Predicting with Linear Models

GOAL 1 **DECIDING WHEN TO USE A LINEAR MODEL**

Two of your major goals in this course are to learn about different types of models and to learn which type of model to use in a specific real-life situation.

In this lesson you will learn how to decide when a linear model can be used to represent real-life data.

EXAMPLE 1 *Which Data Set is More Linear?*

The amount (in millions of dollars) spent on advertising in broadcast television and on the Internet from 1995 through 2001 is given in the table. Which data are better modeled with a linear model?

Year	1995	1997	1999	2001
Broadcast television advertising expenditures (in millions)	32,720	36,893	41,230*	46,140*
Internet advertising expenditures (in millions)	24	1300	15,000*	46,000*

▶ Sources: Veronis, Suhler & Associates Communications Industry Forecast and *Activ* Media
*Projected data

SOLUTION

A good way to decide whether data can be represented by a linear model is to draw a scatter plot of the data.
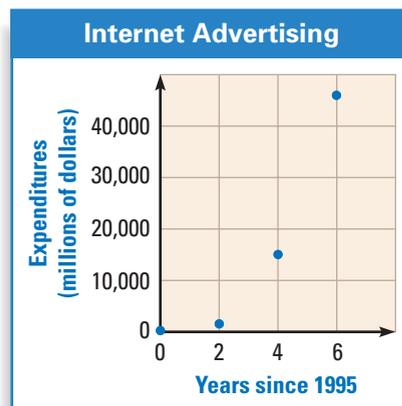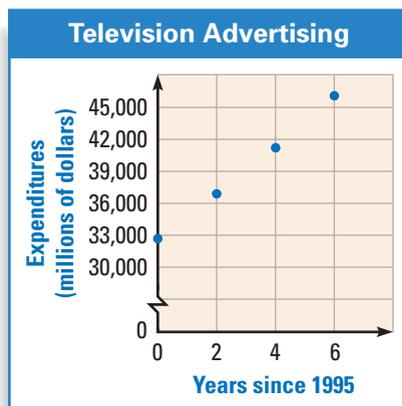
From the two scatter plots, you can see that the broadcast television data fall almost exactly on a line. The Internet data are much less linear. That is, the data points don't lie as close to a straight line.

REAL LIFE

Advertising

EXAMPLE 2 *Writing a Linear Model*

Write a linear model for the amounts spent on television advertising given in the table in Example 1.

SOLUTION

METHOD 1 Make a scatter plot of the data and draw the line that best fits the points. The line does not need to pass through any of the data points.

Television Advertising

Years since 1995
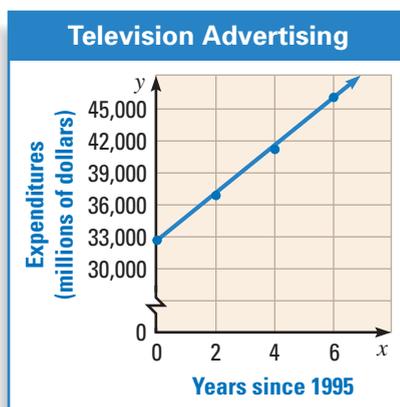
Find two points on the best-fitting line such as (0, 32,720) and (6, 46,140). Use these points to find the slope of the best-fitting line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{46{,}140 - 32{,}720}{6 - 0} \approx 2236.67 \approx \mathbf{2237}$$

Using a y-intercept of $b = 32{,}720$ and a slope of $m = 2237$ you can write an equation of the line.

$y = \mathbf{m}x + \mathbf{b}$ **Write slope-intercept form.**

$y = \mathbf{2237}x + \mathbf{32{,}720}$ **Substitute 2237 for *m* and 32,720 for *b*.**

▶ A linear model for the amounts spent on television advertising is $y = 2237x + 32{,}720$.

METHOD 2 You can use a graphing calculator and follow the procedure given on page 299.

