7.2 Solving Linear Systems by Substitution

What you should learn

GOAL 1 Use substitution to solve a linear system.

GOAL 2 Model a **real-life** situation using a linear system, such as the average number of visitors to a Museum in **Example 3**.

Why you should learn it

▼ To solve **real-life** problems such as finding a distance run in **Exs. 46** and **47**.

GOAL 1 USING SUBSTITUTION

In this lesson you will study an algebraic method for solving a linear system.

SOLVING A LINEAR SYSTEM BY SUBSTITUTION

STEP 1 Solve one of the equations for one of its variables.

STEP 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

STEP 3 Substitute the value from Step 2 into the revised equation from Step 1 and solve.

STEP 4 Check the solution in each of the original equations.

EXAMPLE 1 *The Substitution Method*

Solve the linear system. $-x + y = 1$ **Equation 1**; $2x + y = -2$ **Equation 2**

SOLUTION

1 Solve for y in Equation 1.

$y = x + 1$ **Revised Equation 1**

2 Substitute $x + 1$ for y in Equation 2 and solve for x.

$2x + y = -2$ **Write Equation 2.**

$2x + (x + 1) = -2$ **Substitute $x + 1$ for y.**

$3x + 1 = -2$ **Simplify.**

$3x = -3$ **Subtract 1 from each side.**

$x = -1$ **Solve for x.**

3 To find the value of y, substitute -1 for x in the revised Equation 1.

$y = x + 1$ **Write revised Equation 1.**

$y = -1 + 1$ **Substitute -1 for x.**

$y = 0$ **Solve for y.**

4 Check that $(-1, 0)$ is a solution by substituting -1 for x and 0 for y in each of the original equations.

▶ The solution is $(-1, 0)$.

EXAMPLE 2 The Substitution Method

Solve the linear system.

$$2x + 2y = 3 \quad \text{Equation 1}$$
$$x - 4y = -1 \quad \text{Equation 2}$$

STUDENT HELP

Study Tip

When using substitution, you will get the same solution whether you solve for y first or x first. You should begin by solving for the variable that is easier to isolate.

SOLUTION

Solve for x in Equation 2 because it is easy to isolate x.

$$x = 4y - 1 \quad \text{Revised Equation 2}$$

Substitute $4y - 1$ for x in Equation 1 and solve for y.

$$2x + 2y = 3 \quad \text{Write Equation 1.}$$
$$2(4y - 1) + 2y = 3 \quad \text{Substitute } 4y - 1 \text{ for } x.$$
$$8y - 2 + 2y = 3 \quad \text{Distribute the 2.}$$
$$10y - 2 = 3 \quad \text{Simplify.}$$
$$10y = 5 \quad \text{Add 2 to each side.}$$
$$y = \frac{1}{2} \quad \text{Solve for } y.$$

Substitute $\frac{1}{2}$ for y in the revised Equation 2 to find the value of x.

$$x = 4y - 1 \quad \text{Write revised Equation 2.}$$
$$x = 4\left(\frac{1}{2}\right) - 1 \quad \text{Substitute } \frac{1}{2} \text{ for } y.$$
$$x = 1 \quad \text{Solve for } x.$$

Check by substituting 1 for x and $\frac{1}{2}$ for y in each of the original equations.

EQUATION 1	EQUATION 2
$2x + 2y = 3$	$x - 4y = -1$
$2(1) + 2\left(\frac{1}{2}\right) \stackrel{?}{=} 3$	$1 - 4\left(\frac{1}{2}\right) \stackrel{?}{=} -1$
$2 + 1 \stackrel{?}{=} 3$	$1 - 2 \stackrel{?}{=} -1$
$3 = 3$	$-1 = -1$

▶ The solution is $\left(1, \frac{1}{2}\right)$.

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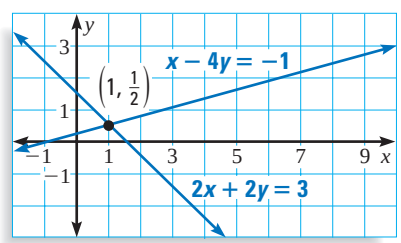
When you use the substitution method, you can still use a graph to check the reasonableness of your solution. For instance, the graph at the right shows a graphic check for Example 2.



GOAL 2 MODELING A REAL-LIFE PROBLEM

EXAMPLE 3 *Writing and Using a Linear System*

MUSEUM ADMISSIONS In one day the National Civil Rights Museum in Memphis, Tennessee, collected $1590 from 321 people admitted to the museum. The price of each adult admission is $6. People with the ages of 4–17 pay the child admission, $4. Estimate how many adults and how many children were admitted that day.

