8.2 Zero and Negative Exponents

GOAL 1 Using Zero and Negative Exponents

In this lesson you will study how to use the multiplication properties of exponents when working with negative exponents.

What you should learn

GOAL 1 Evaluate powers that have zero and negative exponents.

GOAL 2 Graph exponential functions.

Why you should learn it

▼ To solve **real-life** problems such as predicting a player's average score per game in **Example 6**.

ACTIVITY Developing Concepts

Investigating Zero and Negative Exponents

1. Copy the table and discuss any patterns you see. Use the patterns to complete the table. Write non-integers as fractions in simplest form.

Exponent, n	3	2	1	0	-1	-2	-3
Power, 2^n	8	4	2	?	?	?	?
Power, 3^n	27	9	3	?	?	?	?
Power, 4^n	64	16	4	?	?	?	?

2. What appears to be the value of a^0 for any number a?
3. How can you evaluate an expression of the form a^{-n}?

DEFINITION OF ZERO AND NEGATIVE EXPONENTS

Let a be a nonzero number and let n be a positive integer.

- A nonzero number to the zero power is 1: $a^0 = 1$, $a \neq 0$.
- a^{-n} is the reciprocal of a^n: $a^{-n} = \frac{1}{a^n}$, $a \neq 0$.

EXAMPLE 1 Powers with Zero and Negative Exponents

a. $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ — 2^{-2} is the reciprocal of 2^2.

b. $(-2)^0 = 1$ — a^0 is 1.

c. $5^{-x} = \frac{1}{5^x}$ — 5^{-x} is the reciprocal of 5^x.

d. $\left(\frac{1}{3}\right)^{-1} = 3$ — The reciprocal of $\frac{1}{3}$ is 3.

e. $0^{-3} = \frac{1}{0^3}$ (Undefined) — Zero has no reciprocal.

STUDENT HELP

HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 Simplifying Exponential Expressions

STUDENT HELP

Study Tip

Informally, you can think of rewriting expressions with positive exponents as "moving factors" from the denominator to the numerator or vice versa.

$$\frac{1}{3x^{-4}} = \frac{x^4}{3}$$

Rewrite with positive exponents.

a. $5(2^{-x})$ **b.** $2x^{-2}y^{-3}$

SOLUTION

a. $5(2^{-x}) = 5\left(\frac{1}{2^x}\right) = \frac{5}{2^x}$

b. $2x^{-2}y^{-3} = 2 \cdot \frac{1}{x^2} \cdot \frac{1}{y^3} = \frac{2}{x^2y^3}$

EXAMPLE 3 Evaluating Exponential Expressions

Evaluate the expression.

a. $3^{-2} \cdot 3^2$ **b.** $(2^{-3})^{-2}$ **c.** 3^{-4}

SOLUTION

a. $3^{-2} \cdot 3^2 = 3^{-2+2}$ Use product of powers property.

$= 3^0$ Add exponents.

$= 1$ a^0 is 1.

b. $(2^{-3})^{-2} = 2^{-3 \cdot (-2)}$ Use power of a power property.

$= 2^6$ Multiply exponents.

$= 64$ Evaluate.

STUDENT HELP

KEYSTROKE HELP

Use [y^x] or [^] to input the exponent −4.

c. You might want to evaluate 3^{-4} with a calculator.

KEYSTROKES	DISPLAY
3 [y^x] 4 [+/−] [=]	.01234568

EXAMPLE 4 Simplifying Exponential Expressions

Rewrite with positive exponents.

a. $(5a)^{-2}$ **b.** $\frac{1}{d^{-3n}}$

SOLUTION

a. $(5a)^{-2} = 5^{-2} \cdot a^{-2}$ Use power of a product property.

$= \frac{1}{5^2} \cdot \frac{1}{a^2}$ Write reciprocals of 5^2 and a^2.

$= \frac{1}{25a^2}$ Multiply fractions.

b. $\frac{1}{d^{-3n}} = (d^{-3n})^{-1}$ Use definition of negative exponents.

$= d^{(-3n) \cdot (-1)}$ Use power of a power property.

$= d^{3n}$ Multiply exponents.

GOAL 2 GRAPHING EXPONENTIAL FUNCTIONS

So far we have used expressions of the form b^n where $b \neq 0$ and n is an integer. To model some situations, we need an **exponential function** of the form $y = a \cdot b^x$ where x is a real number, $b > 0$, and $b \neq 1$. To graph the exponential function $y = a \cdot b^x$, make a table using integer values for x and plot the corresponding points. To complete the graph for all real values of x, connect the points with a smooth curve.

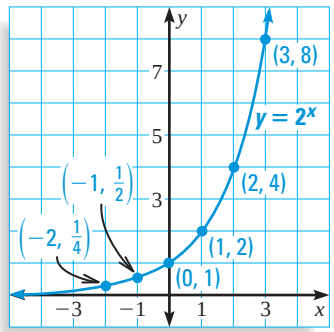
EXAMPLE 5 *Graphing an Exponential Function*

To sketch the graph of $y = 2^x$, make a table that includes negative x-values.

x	-2	-1	0	1	2	3
2^x	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	2	4	8

Draw a coordinate plane and plot the six points given by the table. Then draw a smooth curve through the points.

