7.5 Special Types of Linear Systems

What *you should learn*

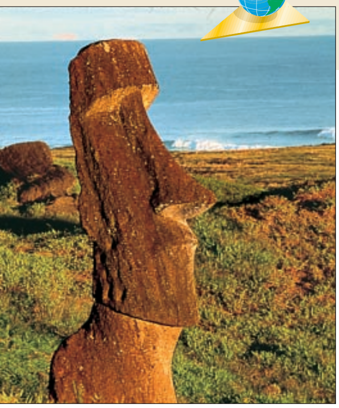
GOAL 1 Identify linear systems as having one solution, no solution, or infinitely many solutions.

GOAL 2 Model **real-life** problems using a linear system, as in **Ex. 30**.

Why *you should learn it*

▼ To solve **real-life** problems such as finding the heights of statues on Easter Island in **Example 4**.

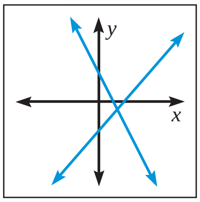
REAL LIFE



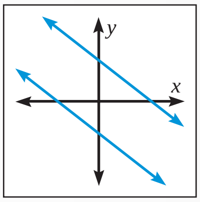
GOAL 1 IDENTIFYING THE NUMBER OF SOLUTIONS

In Lessons 7.1 through 7.4, each linear system in two variables has exactly one solution. The summary below shows that there are two other possibilities.

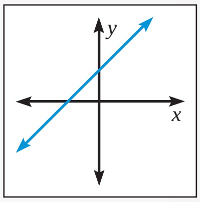
CONCEPT SUMMARY — **NUMBER OF SOLUTIONS OF A LINEAR SYSTEM**



Lines intersect
one solution



Lines are parallel
no solution



Lines coincide
infinitely many solutions (the coordinates of every point on the line)

EXAMPLE 1 *A Linear System with No Solution*

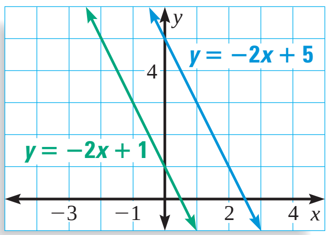
Show that the linear system has no solution.

$2x + y = 5$ **Equation 1**

$2x + y = 1$ **Equation 2**

SOLUTION

Method 1: **GRAPHING** Rewrite each equation in slope-intercept form. Then graph the linear system.



$y = -2x + 5$ **Revised Equation 1**

$y = -2x + 1$ **Revised Equation 2**

▶ Because the lines have the same slope but different y-intercepts, they are parallel. Parallel lines do not intersect, so the system has no solution.

Method 2: **SUBSTITUTION** Because Equation 2 can be revised to $y = -2x + 1$, you can substitute $-2x + 1$ for y in Equation 1.

$2x + y = 5$ **Write Equation 1.**

$2x + (-2x + 1) = 5$ **Substitute $-2x + 1$ for y.**

$1 = 5$ **Simplify. False statement**

▶ The variables are eliminated and you are left with a statement that is not true regardless of the values of x and y. This tells you the system has no solution.

STUDENT HELP

Look Back
For help with equations in one variable with no solution, see p. 155.

EXAMPLE 2 *A Linear System with Many Solutions*

Show that the linear system has infinitely many solutions.

$-2x + y = 3$ **Equation 1**

$-4x + 2y = 6$ **Equation 2**

STUDENT HELP

Study Tip

When both equations of a system are in the form $y = mx + b$, you can determine the number of solutions.

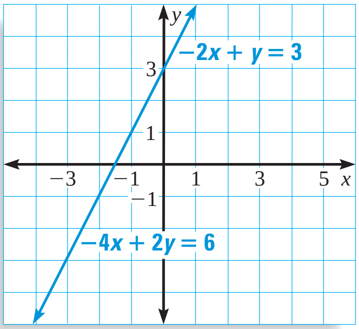
- different slopes: one solution
- same slopes, different y-intercepts: no solution
- same slopes, same y-intercepts: infinitely many solutions

SOLUTION

Method 1: **GRAPHING** Rewrite each equation in slope-intercept form.

$y = 2x + 3$ **Revised Equation 1**

$y = 2x + 3$ **Revised Equation 2**



▶ From these equations you can see that the equations represent the same line. Any point on the line is a solution.

Method 2: **LINEAR COMBINATIONS** You can multiply Equation 1 by -2.