SOLUTION

Use a verbal model to find the number of adults and children admitted to the museum that day.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\text{Number of adults} + \text{Number of children} = \text{Total number admitted}$$

$$\text{Number of adults} \cdot \text{Price of adult admission} + \text{Number of children} \cdot \text{Price of child admission} = \text{Total amount collected}$$

LABELS

Number of adults = x (people)

Number of children = y (people)

Total number admitted = **321** (people)

Price of adult admission = **6** (dollars per person)

Price of child admission = **4** (dollars per person)

Total amount collected = **1590** (dollars)

ALGEBRAIC MODEL

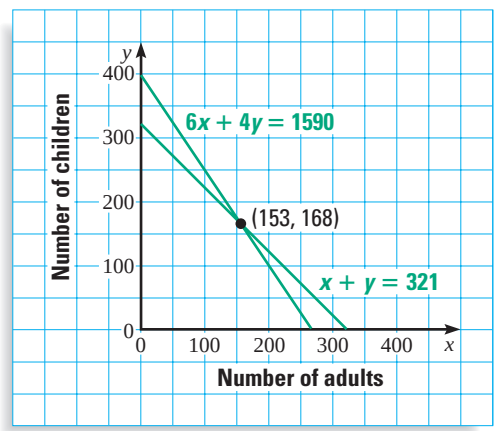
$x + y = 321$ Equation 1 (Number admitted)

$6x + 4y = 1590$ Equation 2 (Amount collected)

Using the substitution method, you can determine that $x = 153$ when $y = 168$. The solution is the ordered pair (153, 168).

The graphs of the two equations appear to intersect at (153, 168). The solution checks graphically.

▶ You can conclude that 153 adults were admitted and 168 children were admitted to the National Civil Rights Museum that day.



FOCUS ON APPLICATIONS

REAL LIFE **NATIONAL CIVIL RIGHTS MUSEUM**

The National Civil Rights Museum educates people about the history of the civil rights movement through its unique collections and powerful exhibits.

INTERNET **APPLICATION LINK**

www.mcdougallittell.com

GUIDED PRACTICE

Concept Check ✓ **In Exercises 1–5, use the linear system below.**

$$-x + y = 5 \quad \text{Equation 1}$$
$$\frac{1}{2}x + y = 8 \quad \text{Equation 2}$$

1. Which equation would you choose to solve for y? Why?
2. Solve for y in the equation that you chose.
3. Substitute the expression into the other equation and solve for x.
4. Substitute the value of x into your equation from Exercise 2. What is the solution of the linear system?
5. Explain how you can check the solution algebraically and graphically.

Skill Check ✓ **Use substitution to solve the linear system.**

6. $3x + y = 3$, $7x + 2y = 1$

7. $2x - y = -1$, $2x + y = -7$

8. $3x - y = 0$, $5y = 15$

9. $2x + y = 4$, $-x + y = 1$

10. $x - y = 0$, $x + y = 2$

11. $x + y = 1$, $2x - y = 2$

12. $-x + 4y = 10$, $x - 3y = 11$

13. $x + y = 1$, $x - y = 2$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 803.

REASONING **Tell which equation you would use to isolate a variable. Explain your reasoning.**

14. $2x + y = -10$, $3x - y = 0$

15. $m + 4n = 30$, $m - 2n = 0$

16. $5c + 3d = 11$, $5c - d = 5$

SOLVING LINEAR SYSTEMS **Use the substitution method to solve the linear system.**

17. $y = x - 4$, $4x + y = 26$

18. $s = t + 4$, $2t + s = 19$

19. $2c - d = -2$, $4c + d = 20$

20. $2a = 8$, $a + b = 2$

21. $2x + 3y = 31$, $y = x + 7$

22. $p + q = 4$, $4p + q = 1$

23. $x - 2y = -25$, $3x - y = 0$

24. $u - v = 0$, $7u + v = 0$

25. $x - y = 0$, $12x - 5y = -21$

26. $m + 2n = 1$, $5m + 3n = -23$

27. $x - y = -5$, $x + 4 = 16$

28. $-3a + b = 4$, $-9a + 5b = -1$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 14–34
Example 2: Exs. 14–34
Example 3: Exs. 42–44

29. $3w - 2u = 12$, $w - u = 60$

30. $y = 3x$, $x = 3y$

31. $x + y = 5$, $0.5x + 6.0y = 8.0$

32. $x + y = 12$, $x + \frac{3}{2}y = \frac{3}{2}$

33. $7g + h = -2$, $g - 2h = 9$

34. $\frac{1}{8}p + \frac{3}{4}q = 7$, $\frac{3}{2}p - q = 4$

35. ERROR ANALYSIS You are helping a friend with tonight's math homework. Answer your friend's questions.

$3x + y = 9$ ← Equation 1
$-2x + y = 4$ ← Equation 2

$3x + y = 9$
$y = -3x + 9$

$3x + (-3x + 9) = 9$
$3x - 3x + 9 = 9$
$9 = 9$

What does this mean?

How do I find the answer?

FOCUS ON APPLICATIONS

REAL LIFE SOFTBALL SIZES Softballs are measured by their circumference. There are three official softball sizes: 11 inches, 12 inches, and 16 inches.

SOLVING LINEAR SYSTEMS In Exercises 36–41, use substitution to solve the linear system. Then use a graphing calculator or a computer to check your solution.

