6.5 Graphing Linear Inequalities in Two Variables

What you should learn

GOAL 1 Graph a linear inequality in two variables.

GOAL 2 Model a **real-life** situation using a linear inequality in two variables, such as purchasing produce in **Ex. 64**.

Why you should learn it

▼ To model **real-life** situations, such as salvaging coins from a shipwreck in **Example 5**.

GOAL 1 GRAPHING LINEAR INEQUALITIES

A **linear inequality** in x and y is an inequality that can be written as follows.

$ax + by < c$ $\quad ax + by \leq c$ $\quad ax + by > c$ $\quad ax + by \geq c$

An ordered pair (x, y) is a **solution** of a linear inequality if the inequality is true when the values of x and y are substituted into the inequality.

EXAMPLE 1 Checking Solutions of a Linear Inequality

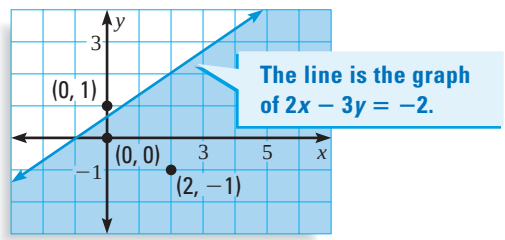
Check whether the ordered pair is a solution of $2x - 3y \geq -2$.

a. $(0, 0)$ **b.** $(0, 1)$ **c.** $(2, -1)$

SOLUTION

(x, y)	$2x - 3y \geq -2$	Conclusion
a. $(0, 0)$	$2(0) - 3(0) = 0 \geq -2$	$(0, 0)$ is a solution.
b. $(0, 1)$	$2(0) - 3(1) = -3 \not\geq -2$	$(0, 1)$ is not a solution.
c. $(2, -1)$	$2(2) - 3(-1) = 7 \geq -2$	$(2, -1)$ is a solution.

The **graph** of a linear inequality in two variables is the graph of the solutions of the inequality. For instance, the graph of $2x - 3y \geq -2$ is shown. Every point in the shaded region and on the line is a solution of the inequality. Every other point in the plane is not a solution.



GRAPHING A LINEAR INEQUALITY

STEP 1 Graph the corresponding equation. Use a *dashed* line for inequalities with $>$ or $<$ to show that the points on the line are not solutions. Use a *solid* line for inequalities with $\geq$ or $\leq$ to show that the points on the line are solutions.

STEP 2 The line you drew separates the coordinate plane into two **half-planes.** Test a point in one of the half-planes to find whether it is a solution of the inequality.

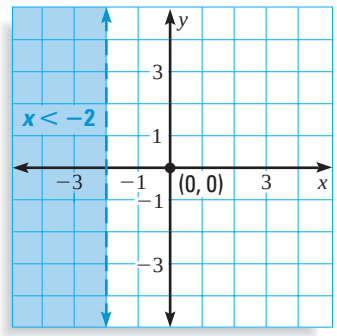
STEP 3 If the test point is a solution, shade the half-plane it is in. If not, shade the other half-plane.

EXAMPLE 2 Graphing a Linear Inequality

Sketch the graph of $x < -2$.

SOLUTION

1. Graph the corresponding equation $x = -2$, a vertical line. Use a dashed line.
2. Test a point. The origin $(0, 0)$ is *not* a solution and it lies to the right of the line. So, the graph of $x < -2$ is all points to the left of the line $x = -2$.
3. Shade the region to the left of the line.

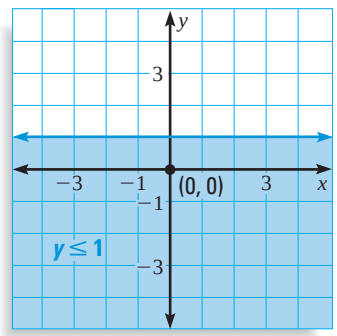


EXAMPLE 3 Graphing a Linear Inequality

Sketch the graph of $y \leq 1$.