The linear regression program gives you the results shown at the right. A linear model for spending on television advertising is $y = 2229.85x + 32{,}556.2$. The r-value of about 0.999 indicates that the linear model is a very good fit.
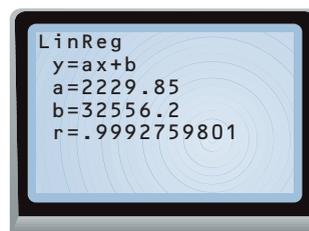
```
LinReg
 y=ax+b
 a=2229.85
 b=32556.2
 r=.9992759801
```

INTERPOLATION AND EXTRAPOLATION When you use a linear model to estimate data points that are not given, you are using *linear interpolation* or *linear extrapolation*. **Linear interpolation** is a method of estimating the coordinates of a point that lies between two given data points. **Linear extrapolation** is a method of estimating the coordinates of a point that lies to the right or left of all of the given data points.

EXAMPLE 3 *Linear Interpolation and Linear Extrapolation*

ADVERTISING Use the linear model you found in Example 2 to estimate the amount spent on advertising in broadcast television in the given year. Tell whether you use *linear interpolation* or *linear extrapolation*.

a. 1996   **b.** 2015

Television Advertising
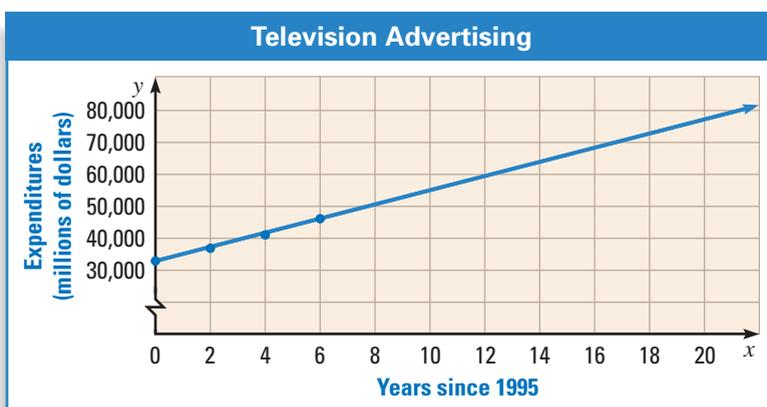
a. You used data for 1995 and 2001 to write the model. Because 1996 is between these two dates, you will use *linear interpolation*. Because x represents the number of years since 1995, you can estimate the amount spent in 1996 by substituting $x = 1$ into the linear model from Example 2.

$$y = 2237x + 32{,}720 \qquad \text{Write linear model.}$$
$$= 2237(1) + 32{,}720 \qquad \text{Substitute 1 for } x.$$
$$= 34{,}957 \qquad \text{Simplify.}$$

▶ The model predicts that almost \$35,000 million, or \$35 billion, was spent in 1996.

b. Because 2015 is to the right of all of the given data, you will use *linear extrapolation*. You can estimate the amount spent in 2015 by substituting $x = 20$ into the linear model from Example 2.

$$y = 2237x + 32{,}720 \qquad \text{Write linear model.}$$
$$= 2237(20) + 32{,}720 \qquad \text{Substitute 20 for } x.$$
$$= 77{,}460 \qquad \text{Simplify.}$$

▶ The model predicts that about \$77,000 million, or about \$77 billion, will be spent in 2015.

GUIDED PRACTICE

1. Explain the difference between linear interpolation and linear extrapolation.

2. Explain how to decide whether a data set can be represented by a linear model.

Tell whether it is reasonable for the graph to be represented by a linear model.

3.

4.

5.
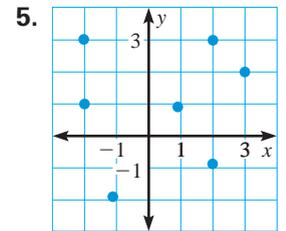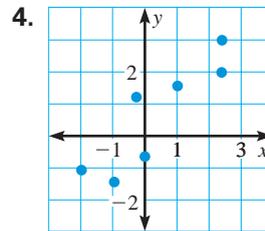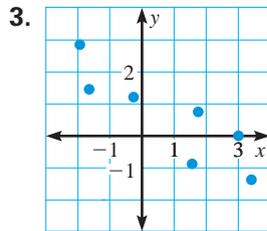
🌐 **BOTTLED WATER** **In Exercises 6–10, use the data on bottled water consumption per person.**

6. Make a scatter plot of the data.

7. Write a linear model for the data.

Use your model to estimate the consumption of bottled water in the given year. Tell whether you use *linear interpolation* or *linear extrapolation*.

