7.6 Solving Systems of Linear Inequalities

What you should learn

GOAL 1 Solve a system of linear inequalities by graphing.

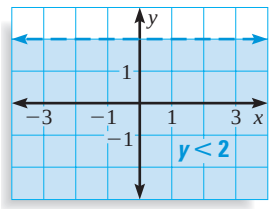
GOAL 2 Use a system of linear inequalities to model a **real-life** situation such as earnings from two jobs in **Exs. 38** and **39**.

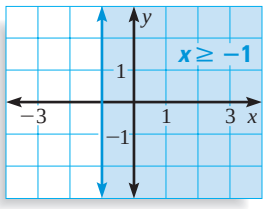
Why you should learn it

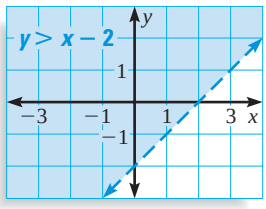
▼ To model **real-life** situations, such as buying spotlights for a concert hall in **Example 4**.

GOAL 1 SOLVING INEQUALITIES BY GRAPHING

From Lesson 6.5, remember that the graph of a linear inequality in two variables is a half-plane. The boundary line of the half-plane is dashed if the inequality is $<$ or $>$ and solid if the inequality is $\leq$ or $\geq$.







Two or more linear inequalities form a **system of linear inequalities** or simply a *system of inequalities*. A **solution** of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system. The **graph** of a system of linear inequalities is the graph of *all solutions* of the system.

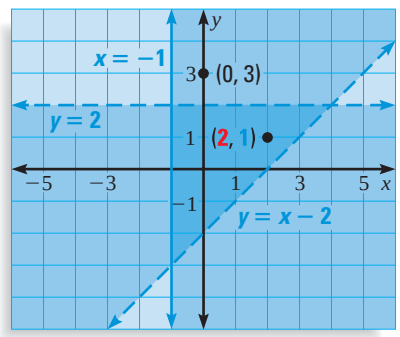
EXAMPLE 1 A Triangular Solution Region

Graph the system of linear inequalities.

$y < 2$ **Inequality 1**
$x \geq -1$ **Inequality 2**
$y > x - 2$ **Inequality 3**

SOLUTION

Graph all three inequalities in the same coordinate plane. The graph of the system is the overlap, or *intersection*, of the three half-planes shown.



✓ **CHECK** You can see from the graph that the point (2, 1) is a solution of the system. To check this, substitute the point into each inequality.

$1 < 2$ **True**
$2 \geq -1$ **True**
$1 > 2 - 2$ **True**

The point (0, 3) is not in the graph of the system. Notice (0, 3) is not a solution of Inequality 1. This point is not a solution of the system.

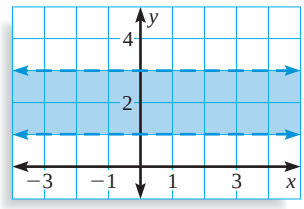
When graphing a system of inequalities, it is helpful to find each corner point (or *vertex*). For instance, the graph in Example 1 has three corner points: $(-1, 2)$, $(-1, -3)$, and $(4, 2)$.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 Solution Region Between Parallel Lines

Write a system of inequalities that defines the shaded region at the right.



SOLUTION

The graph of one inequality is the half-plane *below* the line $y = 3$.

The graph of the other inequality is the half-plane *above* the line $y = 1$.

The shaded region of the graph is the horizontal band that lies *between* the two horizontal lines, $y = 3$ and $y = 1$, but not *on* the lines.

▶ The system of linear inequalities at the right defines the shaded region.

$y < 3$ **Inequality 1**
$y > 1$ **Inequality 2**

EXAMPLE 3 A Quadrilateral Solution Region

STUDENT HELP

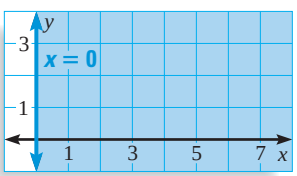
Study Tip
When you are graphing a system of inequalities, try shading with several colored pencils. The graph of the system is the region that has been shaded by every color.