SOLUTION

1. Graph the corresponding equation $y = 1$, a horizontal line. Use a solid line.
2. Test a point. The origin $(0, 0)$ *is* a solution and it lies below the line. So, the graph of $y \leq 1$ is all points on or below the line $y = 1$.
3. Shade the region below the line.



EXAMPLE 4 Writing in Slope-Intercept Form

Sketch the graph of $x + y > 3$.

SOLUTION

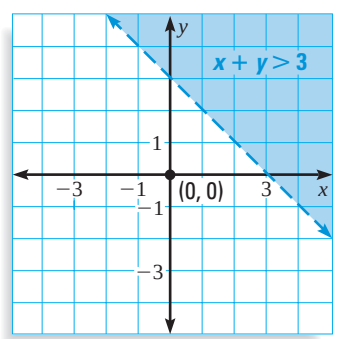
The corresponding equation is $x + y = 3$. To graph this line, you can first write the equation in slope-intercept form.

$$y = -x + 3$$

Then graph the line that has a slope of -1 and a y-intercept of 3. Use a dashed line.

The origin $(0, 0)$ is *not* a solution and it lies below the line. So, the graph of $x + y > 3$ is all points above the line $y = -x + 3$.

✓ **CHECK** Test any point above the line. Any point you choose will satisfy the inequality.



STUDENT HELP

Study Tip

You can use any point that is not on the line as a test point. The origin is often used because it is convenient to evaluate expressions in which 0 is substituted for each variable.

GOAL 2 MODELING A REAL-LIFE SITUATION

REAL LIFE *Treasure Diving*

EXAMPLE 5 *Modeling with a Linear Inequality*

You are on a treasure-diving ship that is hunting for gold and silver coins. Objects collected by the divers are placed in a wire basket. One of the divers signals you to reel in the basket. It feels as if it contains no more than 50 pounds of material.

If each gold coin weighs about 0.5 ounce and each silver coin weighs about 0.25 ounce, what are the different amounts of coins that could be in the basket?

SOLUTION

Find the number of ounces in the basket. There are 16 ounces in a pound.

$$16 \cdot 50 = 800$$

Write an algebraic model.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Weight per gold coin}} \cdot \boxed{\text{Number of gold coins}} + \boxed{\text{Weight per silver coin}} \cdot \boxed{\text{Number of silver coins}} \leq \boxed{\text{Weight in basket}}$$

LABELS

Weight per gold coin = **0.5** (ounces per coin)

Number of gold coins = x (coins)

Weight per silver coin = **0.25** (ounces per coin)

Number of silver coins = y (coins)

Maximum weight in basket = **800** (ounces)

ALGEBRAIC MODEL

$$0.5x + 0.25y \leq 800$$ Write algebraic model.

Graph the inequality to see the possible solutions. To make a quick graph of the corresponding equation, find the x-intercept and the y-intercept. The x-intercept is (1600, 0). The y-intercept is (0, 3200). Then graph the line, test the origin, and shade the graph of the inequality.

FOCUS ON APPLICATIONS

REAL LIFE **GOLD** found in the ocean can look brand new. Most other metals deteriorate quickly in salt water.

INTERNET APPLICATION LINK
www.mcdougallittell.com

The graph shows all the solutions of the inequality. The possible numbers of gold and silver coins, however, are only the ordered pairs of integers in the graph.

One solution is all gold coins.

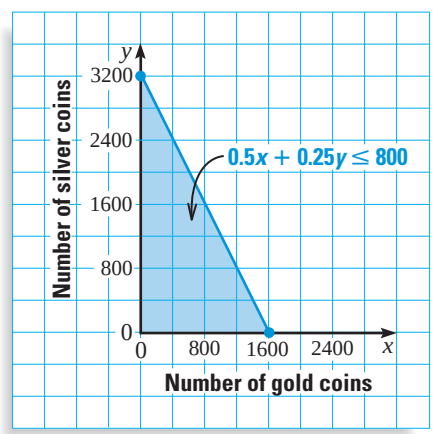
(1600, 0)

Another solution is all silver coins.

(0, 3200)

There are many other solutions, including no gold or silver coins.