Notice that the graph has a y-intercept of 1, and that it gets closer to the negative side of the x-axis as the x-values get smaller.



In real-life applications of $y = ab^x$, x is often the time period.

EXAMPLE 6 *Evaluating an Exponential Function*

REAL LIFE *Basketball*

A professional basketball player's first year in the NBA was 1998. Suppose it were estimated that from 1998 to 2010 his average points per game could be modeled by $P = 18.5(1.038)^t$, where $t = 0$ represents the year 2000.

a. Estimate the player's average points per game in 1998.

b. Estimate his average points per game in the year 2000.

SOLUTION

a. $P = 18.5 \cdot 1.038^{-2}$ Substitute -2 for t.

≈ 17.17 Use a calculator.

▶ In the year 1998 his average points per game was about 17.2 points.

b. $P = 18.5 \cdot 1.038^{0}$ Substitute 0 for t.

$= 18.5$ a^0 is 1.

▶ In the year 2000 his average points per game was about 18.5 points.

GUIDED PRACTICE

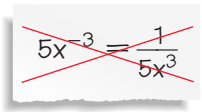
Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

1. The function $y = ab^x$ is a(n) ? function.

2. ERROR ANALYSIS Describe the error at the right.



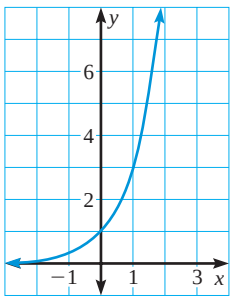
Evaluate the expression.

3. 3^{-1} **4.** 0^{-4} **5.** 0^0 **6.** $6 \cdot 3^0$

Rewrite as an expression with positive exponents.

7. m^{-2} **8.** $3c^{-5}$ **9.** a^5b^{-8} **10.** $\frac{1}{(2x)^{-3}}$

11. Does the graph at the right appear to be the graph of the exponential function $y = 3^x$?



Ex. 11

12. Tell whether the following statement is true.

If a is positive, then a^{-n} is positive.

Explain your reasoning.

13. BASKETBALL Use the information in Example 6. If the player's final year in the NBA is 2010, estimate his average points per game in his final year.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

EVALUATING EXPRESSIONS **Evaluate the exponential expression. Write fractions in simplest form.**

14. 4^{-2} **15.** 3^{-4} **16.** $\left(\frac{1}{5}\right)^{-1}$ **17.** $8\left(\frac{1}{4}\right)^{-1}$

18. $4(4^{-2})$ **19.** $\left(\frac{1}{10}\right)^{-2}$ **20.** $-6^0 \cdot \frac{1}{3^{-2}}$ **21.** $2^{-3} \cdot 2^2$

22. $8^3 \cdot 0^{-1}$ **23.** $7^4 \cdot 7^{-4}$ **24.** $8^{-7} \cdot 8^7$ **25.** $-4 \cdot (-4)^{-1}$

26. $(5^{-3})^2$ **27.** $(-3^{-2})^{-1}$ **28.** $11 \cdot 11^{-1}$ **29.** $4^0 \cdot 5^{-3}$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 14–29
Example 2: Exs. 30–45
Example 3: Exs. 14–29, 49–52
Example 4: Exs. 30–45
Example 5: Exs. 53–63
Example 6: Exs. 64, 65

SIMPLIFYING EXPRESSIONS **Rewrite the expression with positive exponents.**

30. x^{-5} **31.** $3x^{-4}$ **32.** $\frac{1}{2x^{-5}}$ **33.** $x^{-2}y^4$

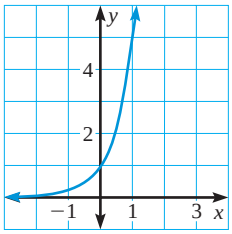
34. x^4y^{-7} **35.** $8x^{-2}y^{-6}$ **36.** $\frac{1}{9x^{-3}y^{-1}}$ **37.** $\frac{1}{4x^{-10}y^{14}}$

38. $(-9)^0x$ **39.** $(-4x)^{-3}$ **40.** $(-10a)^0$ **41.** $(3xy)^{-2}$

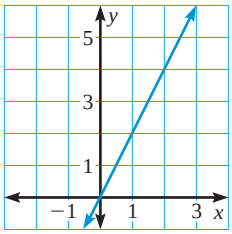
42. $(6a^{-3})^3$ **43.** $\frac{8}{m^{-2}}$ **44.** $\frac{1}{(4x)^{-5}}$ **45.** $\left(\frac{-4x^2}{2x^{-1}}\right)^{-1}$

MATCHING THE GRAPH Match the equation with its graph.

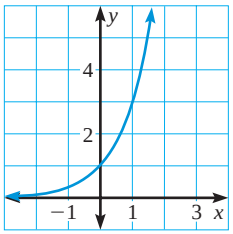
A.



B.



C.



