EXPLORING DATA AND STATISTICS

7.4 Applications of Linear Systems

GOAL 1 CHOOSING A SOLUTION METHOD

What you should learn

GOAL 1 Choose the best method to solve a system of linear equations.

GOAL 2 Use a system to model **real-life** problems, such as the mixture problem in **Example 2**.

Why you should learn it

▼ To solve **real-life** problems such as choosing between two jobs in **Example 3**.

CONCEPT SUMMARY — WAYS TO SOLVE A SYSTEM OF LINEAR EQUATIONS

GRAPHING: A useful method for approximating a solution, checking the reasonableness of a solution, and providing a visual model. **(Examples 1–3, pp. 398–400)**

SUBSTITUTION: A useful method when one of the variables has a coefficient of 1 or −1. **(Examples 1–3, pp. 405–407)**

LINEAR COMBINATIONS: A useful method when none of the variables has a coefficient of 1 or −1. **(Examples 1–4, pp. 411–413)**

EXAMPLE 1 *Choosing a Solution Method*

SELLING SHOES A store sold 28 pairs of cross-trainer shoes for a total of \$2220. Style A sold for \$70 per pair and Style B sold for \$90 per pair. How many of each style were sold?

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Number of Style A + Number of Style B = Total number sold

Price of Style A · Number of Style A + Price of Style B · Number of Style B = Total receipts

LABELS

Number of Style A	$= x$	(pairs of shoes)
Number of Style B	$= y$	(pairs of shoes)
Total number sold	$= 28$	(pairs of shoes)
Price of Style A	$= 70$	(dollars per pair)
Price of Style B	$= 90$	(dollars per pair)
Total receipts	$= 2220$	(dollars)

ALGEBRAIC MODEL

$x + y = 28$ **Equation 1**

$70x + 90y = 2220$ **Equation 2**

Because the coefficients of x and y are 1 in Equation 1, substitution is most convenient. Solve Equation 1 for x and substitute the result into Equation 2. Simplify to obtain $y = 13$. Substitute 13 for y in Equation 1 and solve for x.

▶ The solution is 15 pairs of Style A and 13 pairs of Style B.

STUDENT HELP

Study Tip

Examples 1 and 2 are called *mixture problems*. Mixture problems often have one equation of the form

$x + y =$ amount

and another equation in which the coefficients of x and y are not 1.

GOAL 2 SOLVING REAL-LIFE PROBLEMS

EXAMPLE 2 *Solving a Mixture Problem*

CAR MAINTENANCE Your car's manual recommends that you use at least 89-octane gasoline. Your car's 16-gallon gas tank is almost empty. How much regular gasoline (87-octane) do you need to mix with premium gasoline (92-octane) to produce 16 gallons of 89-octane gasoline?

SOLUTION

An octane rating is the percent of isooctane in the gasoline, so 16 gallons of 89-octane gasoline contains 89% of 16, or 14.24, gallons of isooctane.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Volume of regular + Volume of premium = Volume of 89-octane

Isooctane in regular + Isooctane in premium = Isooctane in 89-octane

LABELS

Volume of regular	$= x$	(gallons)
Volume of premium	$= y$	(gallons)
Volume of 89-octane	$= 16$	(gallons)
Isooctane in regular	$= 0.87x$	(gallons)
Isooctane in premium	$= 0.92y$	(gallons)
Isooctane in 89-octane	$= 14.24$	(gallons)

ALGEBRAIC MODEL

$x + y = 16$ **Equation 1**

$0.87x + 0.92y = 14.24$ **Equation 2**

Use substitution. Solve Equation 1 for y and substitute into Equation 2.