(0, 0)



GUIDED PRACTICE

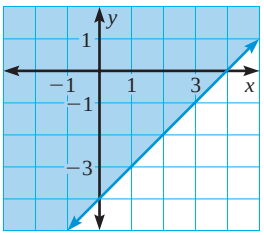
Vocabulary Check ✓

Concept Check ✓

1. Explain what a *solution* of a linear inequality in x and y is.

2. In the graph in Example 5, why is shading shown only in Quadrant 1? Use the real-life context to explain your reasoning.

3. Choose the inequality whose solution is shown by the graph. Explain your reasoning.



A. $x - y > 4$

B. $x - y < 4$

C. $x - y \geq 4$

D. $x - y \leq 4$

Skill Check ✓

Check whether (0, 0) is a solution. Then sketch the graph of the inequality.

4. $y < -2$ **5.** $x > -2$ **6.** $x + y \geq -1$

7. $x - y \leq -2$ **8.** $x + y < 4$ **9.** $x - y \leq 5$

10. $x + y > 3$ **11.** $3x - y < 3$ **12.** $x - 3y \geq 12$

BASKETBALL SCORES **With two minutes left in a basketball game, your team is 12 points behind. What are two different numbers of 2-point and 3-point shots your team could score to earn at least 12 points?**

13. Write a verbal model for the situation. Assign labels to each part of the verbal model and write an inequality.

14. Sketch the graph of the inequality. Then name two ways your team could score at least 12 points.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 802.

CHECKING SOLUTIONS **Is each ordered pair a solution of the inequality?**

15. $x + y > -3$; (0, 0), (−6, 3) **16.** $2x + 2y \leq 0$; (−1, −1), (1, 1)

17. $2x + 5y \geq 10$; (1, 2), (6, 1) **18.** $4x + 7y \leq 26$; (3, 2), (2, 3)

19. $0.6x + 0.6y > 2.4$; (2, 2), (3, −3) **20.** $1.8x - 3.8y \geq 5$; (0, 0), (1, −1)

21. $\frac{3}{4}x - \frac{3}{4}y < 2$; (8, 8), (8, −8) **22.** $\frac{5}{6}x + \frac{5}{3}y > 4$; (6, −12), (8, −8)

SKETCHING GRAPHS **Sketch the graph of the inequality.**

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–22
Example 2: Exs. 23–42
Example 3: Exs. 23–42
Example 4: Exs. 43–60
Example 5: Exs. 64–66

23. $x \geq 4$ **24.** $x \leq 5$ **25.** $y > -3$ **26.** $y < 9$

27. $x + 3 > -2$ **28.** $7 - x \leq 16$ **29.** $y + 6 > 5$ **30.** $8 - y \leq 0$

31. $4x < -12$ **32.** $-2x \geq 10$ **33.** $5y \leq -25$ **34.** $8y > 24$

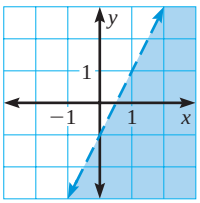
35. $-3x \geq 15$ **36.** $6y \leq 24$ **37.** $-4y < 8$ **38.** $-5x > -10$

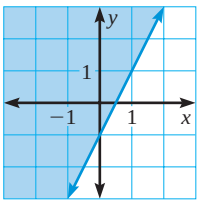
39. $x < \frac{1}{2}$ **40.** $y \geq 1.5$ **41.** $2y > 1$ **42.** $x \leq 3.5$

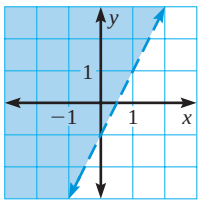
MATCHING GRAPHS **Match the inequality with its graph.**

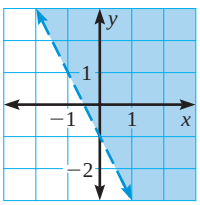
A. $2x - y \leq 1$ **B.** $-2x - y < 1$ **C.** $2x - y \geq 1$

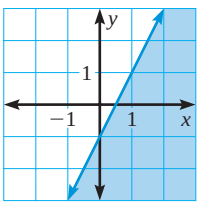
D. $2x - y > 1$ **E.** $2x - y < 1$ **F.** $-2x - y > 1$

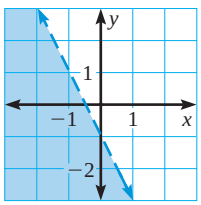
43. 

