CHAPTER 7

Chapter Summary

WHAT did you learn?

WHY did you learn it?

Solve a system of linear equations
- by graphing. (7.1)
- by substitution. (7.2)
- by linear combinations. (7.3)

→ Predict traffic at two Internet sites. (p. 400)
→ Analyze a museum's ticket sales. (p. 407)
→ Model compositions of chemical mixtures. (p. 413)

Choose the best method to solve a linear system. (7.4) → Make a decision about two job offers. (p. 420)

Identify the number of solutions of a linear system. (7.5) → Interpret results when modeling with linear systems. (p. 428)

Use a system of linear equations to model a real-life situation. (7.1–7.5) → Find the speed of an airplane flying against the wind. (p. 415)

Solve a system of linear inequalities. (7.6) → Plan a minimum order for free delivery. (p. 434)

Use a system of linear inequalities to model a real-life situation. (7.6) → Find ways to work within a budget. (p. 436)

How does Chapter 7 fit into the BIGGER PICTURE of algebra?

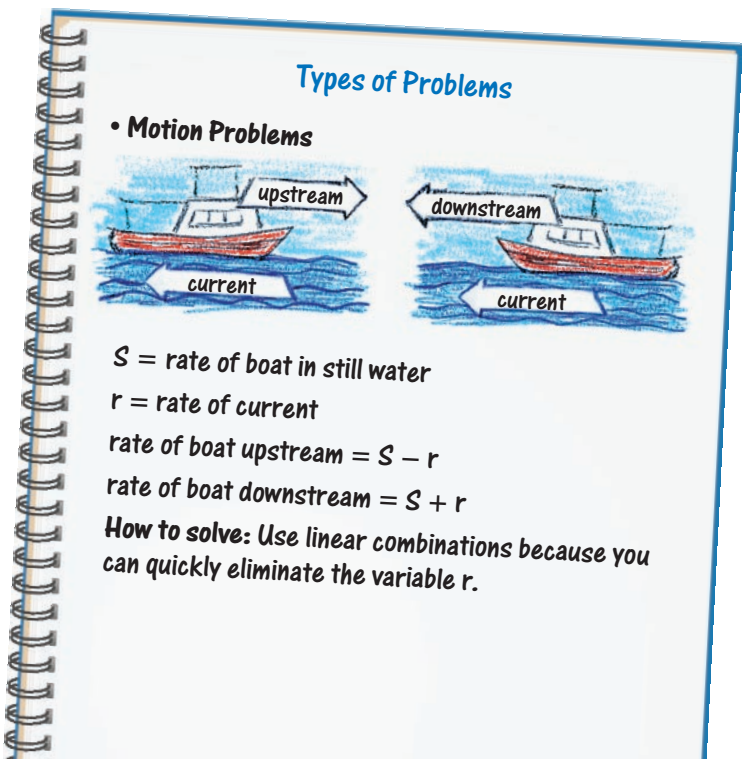
You saw in this chapter how to choose and apply an appropriate method to solve systems of linear equations and systems of linear inequalities. Knowing the advantages and disadvantages of each method is part of becoming an efficient problem solver.

Linear systems often occur as models of real-life situations. The information you gain from studying systems that model real-life problems will help you analyze situations and make decisions.

STUDY STRATEGY

How did you use your list of problem types?

One type of problem that you listed in your notebook, using the **Study Strategy** on page 396, may be this one.



CHAPTER 7

Chapter Review

VOCABULARY

- system of linear equations, or linear system (p. 398)
- solution of a system of linear equations (p. 398)
- linear combination (p. 411)
- system of linear inequalities (p. 432)
- solution of a system of linear inequalities (p. 432)
- graph of a system of linear inequalities (p. 432)

7.1 Solving Linear Systems by Graphing

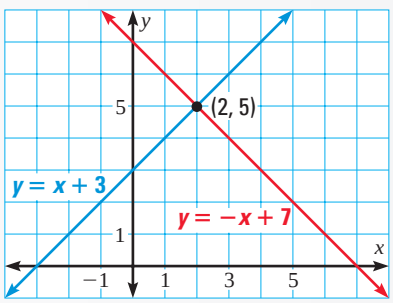
Examples on pp. 398–400

EXAMPLE To solve a linear system by graphing, you can graph each equation using a table of values, the intercepts, or slope-intercept form.

Graph and solve the system.

$y = x + 3$ **Equation 1**

$y = -x + 7$ **Equation 2**



The two lines seem to intersect at the point (2, 5). Check this solution algebraically in both equations.

EQUATION 1:

$y = x + 3$

$5 \stackrel{?}{=} 2 + 3$

$5 = 5$

EQUATION 2:

$y = -x + 7$

$5 \stackrel{?}{=} -(2) + 7$

$5 = 5$

▶ The solution is (2, 5).

Graph and check to solve the linear system.