$4x - 2y = -6$ **Multiply Equation 1 by -2.**

$-4x + 2y = 6$ **Write Equation 2.**

$0 = 0$ **Add equations. True statement**

▶ The variables are eliminated and you are left with a statement that is true regardless of the values of x and y. This result tells you that the linear system has infinitely many solutions.

. . . .

Notice in Example 2 that $-2x + y = 3$ can be transformed to look exactly like $-4x + 2y = 6$ by multiplying by 2. The two equations are *equivalent*. That is why your result is $0 = 0$.

EXAMPLE 3 *Identifying the Number of Solutions*

Solve the system and interpret the results.

a. $3x + y = -1$
$-9x - 3y = 3$

b. $x - 2y = 5$
$-2x + 4y = 2$

c. $2x + y = 4$
$4x - 2y = 0$

SOLUTION

a. Using the substitution method, you get $3 = 3$. This is true regardless of the values of x and y. This linear system has infinitely many solutions.

b. Using linear combinations, you get $0 = 12$. This is not true for any values of x and y. This linear system has no solution.

c. Using the substitution method to eliminate y, you get the equation $x = 1$. This linear system has exactly one solution—the ordered pair $(1, 2)$.

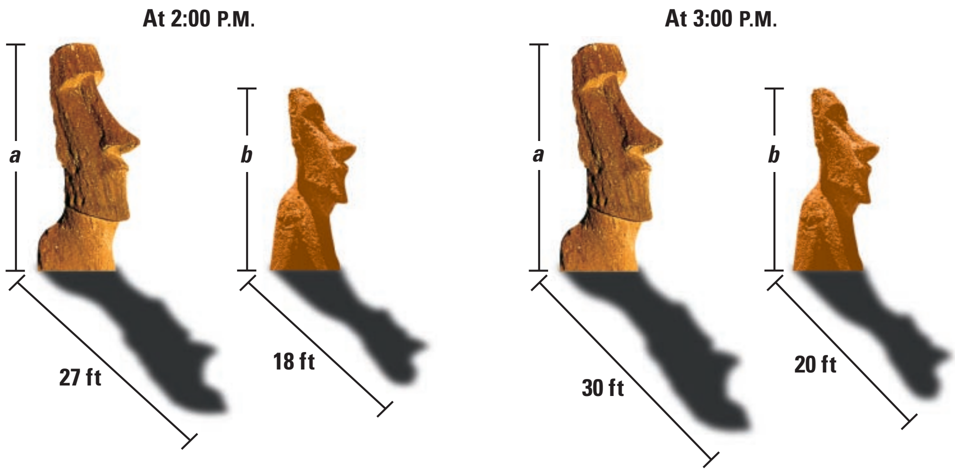
FOCUS ON APPLICATIONS

REAL LIFE **EASTER ISLAND** is located in the South Pacific. On the island there are more than 600 massive stone statues called *moai*. The statues were carved hundreds of years ago, and some are over 40 feet high.

GOAL 2 MODELING A REAL-LIFE PROBLEM

EXAMPLE 4 *Error Analysis*

GEOMETRY CONNECTION Two students are visiting the mysterious statues on Easter Island in the South Pacific. To find the heights of two statues that are too tall to measure, they tried a technique involving proportions. They measured the shadow lengths of the statues at 2:00 P.M. and again at 3:00 P.M. Why were they unable to use their measurements to determine the heights of the statues?



SOLUTION They let a and b represent the heights of the two statues. Because the ratios of corresponding sides of similar triangles are equal, the students wrote the following two equations.

2:00 P.M. $\frac{a}{27} = \frac{b}{18}$

$a = \frac{27}{18}b$

$a = \frac{3}{2}b$

3:00 P.M. $\frac{a}{30} = \frac{b}{20}$

$a = \frac{30}{20}b$

$a = \frac{3}{2}b$

▶ They got stuck because the equations that they wrote are equivalent. All that the students found from these measurements was that $a = \frac{3}{2}b$. In other words, the height of the first statue is one and one half times the height of the second.

To find the heights of the two statues, the students need to measure the shadow length of something else whose height they know or can measure. For instance, if at 2:00 P.M. a student 5 feet tall casts a shadow 3 feet long, the proportion $3:5 = 27:a$ can be used to find the height of the taller statue.

GUIDED PRACTICE

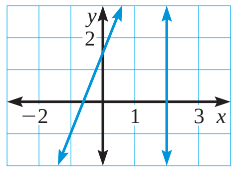
Vocabulary Check ✓

In Exercises 1–3, describe the graph of a linear system that has the given number of solutions. Sketch an example.