44. 

45. 

46. 

47. 

48. 

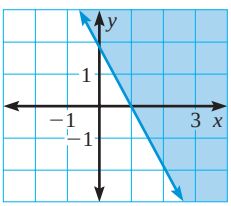
SKETCHING GRAPHS **Sketch the graph of the inequality.**

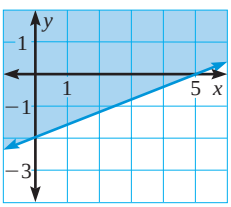
49. $x + y > -8$ **50.** $x - y \geq 4$ **51.** $y - x \leq 11$ **52.** $-x - y < 3$

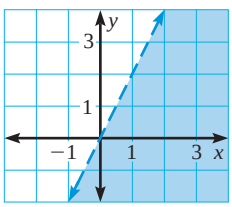
53. $x + 6y \leq 12$ **54.** $3x - y \geq 5$ **55.** $2x + y > 6$ **56.** $y - 5x < 0$

57. $4x + 3y \leq 24$ **58.** $9x - 3y \geq 18$ **59.** $\frac{1}{4}x + \frac{1}{2}y < 1$ **60.** $\frac{1}{3}x - \frac{2}{3}y \geq 2$

WRITING INEQUALITIES FROM GRAPHS **Write an inequality whose solution is shown in the graph.**

61. 

62. 

63. 

64. **PURCHASING PRODUCE** You have \$12 to spend on fruit for a meeting. Grapes cost \$1 per pound and peaches cost \$1.50 per pound. Let x represent the number of pounds of grapes you can buy. Let y represent the number of pounds of peaches you can buy. Write and graph an inequality to model the amounts of grapes and peaches you can buy.

65. **FOOTBALL** In the last quarter of a high school football game, your team is behind by 21 points. A field goal is 3 points and a touchdown (with the point-after-touchdown) is 7 points. Let x represent the number of field goals scored. Let y represent the number of touchdowns scored.

a. Write and graph an inequality that models the different numbers of field goals and touchdowns your team could score and still not win or tie. (Assume the other team scores no more points.)

b. Does every point on the graph represent a solution of the real-life problem? Give examples to support your answer.

STUDENT HELP

INTERNET HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for help with Ex. 65.

FOCUS ON CAREERS

NUTRITIONIST Nutritionists plan food programs and supervise the preparing of meals.

CAREER LINK
www.mcdougallittell.com

66. CAR SALES You are a car dealer. You have $1,408,000 available to purchase compact cars and sport utility vehicles for your lot. The compact car costs $11,000 and the sport utility vehicle costs $22,000. Let x represent the number of compact cars and let y represent the number of sport utility vehicles you purchase. Write an inequality that models the different numbers of compact cars and sport utility vehicles that you could purchase.

NUTRITION In Exercises 67–69, use the following information. You are planning a breakfast that supplies at most 500 calories. The table shows the numbers of calories in certain foods.

Breakfast food	Calories
Bagel, 1 plain	200
Cereal, 1 cup	88
Juice, Apple, 1 glass	120
Juice, Tomato, 1 glass	43
Egg	75
Milk, skim, 1 cup	85
Margarine, 1 tsp	33

67. a. You select tomato juice, some cereal, and milk. What different amounts of cups of cereal and milk could you choose along with one glass of tomato juice to supply at most 500 calories?

b. Describe a reasonable meal of tomato juice, cereal, and milk that supplies at most 500 calories.

c. Find reasonable amounts of cups of cereal and milk, with a glass of tomato juice, to supply at most 600 calories.

68. a. Another morning you choose a bagel with some margarine, and apple juice. Find different numbers of teaspoons of margarine and glasses of apple juice, with a bagel, to supply at most 500 calories.

b. Describe a reasonable meal of a bagel with margarine, and apple juice, that supplies at most 500 calories.