1. $x + y = 6$
$x - y = 12$

2. $4x - y = 3$
$3x + y = 4$

3. $x + 9y = 9$
$3x + 6y = 6$

4. $5x - y = -5$
$3x + 6y = -3$

5. $7x + 8y = 24$
$x - 8y = 8$

6. $2x - 3y = -3$
$x + 6y = -9$

7.2 Solving Linear Systems by Substitution

Examples on pp. 405–407

EXAMPLE To solve the linear system at the right by substitution, first solve one equation for one of its variables.

$4x - 2y = 0$ **Equation 1**

$8x - 2y = 16$ **Equation 2**

$y = 2x$ **Revised Equation 1**

Then substitute $2x$ for y in Equation 2 and solve for x.

$8x - 2(2x) = 16$ **Substitute $2x$ for y.**

$x = 4$ **Solve for x.**

$y = 2(4) = 8$ **Substitute 4 for x in $y = 2x$.**

▶ The solution is (4, 8). Check the solution in the original equations.

Use the substitution method to solve the linear system.

7. $x + 3y = 9$
$4x - 2y = -6$

8. $-2x - 5y = 7$
$7x + y = -8$

9. $4x - 3y = -2$
$4x + y = 4$

10. $-x + 3y = 24$
$5x + 8y = -5$

11. $4x + 9y = 2$
$2x + 6y = 1$

12. $9x + 6y = 3$
$3x - 7y = -26$

7.3 Solving Linear Systems by Linear Combinations

Examples on pp. 411–413

EXAMPLE To solve the linear system, get coefficients for x or y that are opposites.

$2x - 15y = -10$ **Equation 1**
$-4x + 5y = -30$ **Equation 2**

$4x - 30y = -20$ **Multiply Equation 1 by 2.**
$-4x + 5y = -30$ **Write Equation 2.**
$-25y = -50$ **Add the equations.**
$y = 2$ **Solve for *y*.**

Then substitute 2 for y in Equation 2 and solve for x.

$-4x + 5y = -30$ **Write Equation 2.**
$-4x + 5(2) = -30$ **Substitute 2 for *y*.**
$x = 10$ **Solve for *x*.**

▶ The solution is (10, 2). Check the solution in the original equations.

Use linear combinations to solve the linear system.

13. $-4x - 5y = 7$
$x + 5y = 8$

14. $2x + y = 0$
$5x - 4y = 26$

15. $3x + 5y = -16$
$-2x + 6y = -36$

16. $9x + 6y = 3$
$3y + 6x = 18$

17. $2 - 7x = 9y$
$2y - 4x = 6$

18. $4x - 9y = 1$
$-5x + 6y = 4$

7.4 Applications of Linear Systems

Examples on pp. 418–420

EXAMPLES After you model a real-life problem with a linear system, you have a choice of three methods for solving the system.

GRAPHING: Use to approximate a solution.

SUBSTITUTION: Use when one variable has a coefficient of 1 or −1.

LINEAR COMBINATIONS: Use when no variable has a coefficient of 1 or −1.

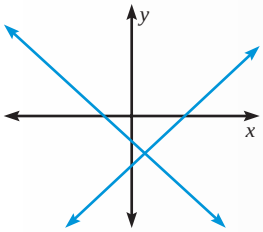
19. **CARNIVAL** You have 50 tickets to ride the Ferris wheel and the roller coaster. If you ride 12 times, using 3 tickets for each Ferris wheel ride and 5 tickets for each roller coaster ride, how many times did you go on each ride?

20. **RENTING MOVIES** You spend $13 to rent five movies for the weekend. Since new releases rent for $3 and regular movies rent for $2, how many regular movies did you rent? How many new releases did you rent?

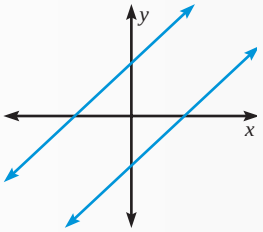
7.5 Special Types of Linear Systems

Examples on pp. 426–428

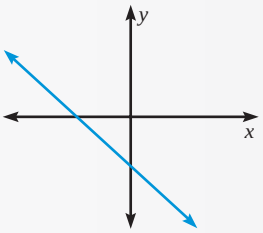
EXAMPLES A system of linear equations in two variables can have exactly one solution, no solution, or infinitely many solutions.



Since the lines intersect, there is one solution.



Since the lines are parallel, there is no solution.



Since the lines coincide, there are infinitely many solutions.

Solve the linear system and tell how many solutions the linear system has.

21. $\frac{1}{3}x + y = 2$
$2x + 6y = 12$

22. $2x - 3y = 1$
$-2x + 3y = 1$

23. $-6x + 5y = 18$
$7x + 2y = 26$

24. $10x + 4y = 25$
$5x + 8y = 11$

25. $14x + 7y = 0$
$-2x + y = 13$

26. $21x + 28y = 14$
$9x + 12y = 6$

7.6 Solving Systems of Linear Inequalities

Examples on pp. 432–434

EXAMPLE The boundary line of the graph of a linear inequality in two variables is dashed if the inequality is $<$ or $>$ and solid if the inequality is $\leq$ or $\geq$.