36. $x - y = 2$
$2x + y = 1$

37. $2y = x$
$4y = 300 - x$

38. $x - 2y = 9$
$1.5x + 0.5y = 6.5$

39. $0.50x + 0.25y = 2.00$
$x + y = 1$

40. $x + y = 20$
$\frac{1}{5}x + \frac{1}{2}y = 8$

41. $1.5x - y = 40.0$
$0.5x + 0.5y = 10.0$

42. TICKET SALES You are selling tickets for a high school play. Student tickets cost \$4 and general admission tickets cost \$6. You sell 525 tickets and collect \$2876. How many of each type of ticket did you sell?

43. ORDERING SOFTBALLS You are ordering softballs for two softball leagues. The Pony League uses an 11-inch softball priced at \$2.75. The Junior League uses a 12-inch softball priced at \$3.25. The bill smeared in the rain, but you know the total was 80 softballs for \$245. How many of each size did you order?

44. MATH TEST Your math teacher tells you that next week's test is worth 100 points and contains 38 problems. Each problem is worth either 5 points or 2 points. Because you are studying systems of linear equations, your teacher says that for extra credit you can figure out how many problems of each value are on the test. How many of each value are there?

45. INVESTING IN STOCKS The value of your EFG stock is three times the value of your PQR stock. If the total value of the stocks is \$4500, how much is invested in each company?

RUNNING In Exercises 46 and 47, you can run 250 meters per minute downhill and 180 meters per minute uphill. One day you run a total of 1557 meters in 7.6 minutes on a route that goes both uphill and downhill.

46. Assign labels to the verbal model below. Then write an algebraic model.

$$\boxed{\text{Meters uphill}} + \boxed{\text{Meters downhill}} = \boxed{\text{Total meters}}$$

$$\frac{\boxed{\text{Meters uphill}}}{\boxed{\text{Rate uphill}}} + \frac{\boxed{\text{Meters downhill}}}{\boxed{\text{Rate downhill}}} = \boxed{\text{Total time}}$$

47. Find the number of meters you ran uphill and the number of meters you ran downhill.

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 46 and 47.

Test Preparation

MULTI-STEP PROBLEM **In Exercises 48–51, use the linear system below.**

$y = x + 3$
$y = 2x + 3$

48. Graph the system. Explain what the graph shows.

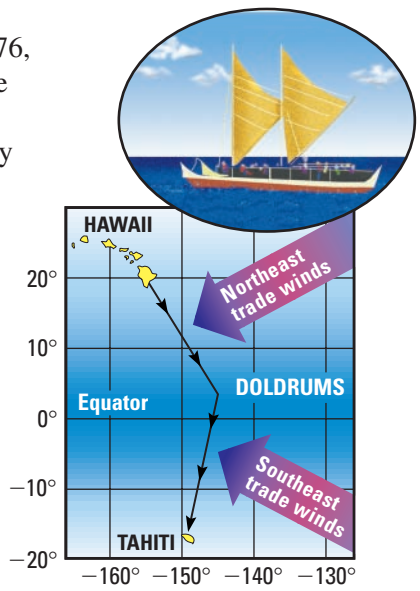
49. Solve the linear system using substitution. What does the solution mean?

50. Which method do you think is easier for solving this linear system?

51. *Writing* Describe the advantages and disadvantages of each method.

Challenge

HISTORY CONNECTION **In Exercises 52 and 53, use the following information.** On May 1, 1976, a team of adventurers set sail to discover how the ancient Polynesians regularly navigated the 3000-nautical-mile route shown at the right. They sailed from Hawaii to Tahiti on a traditional twin-hulled canoe called the *Hokule'a*.



Sailing into northeast trade winds, the crew maintained a course represented by

$y = -\frac{3}{2}x - 215.$

Sailing into southeast trade winds, the crew maintained a course represented by

$y = 7x + 1026.$

At the point of intersection, the team became caught in the "doldrums" and made little headway for 5 to 6 days.

52. Find the coordinates of the point where the team was caught in the doldrums.

EXTRA CHALLENGE
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53. The equation of the straight line passing through Hawaii and Tahiti is $36x + 7y = -5483$. Find the coordinates of Hawaii and Tahiti.

MIXED REVIEW

SIMPLIFYING EXPRESSIONS **Simplify the variable expression.** (Review 2.6 for 7.3)

54. $4g + 3h + 2g - 3h$

55. $3x + 2y - (5x + 2y)$

56. $6(2p - m) - 3m - 12p$

57. $4(3x + 5y) + 3(-4x + 2y)$

GRAPHING LINES **Write the equation in slope-intercept form. Then graph the equation.** (Review 4.6)

58. $6x + y = 0$

59. $8x - 4y + 16 = 0$

60. $3x + y + 5 = 0$

61. $5x + 3y = 3$

62. $x + y = 0$

63. $y = -2$

SOLVING AND GRAPHING **Solve the inequality. Then graph its solution.** (Review 6.3, 6.4)

64. $-5 < -x \leq -1$

65. $|x + 5| \leq 14$

66. $3 > -x > -1$

67. $2x - 6 < -7$ or $2x - 6 > 5$

68. $3x - 2 > 4$ or $3x - 2 < -5$