Graph the system of linear inequalities. Label each vertex of the solution region. Describe the shape of the region.

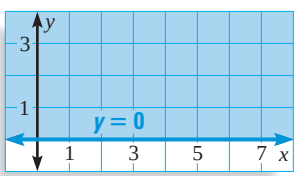
$x \geq 0$ **Inequality 1**
$y \geq 0$ **Inequality 2**
$y \leq 2$ **Inequality 3**
$y \leq -\frac{1}{2}x + 3$ **Inequality 4**

SOLUTION

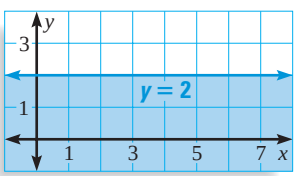
The graph of the first inequality is the half-plane *on and to the right* of the y-axis.



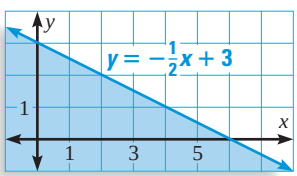
The graph of the second inequality is the half-plane *on and above* the x-axis.



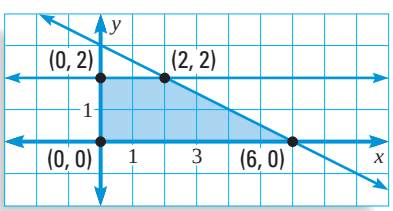
The graph of the third inequality is the half-plane *on and below* the horizontal line $y = 2$.



The graph of the fourth inequality is the half-plane *on and below* the line $y = -\frac{1}{2}x + 3$.



▶ The region that lies in all four half-planes is a *quadrilateral* with vertices at (0, 2), (0, 0), (6, 0), and (2, 2). Note that the point (0, 3) is not a vertex of the solution region even though two boundary lines meet at that point.



GOAL 2 MODELING A REAL-LIFE PROBLEM

EXAMPLE 4 *Writing a System of Linear Inequalities*

LIGHTING You are ordering lighting for a theater so the spotlights can follow the performers. The lighting technician needs at least 3 medium-throw spotlights and at least 1 long-throw spotlight. A medium-throw spotlight costs $1000 and a long-throw spotlight costs $3500. The minimum order for free delivery is $10,000.

a. Write and graph a system of linear inequalities that shows how many medium-throw spotlights and long-throw spotlights should be ordered to get the free delivery.

b. Will an order of 4 medium-throw spotlights and 1 long-throw spotlight be delivered free?

SOLUTION

PROBLEM SOLVING STRATEGY

a. **VERBAL MODEL**

Number of medium-throws $\geq$ 3

Number of long-throws $\geq$ 1

Number of medium-throws $\cdot$ Price of a medium-throw $+$ Number of long-throws $\cdot$ Price of a long-throw $\geq$ 10,000

LABELS

Number of medium-throws = x (no units)

Number of long-throws = y (no units)

Price of a medium-throw = 1000 (dollars)

Price of a long-throw = 3500 (dollars)

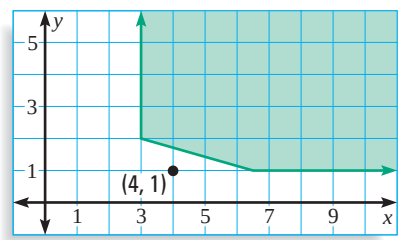
ALGEBRAIC MODEL

$x \geq 3$ Inequality 1

$y \geq 1$ Inequality 2

$1000x + 3500y \geq 10{,}000$ Inequality 3

The graph of the system of inequalities is shown. Any point in the shaded region of the graph is a solution of the system. A fraction of a spotlight cannot be ordered, so only ordered pairs of integers in the shaded region will correctly answer the problem.



b. The point (4, 1) is outside the solution region, so an order of 4 medium-throw spotlights and 1 long-throw spotlight would not be delivered free.

FOCUS ON CAREERS

LIGHTING TECHNICIANS use extra spotlights on each area of the stage to do color changes for the whole stage. Lighting control boards can be programmed in advance to produce complicated lighting sequences.