69. *Writing* Describe two methods you could use to solve part (a) of Exercise 68.

Test Preparation

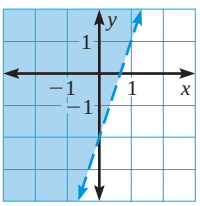
70. MULTIPLE CHOICE Choose the inequality whose graph is shown.

Ⓐ $2y - 6x < -4$

Ⓑ $2y - 6x \leq -4$

Ⓒ $2y - 6x > -4$

Ⓓ $6x + 2y > 4$



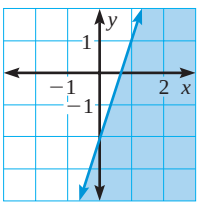
71. MULTIPLE CHOICE Choose the inequality whose graph is shown.

Ⓐ $2y - 6x < -4$

Ⓑ $2y - 6x \leq -4$

Ⓒ $2y - 6x \geq -4$

Ⓓ $6x + 2y > 4$



Challenge

72. Sketch the graph of $|y| < 2$ in a coordinate plane. Find the domain and the range.

73. Sketch the graph of $|x + 3| \leq y$ in a coordinate plane.

MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 1.3 for 6.6)

74. $\frac{16 + 11 + 18}{3}$ **75.** $\frac{20 + 15 + 22 + 19}{4}$ **76.** $\frac{37 + 65 + 89 + 72 + 82}{5}$

SOLVING A FORMULA **Solve for the indicated variable.** (Review 3.7)

77. Distance traveled
Solve for r: $d = rt$

78. Volume of a pyramid
Solve for h: $V = \frac{1}{3}Bh$

GRAPHING EQUATIONS **Use a table of values to graph the equation.** (Review 4.2)

79. $y = -6x + 7$ **80.** $y = 3x - 7$

81. $-2x + 2y = 5$ **82.** $3x + 4y = 20$

FINDING SLOPES AND *y*-INTERCEPTS **Find the slope and the *y*-intercept of the line.** (Review 4.6)

83. $y = -5x + 2$ **84.** $y = \frac{x}{2} - 2$ **85.** $5x - 5y = 1$

86. $6x + 2y = 14$ **87.** $y = -2$ **88.** $y = 5$

89. $y = 3x - 6.5$ **90.** $y = -4x + \frac{1}{2}$ **91.** $3x + 2y = 10$

QUIZ 2

Self-Test for Lessons 6.4 and 6.5

Solve the equation. (Lesson 6.4)

1. $|x| = 15$ **2.** $|x| = 22$ **3.** $|x + 3| = 6$

4. $|x - 2| = 10$ **5.** $|2x + 7| = 7$ **6.** $|3x - 2| - 2 = 5$

Solve the inequality. Then graph its solution on a number line. (Lesson 6.4)

7. $|x - 4| > 1$ **8.** $|x + 7| < 2$ **9.** $|x - 12| \leq 9$

10. $\left|x - \frac{1}{4}\right| > \frac{9}{4}$ **11.** $|2x + 7| \leq 25$ **12.** $|4x + 2| - 5 > 17$

Is each ordered pair a solution of the inequality? (Lesson 6.5)

13. $x + y \leq 4$; $(0, -1)$, $(2, 2)$ **14.** $y - 3x > 0$; $(0, 0)$, $(-4, 1)$

15. $-2x + 5y \geq 5$; $(2, 1)$, $(-1, 2)$ **16.** $-x - 2y < 4$; $(1, -1)$, $(2, -3)$

Sketch the graph of the inequality in a coordinate plane. (Lesson 6.5)

17. $y - 5x > 0$ **18.** $y \geq 3$ **19.** $x \leq -4$

20. $y < -2x$ **21.** $2x - y \geq 10$ **22.** $3x + y > 15$

23. **STOCK MARKET** You purchase shares of a technology company's stock and hold onto them for one year. During that time period, the price per share ranged from \$65 to \$143. Write an absolute-value inequality that shows the range of stock prices for the year. (Lesson 6.4)