$y = 16 - x$ **Solve Equation 1 for *y*.**

$0.87x + 0.92(16 - x) = 14.24$ **Substitute 16 − *x* for *y* in Equation 2.**

$-0.05x = -0.48$ **Simplify.**

$x = 9.6$ **Solve for *x*.**

$9.6 + y = 16$ **Substitute 9.6 for *x* in Equation 1.**

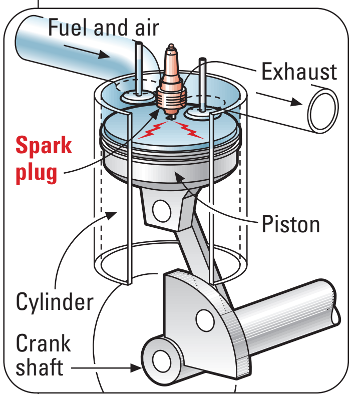
$y = 6.4$ **Solve for *y*.**

✓**CHECK** Substitute in each equation to check your result.

EQUATION 1	EQUATION 2
$9.6 + 6.4 \stackrel{?}{=} 16$	$0.87(9.6) + 0.92(6.4) \stackrel{?}{=} 14.24$
$16.0 = 16$	$8.352 + 5.888 = 14.24$

▶ You need to mix 9.6 gallons of regular gasoline with 6.4 gallons of premium gasoline to get 16 gallons of 89-octane gasoline.

FOCUS ON APPLICATIONS



REAL LIFE **CAR ENGINE** In a car engine, fuel and air in the cylinder are ignited by a spark to provide power to drive the engine. Using a fuel with the correct octane rating makes the engine run more smoothly.

INTERNET **APPLICATION LINK** www.mcdougallittell.com

REAL LIFE
Career Decisions

EXAMPLE 3 *Making a Decision*

You are offered two different sales jobs. Job A offers an annual salary of \$30,000 plus a year-end bonus of 1% of your total sales. Job B offers an annual salary of \$24,000 plus a year-end bonus of 2% of your total sales.

a. How much would you have to sell to earn the same amount in each job?

b. You believe you can sell between \$500,000 and \$800,000 of merchandise per year. Which job should you choose?

Solution

a. First find a linear system that models the situation.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Total earnings = Job A salary + 1% • Total sales

Total earnings = Job B salary + 2% • Total sales

LABELS

Total earnings = y (dollars)

Total sales = x (dollars)

Salary for Job A = **30,000** (dollars)

Salary for Job B = **24,000** (dollars)

ALGEBRAIC MODEL

$y = 30{,}000 + 0.01x$ **Equation 1 (Job A)**

$y = 24{,}000 + 0.02x$ **Equation 2 (Job B)**

Use linear combinations to solve this system of linear equations because you can easily get the coefficients of y to be opposites.

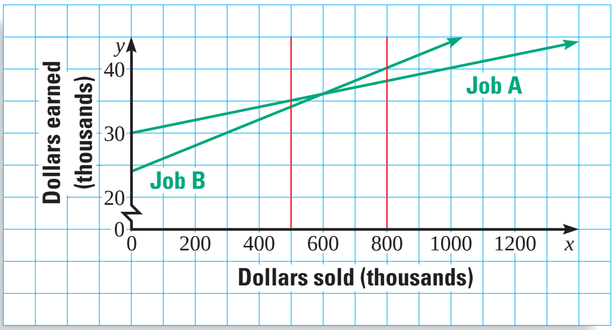
$y = 30{,}000 + 0.01x$	Write Equation 1.
$-y = -24{,}000 - 0.02x$	Multiply Equation 2 by -1.
$0 = 6000 - 0.01x$	Add equations.
$0.01x = 6000$	Add $0.01x$ to each side.
$x = 600{,}000$	Solve for x.

The solution of the linear system is $x = 600{,}000$ and $y = 36{,}000$.

▶ You would have to sell \$600,000 worth of merchandise to earn the same amount of money in each job.

b. One way to decide which job to choose is to draw a graph of the linear system.

▶ From the graph you can see that if your sales are greater than \$600,000, Job B would pay better than Job A.