INTERNET CAREER LINK
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

1. Determine whether the following statement is *true* or *false*. *A solution of a system of linear inequalities is an ordered pair that is a solution of any one of the inequalities in the system.* Explain.

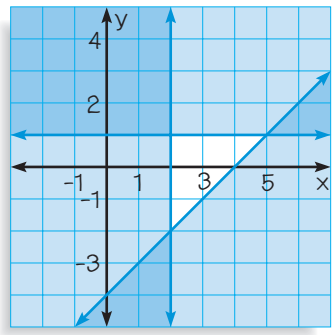
Concept Check ✓

2. How is graphing a system of linear inequalities the same as graphing a system of linear equations? How is it different?
3. After you graph a system of linear inequalities, how can you use algebra to check whether the correct region is shaded?

Skill Check ✓

In Exercises 4 and 5, use both the student graph at the right and the system of linear inequalities shown below.

$y > -1$
$x \geq 2$
$y > x - 4$



Ex. 4

4. Describe the errors the student made while graphing the system.
5. Graph the system correctly.

Graph the system of linear inequalities.

6. $y > -2x + 2$
   $y \leq -1$
7. $y > x$
   $x \leq 1$
8. $x + 1 > y$
   $y \geq 0$

PRACTICE AND APPLICATIONS

STUDENT HELP
Extra Practice to help you master skills is on p. 803.

MATCHING GRAPHS **Match the system of linear inequalities with its graph.**

A. $2x + y < 4$
$-2x + y \leq 4$

B. $2x + y \geq -4$
$x - 2y < 4$

C. $2x + y \leq 4$
$2x + y \geq -4$

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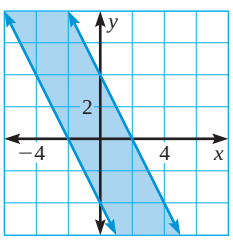

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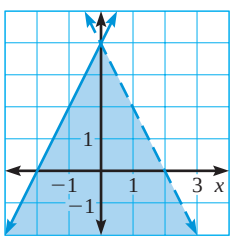

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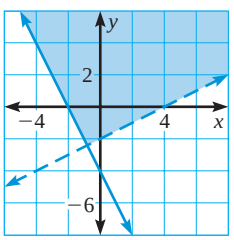
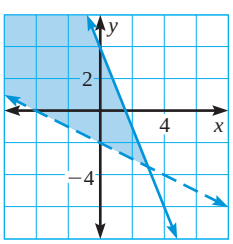


WRITING A SYSTEM **Write a system of linear inequalities that defines the shaded region.**

STUDENT HELP
HOMEWORK HELP
Example 1: Exs. 9–11, 15–26
Example 2: Exs. 12–14
Example 3: Exs. 15–26
Example 4: Exs. 34–38

12.


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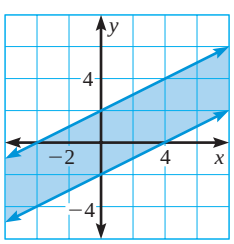

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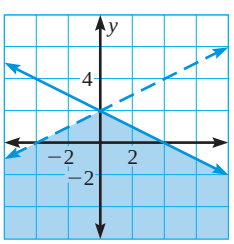


GRAPHING A SYSTEM **Graph the system of linear inequalities.**

15. $2x + y > 2$
$6x + 3y < 12$

16. $2x - 2y < 6$
$x - y < 9$

17. $x - 3y \geq 12$
$x - 6y \leq 12$

18. $x + y < 4$
$x + y > -2$

19. $x + y \leq 6$
$x \geq 1$
$y \geq 0$

20. $\frac{3}{2}x + y < 3$
$x > 0$
$y > 0$

21. $x \geq 0$
$y \geq 0$
$x \leq 3$
$y \leq 5$

22. $x > -2$
$y \geq -2$
$x \leq 1$
$y \leq 4$

23. $-\frac{3}{2}x + y \leq 3$
$\frac{1}{4}x + y > -\frac{1}{2}$
$4x + y < 2$

24. $2x + y \geq 2$
$x \leq 3$
$y \leq \frac{1}{2}$

25. $x \geq 2$
$x - 2y \geq 3$
$3x + y \geq 9$
$x + y \leq 7$

26. $x \geq 1$
$x - 2y \leq 3$
$3x + 2y \geq 9$
$x + y \leq 6$

GRAPHING A SYSTEM **Use a graphing calculator or a computer to graph the system of inequalities. Give the coordinates of each vertex of the solution region.**