8. 1994　　　　**9.** 2010　　　　**10.** 1979

Year	Water (in gallons)
1980	2.4
1985	4.5
1990	8.0
1995	11.6

▶ Source: *Statistical Abstract of the United States*

PRACTICE AND APPLICATIONS

REPRESENTING DATA **Tell whether it is reasonable for the graph to be represented by a linear model.**
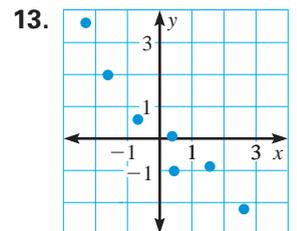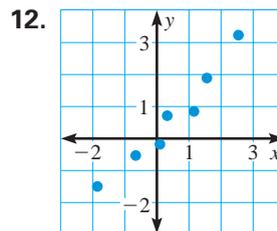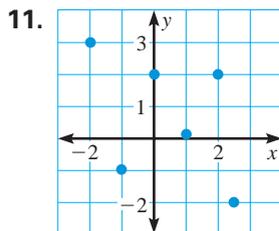
11.

12.

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16.
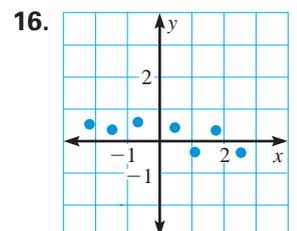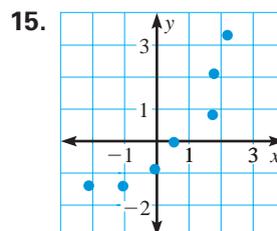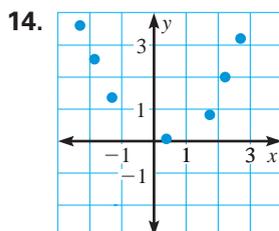
🌎 **MOVIE THEATERS** In Exercises 17–22, use the table which shows the number of movie theater screens (in thousands) from 1975 to 1995.

Year	1975	1980	1985	1990	1995
Indoor screens (in thousands)	11	14	18	23	27
Drive-in screens (in thousands)	4	4	3	1	1

🔄 **DATA UPDATE** of Motion Picture Association of America, Inc. data at www.mcdougallittell.com

17. Make a scatter plot of the number of indoor movie screens in terms of the year t. Let t represent the number of years since 1975.

18. Make a scatter plot of the number of drive-in movie screens in terms of the year t. Let t represent the number of years since 1975.

19. Which data are better modeled with a linear model?

20. Write a linear model for the number of indoor movie screens.

21. Use the linear model to estimate the number of indoor movie screens in 2005.

22. Use the linear model to estimate the number of indoor movie screens in 1989. Did you use *linear interpolation* or *linear extrapolation*?

🌎 **BOOKS AND MAPS** In Exercises 23 and 24, use the table which shows the number of dollars (in billions) spent on books and maps in the United States from 1990 through 1995.

Years since 1990	0	1	2	3	4	5
Billions of dollars	16.5	16.9	17.7	19.0	20.1	20.9

▶ Source: U.S. Bureau of Economic Analysis

23. Write a linear model for the amount spent on books and maps.

24. Use the linear model to estimate the amount spent in 2005. Did you use *linear interpolation* or *linear extrapolation*?

🌎 **TOYS AND SPORT SUPPLIES** In Exercises 25–27, use the table which shows the number of dollars (in billions) spent on toys and sport supplies in the United States from 1990 through 1995.

Years since 1990	Billions of dollars
0	31.6
1	32.8
2	?
3	36.5
4	40.1
5	42.7

▶ Source: U.S. Bureau of Economic Analysis

25. Write a linear model for the amount spent on toys and sport supplies.

26. Use the linear model to estimate the amount spent in 1992. Did you use *linear interpolation* or *linear extrapolation*?

27. Use the linear model to estimate the amount spent in 2005. Did you use *linear interpolation* or *linear extrapolation*?

COLLEGE TUITION In Exercises 28–30, use the table which shows the average tuition for attending a private and a public four-year college.