GUIDED PRACTICE

Concept Check ✓

1. Solve the linear system in Example 3 using the substitution method.

2. You have seen three ways to solve the linear system in Example 3. Which method or methods do you prefer for solving the linear system in Example 3? Why?

Skill Check ✓

Choose a method to solve the linear system. Explain your choice.

3. $x + y = 300$
$x + 3y = 18$

4. $3x + 5y = 25$
$2x - 6y = 12$

5. $2x + y = 0$
$x + y = 5$

PRICE PER GALLON **In Exercises 6–9, use the following problem.** The total cost of 10 gallons of regular gasoline and 15 gallons of premium gasoline is \$32.75. Premium costs \$.20 more per gallon than regular. What is the cost per gallon of each type of gasoline?

6. Write a verbal model for the problem.

7. Assign labels to the verbal model.

8. Write a linear system as an algebraic model.

9. Solve the system and answer the question.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 803.

COMPARING METHODS **Solve the linear system using all three methods on page 418.**

10. $x + y = 2$
$6x + y = 2$

11. $x - y = 1$
$x + y = 5$

12. $3x - y = 3$
$-x + y = 3$

CHOOSING A SOLUTION METHOD **Choose a method to solve the linear system. Explain your choice.**

13. $6x + y = 2$
$9x - y = 5$

14. $2x + 3y = 3$
$5x + 5y = 10$

15. $-3x = 36$
$-6x + y = 1$

16. $2x - 5y = 0$
$x - y = 3$

17. $3x + 2y = 10$
$2x + 5y = 3$

18. $6.2x - 0.5y = -27.8$
$0.3x + 0.4y = 68.7$

SOLVING LINEAR SYSTEMS **Choose a method to solve the linear system. Explain your choice, and then solve the system.**

19. $2x + y = 5$
$x - y = 1$

20. $2x - y = 3$
$4x + 3y = 21$

21. $x - 2y = 4$
$6x + 2y = 10$

22. $3x + 6y = 8$
$-6x + 3y = 2$

23. $x + y = 0$
$3x + 2y = 1$

24. $2x - 3y = -7$
$3x + y = -5$

25. $3y + 4x = 5$
$x + y = 1$

26. $8x + y = 15$
$9 = 2y + 2x$

27. $100 - 9x = 5y$
$0 = 5y - 9x$

28. $x + 2y = 2$
$x + 4y = -2$

29. $-y = -4$
$x + 2y = 4$

30. $0.2x - 0.5y = -3.8$
$0.3x + 0.4y = 10.4$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 10–48
Example 2: Exs. 10–48
Example 3: Exs. 10–48

Focus on Applications

URBAN GARDENS Cities are blossoming with thousands of gardens. They provide green areas of plant life amid acres of steel and concrete.

APPLICATION LINK www.mcdougallittell.com

CHOOSING A METHOD **In Exercises 31–45, solve the linear system.**

31. $6x - y = 18$
$8x + y = 24$

32. $x - y = -4$
$2y + x = 5$

33. $x + 2y = 8$
$3x - 2y = 8$

34. $x + 2y = 1$
$5x - 4y = -23$

35. $8x + 4y = 8$
$-2x + 3y = 12$

36. $3x - 5y = 8$
$-2x + 3y = 3$

37. $2x - y = 6$
$y - x = 0$

38. $8x + 9y = 42$
$6x - y = 16$

39. $7x + 4y = 22$
$-5x - 9y = 15$

40. $3x - 5y = 3$
$9x - 20y = 6$

41. $0.5x + 2.2y = 9$
$6x + 0.4y = -22$

42. $1.5x - 2.5y = 8.5$
$6x + 30y = 24$

43. $3x + 9y = 1$
$2x + 3y = \frac{2}{3}$

44. $3x - 2y = 8$
$x + \frac{3}{2}y = 20$

45. $\frac{1}{2}x - y = -5$
$x - \frac{1}{3}y = 0$

46. **GARDEN PLANTS** In early spring, you buy 6 potted tomato plants for your container garden. The plants contained in 8-inch pots sell for \$5 and the plants contained in 10-inch pots sell for \$8. If you spend \$36, how many of each size are you buying?