27. $x + y \leq 11$
$5x - 3y \leq 15$
$x \geq 0$

28. $5x - 3y \geq -7$
$3x + y \leq 7$
$x + 5y \geq -7$

29. $x + 2y \geq 0$
$5x - 2y \leq 0$
$-x + y \leq 3$

GEOMETRY CONNECTION **In Exercises 30–33, plot the points and draw line segments connecting the points to create the polygon. Then write a system of linear inequalities that defines the polygonal region.**

30. Triangle: $(-2, 0)$, $(2, 0)$, $(0, 2)$

31. Rectangle: $(1, 1)$, $(7, 1)$, $(7, 6)$, $(1, 6)$

32. Triangle: $(0, 0)$, $(-7, 0)$, $(-3, 5)$

33. Trapezoid: $(-1, 1)$, $(1, 3)$, $(4, 3)$, $(6, 1)$

FOCUS ON CAREERS

REAL LIFE CHEF
Chefs prepare meals that appeal to both the taste buds and the eye. They often direct kitchen workers, estimate food requirements, and order new supplies. Some plan meals and develop menus.

INTERNET CAREER LINK
www.mcdougallittell.com

FOOD BUDGET **In Exercises 34 and 35, use the following information.**
You are planning the menu for your restaurant. Each evening you offer two different meals and have at least 240 customers. For Saturday night you plan to offer roast beef and teriyaki chicken. You expect that fewer customers will order the beef than will order the chicken. The beef costs you \$5 per serving and the chicken costs you \$3 per serving. You have a budget of at most \$1200 for meat for Saturday night.

34. Write a system of linear inequalities that shows the various numbers of roast beef and teriyaki chicken meals you could make for Saturday night.

35 Graph the system of linear inequalities.

36. **LOCAL MAGAZINE** A monthly magazine is hiring reporters to cover school events and local events. In each magazine, the managing editor wants at least 4 reporters covering local news and at least 1 reporter covering school news. The budget allows for not more than 9 different reporters' articles to be in one magazine. Graph the region that shows the possible combinations of local and school events covered in a magazine.

37. **GEOMETRY CONNECTION** What is the area of the region described by the system of linear inequalities $x \leq 3$, $y \leq 1$, and $x + y \geq 0$?

EARNING MONEY **In Exercises 38–39, use the following information.**
Suppose you can work a total of no more than 20 hours per week at your two jobs. Baby-sitting pays $5 per hour, and your cashier job pays $6 per hour. You need to earn at least $90 per week to cover your expenses.

38. Write a system of inequalities that shows the various numbers of hours you can work at each job. Graph the result.

39. Give two possible ways you could divide your hours between the two jobs.

EXTENSION: LINEAR PROGRAMMING **In Exercises 40 and 41, use the following information.**
You own a bottle recycling center that receives bottles that are either sorted by color or unsorted. To sort and recycle all of the bottles, you can use up to 4200 hours of human labor and up to 2400 hours of machine time. The system below represents the number of hours your center spends sorting and recycling bottles where x is the number of tons of unsorted bottles and y is the number of tons of sorted bottles.

$4x + y \leq 4200$ **Human labor: up to 4200 hours**
$2x + y \leq 2400$ **Machine time: up to 2400 hours**
$x \geq 0, y \geq 0$ **Cannot have negative number of tons**

40. Graph the system of linear inequalities.

41. a. Find the vertices of your graph.