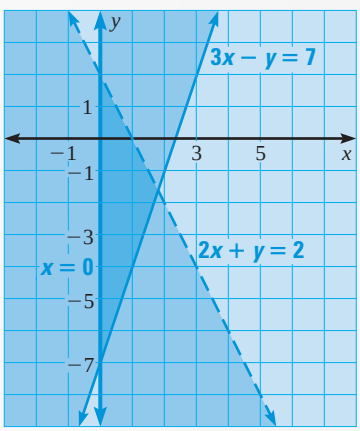
The graph of the system of linear inequalities below is the intersection of the three half-planes shown at the right.

$x \geq 0$ **Inequality 1**
$3x - y \leq 7$ **Inequality 2**
$2x + y < 2$ **Inequality 3**

The graph of the first inequality is the half-plane *on and to the right* of the line $x = 0$.

The graph of the second inequality is the half-plane *on and above* the line $y = 3x - 7$.

The graph of the third inequality is the half-plane *below* the line $y = -2x + 2$.



Graph the system of linear inequalities.

27. $x > -5$
$y < -2$

28. $2x - 10y > 8$
$x - 5y < 12$

29. $-x + 3y \leq 15$
$9x \geq 27$

30. $x < 5$
$y > -2$
$x + 2y > -4$

31. $x + y < 8$
$x - y < 0$
$y \geq -4$

32. $7y > -49$
$-7x + y \geq -14$
$x + y \leq 10$

CHAPTER 7

Chapter Test

Graph and check to solve the linear system.

1. $y = 2x - 3$
$-y = 2x - 1$

2. $6x + 2y = 16$
$-2x + y = -2$

3. $4x - y = 10$
$-2x + 4y = 16$

4. $-4x + y = -10$
$6x + 2y = 22$

5. $3x + 5y = -10$
$-x + 2y = 18$

6. $2x - 3y = 12$
$-x - 3y = -6$

Use the substitution method to solve the linear system.

7. $-4x + 7y = -2$
$-x - y = 5$

8. $7x + 4y = 5$
$x - 6y = -19$

9. $-3x + 6y = 24$
$-2x - y = 1$

10. $5x - y = 7$
$4x + 8y = -12$

11. $x + 6y = 9$
$-x + 4y = 11$

12. $8x + 3y = 0$
$-x - 9y = 92$

Use linear combinations to solve the linear system.

13. $6x + 7y = 5$
$4x - 2y = -10$

14. $-7x + 2y = -5$
$10x - 2y = 6$

15. $-3x + 3y = 12$
$4x + 2y = 20$

16. $3x + 4y = 9$
$4y - 3x = -1$

17. $8x - 2 + y = 0$
$9x - y = 219$

18. $5y - 3x = 1$
$4y + 2x = 80$

Solve the system using the method of your choice and tell how many solutions the system has.

19. $8x + 4y = -4$
$2x - y = -3$

20. $-6x + 3y = -6$
$2x + 6y = 30$

21. $-x + \frac{1}{3}y = -6$
$3x - y = -16$

22. $3x + y = 8$
$4x + 6y = 6$

23. $3x - 4y = 8$
$\frac{9}{2}x - 6y = 12$

24. $6x + y = 12$
$-4x - 2y = 0$

Graph the system of linear inequalities.

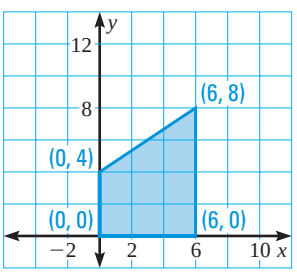
25. $x \leq 4$
$y \geq 1$
$y \leq x + 2$

26. $x < 5$
$y \leq 6$
$y > -2x + 3$

27. $y > \frac{3}{2}x + \frac{3}{2}$
$y < -\frac{1}{4}x - \frac{1}{2}$

Write a system of linear inequalities that defines the shaded region.

28.



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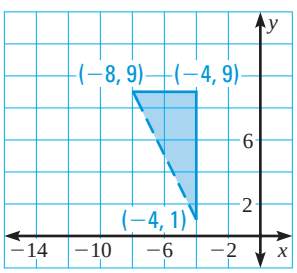
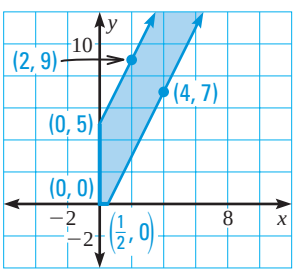


30.



31. **WILD BIRD FOOD** You buy six bags of wild bird food to fill the feeders in your yard. Oyster shell grit, a natural calcium source, sells for $4.00 a bag, and sunflower seeds sell for $4.45 a bag. If you spend $25.80, how many bags of each type of feed are you buying?