$\boxed{\text{Number of 8-inch pots}} + \boxed{\text{Number of 10-inch pots}} = \boxed{\text{Total number buying}}$

$\boxed{\text{Cost of all 8-inch pots}} + \boxed{\text{Cost of all 10-inch pots}} = \boxed{\text{Total cost}}$

47. **SCIENCE CONNECTION** In your chemistry class you have a bottle of 5% boric acid solution and a bottle of 2% boric acid solution. You need 60 milliliters of a 3% boric acid solution for an experiment. How much of each solution do you need to mix together?

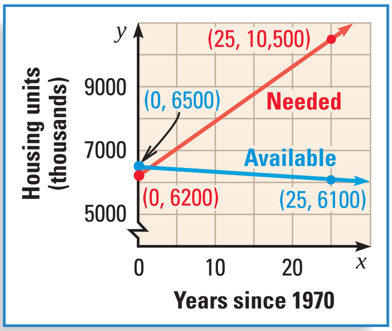
48. **TREE GROWTH** You plant a 14-inch hemlock tree in your backyard that grows at a rate of 4 inches per year and an 8-inch blue spruce tree that grows at a rate of 6 inches per year. In how many years after you plant the trees will the two trees be the same height? How tall will each tree be?

PARTY PLANNING **In Exercises 49 and 50, you are planning a birthday party for your 8-year-old cousin. You can have the party at a pizza place for \$8 per person plus \$30 for favors and a small cake or at a taco place for \$12 per person plus \$14 for a large cake.**

49. How many children would you have to invite to the party for the cost to be the same for both places?

50. How many children would you have to invite to the party for the cost to be less at the pizza place?

51. **LOW-INCOME HOUSING** The graph at the right represents the demands for low-income rental housing in the United States and the number of affordable rental units. Use the coordinates given to write a linear system. Solve the system. What does the point of intersection represent in the context of the situation?



DRAWING PORTRAITS **In Exercises 52–54, you are earning extra money drawing sketches for your friends and family. You receive \$12 per sketch. Your expenses for art supplies are \$50.**

52. Write an equation that models only your income.

53. Suppose you plan to spend an additional \$7 per sketch for a frame. Write an equation that models only your expenses.

54. How many framed sketches do you have to sell in order for your income to equal your expenses?

55. **TREADMILLS** You exercised on a treadmill for 1.5 hours. You ran at 4 miles per hour, then you sprinted at 6 miles per hour. If the treadmill monitor says that you ran and sprinted 7 miles, how long did you run at each speed?

STUDENT HELP

INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for help with Ex. 55.

56. **TRAVEL TIME** You are driving to a concert. To arrive on time, you plan to travel at an average speed of 45 miles per hour. For the first two hours of the trip you travel at 40 miles per hour. Find the expression for the total time you traveled. Then use the following verbal models to find how many hours you traveled at 55 miles per hour to arrive on time.

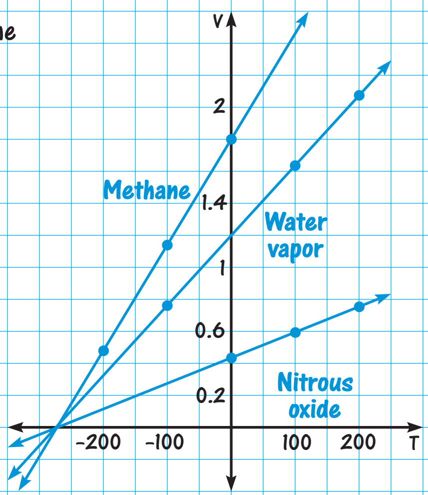
$\boxed{\text{Distance at 40 mi/h}} + \boxed{\text{Distance at 55 mi/h}} = \boxed{\text{Total distance}}$