Year	Public college	Private college
1990	$2,035	$10,348
1991	$2,159	$11,379
1992	$2,410	$12,192
1993	$2,604	$13,055
1994	$2,820	$13,874
1995	$2,977	$14,537
1996	$3,151	$15,581

▶ Source: U.S. National Center for Education Statistics

28. Write a linear model of the tuition for attending a public and of the tuition for attending a private college.

29. Use linear extrapolation to estimate the tuition for attending a public college when you graduate from high school.

30. Use linear extrapolation to estimate the tuition for attending a private college when you graduate from high school.

Test Preparation

MULTI-STEP PROBLEM In Exercises 31–34, use the following information. You are the produce manager at a new grocery store. It is your job to decide how much fruit to order for the week of the grand opening. You find a table that shows yearly per person consumption of pounds of bananas.

31. What was the average *weekly* consumption per person of bananas in 1995? What was the average *monthly* consumption?

32. Write a linear model for the average weekly consumption per person of bananas in pounds.

33. Use the linear model to estimate the average weekly consumption per person of bananas in pounds for this year.

Year	Pounds of bananas
1990	24.4
1991	25.1
1992	27.3
1993	26.8
1994	28.1
1995	27.4

▶ Source: U.S. Department of Agriculture

34. If you expect 1750 customers for the week of the grand opening, how many pounds of bananas should you order?

★ Challenge

In Exercises 35–37, use the data from Exercises 28–30 and a graphing calculator. Let *x* represent years since 1990 and let *y* represent the average tuition at a public college.

35. Enter the data for the tuition for public colleges and graph the equation.

36. Locate the *x*-intercept of the graph. What does this point represent?

37. *Writing* What are the reasonable limits of linear extrapolation in terms of college tuition?

MIXED REVIEW

WRITING INEQUALITIES Graph the numbers on a number line. Then write two inequalities that compare the two numbers. **(Review 2.1 for 6.1)**

38. -7 and 3 **39.** -2 and -1 **40.** 12 and -12

41. $-\dfrac{1}{2}$ and $\dfrac{3}{2}$ **42.** -7 and 2 **43.** -3 and $-\dfrac{7}{2}$

SLOPE-INTERCEPT FORM Write an equation of the line in slope-intercept form. **(Review 5.1)**

44. The slope is 2; the y-intercept is -5. **45.** The slope is $\dfrac{1}{2}$; the y-intercept is -8.

46. The slope is -4; the y-intercept is 7. **47.** The slope is -2; the y-intercept is -6.

WRITING EQUATIONS Write an equation in slope-intercept form of the line that passes through the point and has the given slope. **(Review 5.2)**

48. $(6, 5)$, $m = 2$ **49.** $(-3, 7)$, $m = 7$ **50.** $(8, 2)$, $m = 7$

51. $(8, 4)$, $m = -5$ **52.** $(3, 1)$, $m = 7$ **53.** $(-8, 7)$, $m = 2$

54. FEATHER GROWTH At nine days old, a flight feather of a night heron is 17 millimeters. At 30 days, the flight feather is 150 millimeters. Write a linear model that gives feather length f in terms of age a. **(Review 5.5)**

QUIZ 3

Self-Test for Lessons 5.6 and 5.7

Write the equation in standard form with integer coefficients. **(Lesson 5.6)**

1. $y = \dfrac{1}{2}x + 3$ **2.** $3y + 9x - 18 = 0$ **3.** $6y - 2x = 8$

4. $y = 9x + 12$ **5.** $y = \dfrac{2}{3}x + 6$ **6.** $-8y = \dfrac{1}{2} - 7x$

Write an equation in standard form of the line that passes through the point and has the given slope. **(Lesson 5.6)**

7. $(6, -8)$, $m = -2$ **8.** $(4, 1)$, $m = -\dfrac{1}{2}$ **9.** $(1, 5)$, $m = -3$

10. $(-3, -1)$, $m = 2$ **11.** $(-12, 4)$, $m = 3$ **12.** $(-16, -2)$, $m = 6$

🌎 **LIFE EXPECTANCY** In Exercises 13–15, use the table on projected life expectancy at birth. **(Lesson 5.7)**

13. Write a linear equation of the data.

14. Interpolate the life expectancy for a baby born in 1989.

15. Extrapolate the life expectancy for a baby born in 2015.

Birth year	Life expectancy
1985	74.7
1990	75.4
1995	76.3
2000*	76.7
2005*	77.3
2010*	77.9

▶ Source: U.S. National Center for Health Statistics *Projected