1. no solution **2.** infinitely many solutions **3.** exactly one solution

Concept Check ✓

4. **ERROR ANALYSIS** Patrick says that the graph of the linear system shown at the right has no solution. Why is he wrong?



Ex. 4

5. Explain how you can tell if the system of linear equations has a solution. Then solve the system.

$$x - y = 2$$
$$4x - 4y = 8$$

Skill Check ✓

Graph the system of linear equations. Does the system have *exactly one solution, no solution,* or *infinitely many solutions*?

6. $2x + y = 5$
$-6x - 3y = -15$

7. $-6x + 2y = 4$
$-9x + 3y = 12$

8. $2x + y = 7$
$3x - y = -2$

Use the substitution method or linear combinations to solve the linear system and tell how many solutions the system has.

9. $-x + y = 7$
$2x - 2y = -18$

10. $-4x + y = -8$
$-12x + 3y = -24$

11. $-4x + y = -8$
$2x - 2y = -14$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 803.

MATCHING GRAPHS **Match the graph with its linear system. Does the system have *exactly one solution, no solution,* or *infinitely many solutions*?**

A. $-2x + 4y = 1$
$3x - 6y = 9$

B. $2x - 2y = 4$
$-x + y = -2$

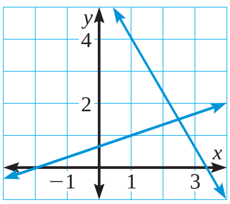
C. $2x + y = 4$
$-4x - 2y = -8$

D. $-x + y = 1$
$x - y = 1$

E. $5x + 3y = 17$
$x - 3y = -2$

F. $x - y = 0$
$5x - 2y = 6$

12.



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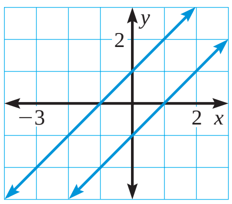

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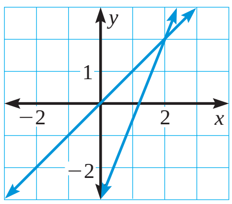

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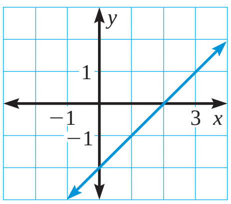

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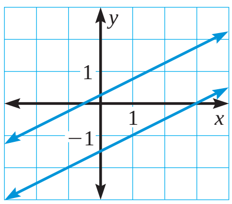

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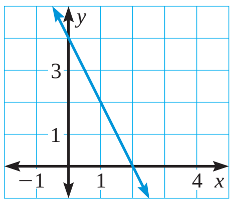

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 12–29
Example 2: Exs. 12–29
Example 3: Exs. 18–29
Example 4: Ex. 30

FOCUS ON CAREERS

REAL LIFE **CARPENTERS** Carpenters know which materials will work best on a variety of projects. Their skills include framing walls, installing doors and windows, and tiling.

INTERNET **CAREER LINK** www.mcdougallittell.com

INTERPRETING ALGEBRAIC RESULTS Use the substitution method or linear combinations to solve the linear system and tell how many solutions the system has.

18. $-7x + 7y = 7$
$2x - 2y = -18$

19. $4x + 4y = -8$
$2x + 2y = -4$

20. $2x + y = -4$
$4x - 2y = 8$

21. $15x - 5y = -20$
$-3x + y = 4$

22. $-6x + 2y = -2$
$-4x - y = 8$

23. $2x + y = -1$
$-6x - 3y = -15$

INTERPRETING GRAPHING RESULTS Use the graphing method to solve the linear system and tell how many solutions the system has.

24. $x + y = 8$
$x + y = -1$

25. $3x - 2y = 3$
$-6x + 4y = -6$

26. $x - y = 2$
$-2x + 2y = 2$

27. $-x + 4y = -20$
$3x - 12y = 48$

28. $6x - 2y = 4$
$-4x + 2y = -\frac{8}{3}$

29. $\frac{3}{4}x + \frac{1}{2}y = 10$
$-\frac{3}{2}x - y = 4$

30. **JEWELRY** You have a necklace and matching bracelet with 2 types of beads. There are 30 small beads and 6 large beads on the necklace. The bracelet has 10 small beads and 2 large beads. The necklace weighs 3.6 grams and the bracelet weighs 1.2 grams. If the chain has no significant weight, can you find the weight of one large bead? Explain.

