8.2)