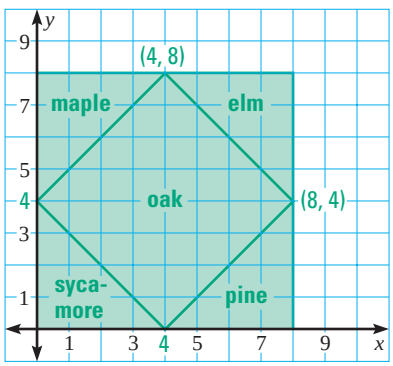
b. You will earn $30 per ton for bottles that are sorted by color and earn $10 per ton for unsorted bottles. Let P be the maximum profit. Substitute the ordered pairs from part (a) into the following equation and solve.

$$P = 10x + 30y$$

c. Assuming that the maximum profit occurs at one of the four vertices, what is the maximum profit?

Test Preparation

42. MULTI-STEP PROBLEM Use the tree farm shown at the right.



a. Write a system of inequalities that defines the region containing maple trees.

b. Write a system of inequalities that defines the region containing sycamore trees.

c. Write a system of inequalities that defines the region containing oak trees.

d. Calculate the area of the oak tree region. Explain your method.

Challenge

43. CRITICAL THINKING Explain why the system of inequalities has no solution.

$2x - y > 4$
$y > 2x - 2$

44. VISUAL THINKING Describe the graph of the system of inequalities.

$2x + 3y > -6$
$2x + 3y \geq 6$

EXTRA CHALLENGE
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MIXED REVIEW

EVALUATING NUMERICAL EXPRESSIONS **Evaluate the expression.** (Review 1.2, 1.3 for 8.1)

45. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

46. $8^2 - 17$

47. $5^3 + 12$

48. $(3^3 - 20)^2$

49. $2^6 - 31$

50. $5 \cdot 5 + 3 \cdot 3 \cdot 3$

EVALUATING ALGEBRAIC EXPRESSIONS **In Exercises 51–56, evaluate the exponential expression.** (Review 1.2)

51. $(x + y)^2$ when $x = 5$ and $y = 2$

52. $(b - c)^2$ when $b = 2$ and $c = 1$

53. $g - h^2$ when $g = 4$ and $h = 8$

54. $x^2 + z$ when $x = 8$ and $z = 12$

55. $m^3 + 1$ when $m = 2$

56. $v^3 + w$ when $v = 3$ and $w = 5$

57. MEAN, MEDIAN, AND MODE Find the mean, the median, and the mode of the collection of numbers shown below. (Review 6.6)

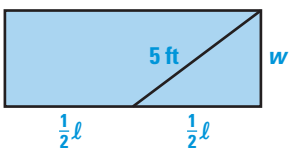
57, 40, 57, 57, 30, 56, 40, 30, 56

58. TEST QUESTIONS Your teacher is giving a test worth 250 points. There are 68 questions. Some questions are worth 5 points and the rest are worth 2 points. How many of each question are on the test? (Review 7.3)

QUIZ 2

Self-Test for Lessons 7.4–7.6

1. GEOMETRY CONNECTION The perimeter of the rectangle is 22 feet. The perimeter of the triangle is 12 feet. Find the dimensions of the rectangle. (Lesson 7.4)



Use the graphing method to solve the linear system and tell how many solutions the system has. (Lesson 7.5)

2. $5x - 2y = 0$
$5x - 2y = -4$

3. $3x + 2y = 12$
$9x + 6y = 18$

4. $-4x + 11y = 44$
$4x - 11y = -44$

5. $3x + y = 9$
$2x + y = 4$

6. $4x + 8y = 8$
$x + y = 1$

7. $3x + 6y = 30$
$4x + 8y = 40$

Graph the system of linear inequalities. (Lesson 7.6)

8. $x - 2y < -6$
$5x - 3y < -9$

9. $-3x + 2y < 6$
$x - 4y > -2$
$2x + y < 3$

10. $x + y < 3$
$x \geq 0$
$y \geq 1$

11. $x \geq 0$
$y \geq 1$
$x \leq 3$
$y \leq 5$

12. $x + y \leq \frac{1}{2}$
$-x + y \leq \frac{1}{2}$
$y \geq 0$

13. $x + y < 26$
$-x + 4y \geq 40$
$2x > 4$

14. INVESTMENTS A total of \$16,000 is invested in two individual retirement accounts paying 5% and 6% annual interest. The combined annual interest is \$860. How much of the \$16,000 is invested in each account? (Lesson 7.4)