46. $y = 2x$ **47.** $y = 3^x$ **48.** $y = 5^x$

EVALUATING EXPONENTIAL EXPRESSIONS **Use a calculator to evaluate the expression. Round your answer to the nearest hundred thousandth.**

49. 2^{-5} **50.** $(1.1)^{-2}$ **51.** $55 \cdot 5^{-6}$ **52.** $3^{-5} \div 0.9$

CHECKING POINTS **Does the graph of the function contain the point (0, 1)?**

53. $y = -3^x$ **54.** $y = 4^x$ **55.** $y = 4 \cdot 1^x$ **56.** $y = 60^x$

GRAPHING FUNCTIONS **In Exercises 57–60, graph the exponential function.**

57. $y = \left(\frac{1}{3}\right)^x$ **58.** $y = \left(\frac{1}{5}\right)^x$ **59.** $y = 4^{-x}$ **60.** $y = 5^x$

61. VISUAL THINKING Sketch the graphs of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$. How are the graphs related?

62. CRITICAL THINKING Sketch the graphs of $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$. Use these graphs and the ones you sketched in Exercise 61 to predict how the graphs of $y = b^x$ and $y = \left(\frac{1}{b}\right)^x$ are related.

63. COMMON POINTS What point do all graphs of the form $y = a^x$ have in common? Is there a point that all graphs of the form $y = 2(a)^x$ have in common? If so, name the point.

64. SAVINGS ACCOUNT You started a savings account in 1990. The balance A is modeled by $A = 450(1.06)^t$, where $t = 0$ represents the year 2000. What is the balance in the account in 1990? in 2000? in 2010?

65. SHIPWRECKS Suppose that from 1860 to 1980 the number of shipwrecks in the Gulf of Mexico increased by about the same percent each year and that the number of shipwrecks S for each decade t can be modeled by $S = 292(1.2)^t$, where $t = 0$ represents the decade 1920 to 1929.

a. Copy and complete the table below.

	1870–1879	1880–1889	1900–1909	1910–1919	1920–1929
t	−5	−4	−2	−1	0
S	?	?	?	?	?

b. Graph the function and check your results.

FOCUS ON APPLICATIONS

REAL LIFE SHIPWRECKS In 1996 the excavation of the *Belle* off the coast of Texas uncovered the hull of the ship, three bronze cannons, millions of glass beads, pottery and even the skeleton of a crew member.

INTERNET APPLICATION LINK
www.mcdougallittell.com

Test Preparation

QUANTITATIVE COMPARISON **In Exercises 66–68, evaluate each function. Then choose the statement below that is true about the given values of *y*.**

(A) The value of y in column A is greater.

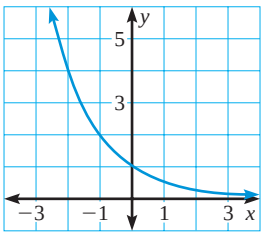
(B) The value of y in column B is greater.

(C) The two values of y are equal.

(D) The relationship cannot be determined from the given information.

		Column A	Column B
66.	When $x = 3$,	$y = 2x$	$y = 2^x$
67.	When $x = 1$,	$y = 2^x$	$y = 2^{-x}$
68.	When $x = 0$,	$y = \left(\frac{1}{2}\right)^x$	$y = \left(\frac{1}{2}\right)^{-x}$

69. MULTIPLE CHOICE What is a possible equation of the graph?



(A) $y = 2^x$ (B) $y = 3^x$

(C) $y = \left(\frac{1}{2}\right)^x$ (D) $y = \left(\frac{1}{3}\right)^x$

★ Challenge

70. ***Writing*** Suppose you did not know that for $b \neq 0$, $b^0 = 1$. Based on the equation $b^2 \cdot b^0 = b^{2+0} = b^2$, explain why you might want to make this definition.

MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 1.3 for 8.3)

71. $\left(\frac{2}{5}\right)^2$ **72.** $\left(\frac{1}{2}\right)^3$ **73.** $\left(-\frac{9}{10}\right)^3$ **74.** $\left(\frac{1}{5}\right)^4$

SOLVE AND GRAPH **Solve the inequality. Then sketch a graph of the solution on a number line.** (Review 6.4)

75. $|5 + x| + 4 \leq 11$ **76.** $|3x + 7| - 4 > 9$ **77.** $|x + 2| - 1 \leq 8$

78. $|3 - x| - 6 > -4$ **79.** $|9 - 2x| + 3 < 4$ **80.** $|3x + 2| + 9 \geq -1$

STATISTICS **Draw a box-and-whisker plot of the data.** (Review 6.7)

81. 48, 10, 48, 25, 40, 42, 44, 23, 21, 13, 50, 17

82. 85, 61, 55, 78, 79, 86, 30, 76, 76, 87, 68, 82

SOLVING SYSTEMS **Use substitution to solve the system.** (Review 7.2)

83. $2x - y = -2$
$4x + y = 5$

84. $-3x + y = 4$
$-9x + 5y = 10$

85. $x + 4y = 300$
$x - 2y = 0$

86. $2x - 3y = 10$
$3x + 3y = 15$

87. $x + 15y = 6$
$-x - 5y = 84$

88. $4x - y = 5$
$2x + 4y = 15$