$\boxed{\text{Average speed}} \cdot \boxed{\text{Total time}} = \boxed{\text{Total distance}}$

SCIENCE CONNECTION **In Exercises 57–60, a chemist is calculating absolute zero using the procedure below. Absolute zero is the lowest possible temperature for all substances. It is believed to be the temperature at which the volume of an ideal gas (at constant pressure) contracts to zero.**

- On page 1811, I found several readings for the volume and the temperature of three gases.
- I made a scatter plot of the data.
- I found a linear equation that relates the volume (in liters) and the temperature (in degrees Celsius) of each gas by fitting a line to the data.

  Methane: $-2T + 302V = 546.4$
  Water: $-T + 228V = 273.2$
  Nitrous oxide: $-3T + 1872V = 819.6$
- I solved the linear system.



STUDENT HELP

INTERNET **APPLICATION LINK**
Visit our Web site www.mcdougallittell.com for more information about absolute zero.

57. What methods could the chemist use to solve the linear system?

58. Use the chemist's graph to predict what absolute zero is on the Celsius scale.

59. Does the chemist need to solve all three equations to find T? Explain.

60. Solve the system algebraically.

MARBLES In Exercises 61–63, consider a bag containing 12 marbles that are either red or blue. A marble is drawn at random. There are three times as many red marbles as there are blue marbles in the bag.

61. Write a linear system to describe this situation.

62. How many red marbles are in the bag?

63. PROBABILITY CONNECTION What is the probability of drawing a red marble?

Test Preparation

64. MULTIPLE CHOICE At what point do the lines $3x - 2y = 0$ and $5x + 2y = 0$ intersect?

(A) (1, 2) (B) (5, 2) (C) (3, 2) (D) (0, 0)

65. MULTIPLE CHOICE Find the x-value of the solution for the linear system below.

$y = 2x - 2$
$y = 3x + 1$

(A) −8 (B) −3 (C) 3 (D) 8

★ Challenge

66. RELAY RACE The total time for a two-member team to complete a 5765-meter relay race was 19 minutes. The first runner averaged 285 meters per minute and the second runner averaged 335 meters per minute. Use the verbal model to find how many minutes the first runner ran.

Time for 1st runner + Time for 2nd runner = Total time

Distance for 1st runner + Distance for 2nd runner = Total distance

EXTRA CHALLENGE
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MIXED REVIEW

PARALLEL LINES Decide whether the graphs of the two equations are parallel lines. (Review 4.6 for 7.5)

67. $y = 4x + 3;\ 2y - 8x = -3$

68. $4y + 5x = 1;\ 10x + 2y = 2$

69. $3x + 9y + 2 = 0;\ 2y = -6x + 3$

70. $4y - 1 = 5;\ 6y + 2 = 8$

GRAPHING FUNCTIONS Graph the function. (Review 4.8)

71. $f(x) = 2x + 3$

72. $h(x) = x + 5$

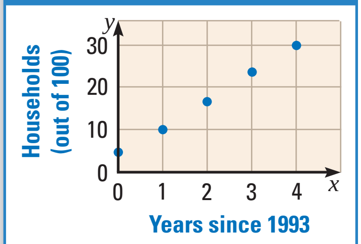
73. $g(x) = -3x - 1$

CD-ROM DRIVES In Exercises 74 and 75, the number of United States households with CD-ROM drives in their computers is shown in the scatter plot at the right. (Review 5.4 and 5.7)

74. Find an equation of the line that you think best fits the data. Notice that x represents the number of years since 1993 and y represents the number of households out of 100.

75. Use a linear model to estimate the number of households that will have computer-installed CD-ROM drives in 2003.

Computers with CD-ROM



DATA UPDATE
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