CARPENTRY SUPPLIES In Exercises 31 and 32, use the following information.

A carpenter is buying supplies for a job. The carpenter needs 4 sheets of oak paneling and 2 sheets of shower tileboard. The carpenter pays \$99.62 for these supplies. For the next job the carpenter buys 12 sheets of oak paneling and 6 sheets of shower tileboard and pays \$298.86.

31. Could you find how much the carpenter is spending on 1 sheet of oak paneling? Explain.

32. If the carpenter later spends a total of \$139.69 for 1 sheet of shower tileboard and 8 sheets of oak paneling, could you find how much 1 sheet of oak paneling costs? Explain.

33. **CRITICAL THINKING** Use linear systems to determine where the x-coordinate is equal to the y-coordinate on the graph of $2x + 3y = 25$.

USING A SPREADSHEET In Exercises 34 and 35, use the spreadsheet at the right to investigate the linear system.

$y = 2x + 3$
$y = 2x - 9$

34. What do the results mean? Explain how you know.

35. Use a spreadsheet or a table of values to verify your answers to Exercises 6–8.

linear system

	A	B	C	D
1	x	y = 2x + 3	y = 2x – 9	col. B – col. C
2	–3	–3	–15	12
3	–2	–1	–13	12
4	–1	1	–11	12
5	0	3	–9	12
6	1	5	–7	12
7	2	7	–5	12
8	3	9	–3	12
9	4	11	–1	12

Exs. 34 and 35

STUDENT HELP

INTERNET **SOFTWARE HELP** Visit our Web site www.mcdougallittell.com to see instructions for several software packages.

Test Preparation

MULTI-STEP PROBLEM **In Exercises 36–38, use the description shown below.**

Choose any number.
Add 10 to the number.
Multiply the result by 2.
Subtract 18 from the result.
Multiply the result by one half.
Subtract the original number.

36. Write a linear equation for the description. Let x represent the number that is chosen and let y represent the final result.

37. To find the number a person would have chosen to obtain a final result of 1, solve the linear system consisting of the equation you wrote in Exercise 36 and the equation $y = 1$. Explain your result.

38. **LOGICAL REASONING** Write a number trick of your own that will give similar results. Explain why your trick works.

★ Challenge

ALTERING LINEAR SYSTEMS **In Exercises 39–42, perform parts (a)–(c).**

a. Find a value of n so that the linear system has infinitely many solutions.

b. Find a value of n so that the linear system has no solution.

c. Graph both results.

EXTRA CHALLENGE
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39. $x - y = 3$
$4x - 4y = n$

40. $x - y = 4$
$-2x + 2y = n$

41. $6x - 9y = n$
$-2x + 3y = 3$

42. $9x + 6y = n$
$1.8x + 1.2y = 3$

MIXED REVIEW

MATCHING GRAPHS **Match the inequality with its graph.** (Review 6.5 for 7.6)

A. $2x + y \le 3$ **B.** $2x + y < 3$ **C.** $2x + y \ge 3$ **D.** $2x + y > 3$

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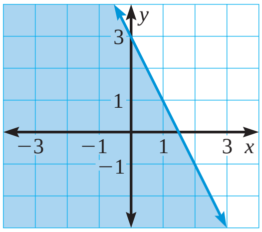

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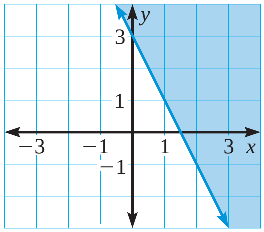

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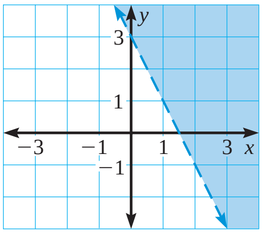

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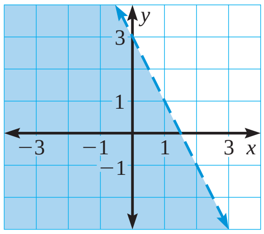

ROCK CLIMBING **In Exercises 47 and 48, you are climbing a 500-foot cliff. By 1:00 P.M. you have climbed 125 feet up the cliff. By 4:00 P.M. you have reached a height of 290 feet.** (Review 4.4)

47. What is your rate of change in height?

48. If you continue climbing the cliff at the same rate, at what time will you reach the top?