5.5 Point-Slope Form of a Linear Equation

What you should learn

GOAL 1 Use the point-slope form to write an equation of a line.

GOAL 2 Use the point-slope form to model a **real-life** situation, such as running pace in **Example 3**.

Why you should learn it

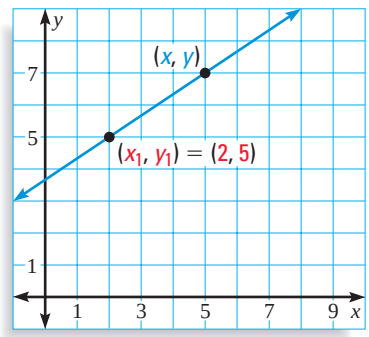
▼ To estimate the **real-life** time it will take a mountain climber to reach the top of a cliff in **Ex. 63**.

GOAL 1 USING THE POINT-SLOPE FORM

In Lesson 5.2 you learned one strategy for writing a linear equation when given the slope and a point on the line. In this lesson you will learn a different strategy—one that uses the **point-slope form** of an equation of a line.

EXAMPLE 1 *Developing the Point-Slope Form*

Write an equation of the line. Use the points (2, 5) and (x, y).



SOLUTION

You are given one point on the line. Let (x, y) be any point on the line. Because (2, 5) and (x, y) are two points on the line, you can write the following expression for the slope of the line.

$$m = \frac{y - 5}{x - 2}$$ Use formula for slope.

The graph shows that the slope is $\frac{2}{3}$. Substitute $\frac{2}{3}$ for m in the formula for slope.

$$\frac{y - 5}{x - 2} = \frac{2}{3}$$ Substitute $\frac{2}{3}$ for m.

$$y - 5 = \frac{2}{3}(x - 2)$$ Multiply each side by $(x - 2)$.

The equation $y - 5 = \frac{2}{3}(x - 2)$ is written in point-slope form.

POINT-SLOPE FORM OF THE EQUATION OF A LINE

The **point-slope form** of the equation of the nonvertical line that passes through a given point (x_1, y_1) with a slope of m is

$$y - y_1 = m(x - x_1).$$

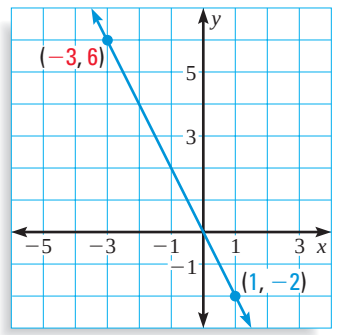
You can use the point-slope form when you are given the slope and a point on the line. In the point-slope form, (x_1, y_1) is the given point and (x, y) is any other point on the line. You can also use the point-slope form when you are given two points on the line. First find the slope. Then use either given point as (x_1, y_1).

EXAMPLE 2 *Using the Point-Slope Form*

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Write an equation of the line shown at the right.



SOLUTION

First find the slope. Use the points $(x_1, y_1) = (-3, 6)$ and $(x_2, y_2) = (1, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 6}{1 - (-3)}$$

$$= \frac{-8}{4}$$

$$= -2$$

STUDENT HELP

Study Tip
The point-slope form $y - y_1 = m(x - x_1)$ has two minus signs. Be sure to account for these signs when the point (x_1, y_1) has negative coordinates.

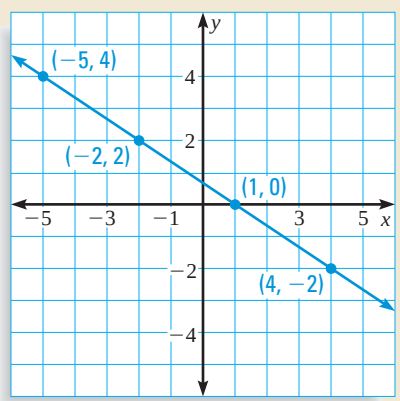
Then use the slope to write the point-slope form. Choose either point as (x_1, y_1).

$y - y_1 = m(x - x_1)$	**Write point-slope form.**
$y - 6 = -2[x - (-3)]$	**Substitute for m, x_1, and y_1.**
$y - 6 = -2(x + 3)$	**Simplify.**
$y - 6 = -2x - 6$	**Use distributive property.**
$y = -2x$	**Add 6 to each side.**

ACTIVITY
Developing Concepts

Investigating the Point-Slope Form

The line shown at the right is labeled with four points.



1. Each person in your group should use a different pair of points to write an equation of the line in point-slope form.

2. Compare the equations of the people in your group. Discuss whether the following statement seems to be always, sometimes, or never true. *Any two points on a line can be used to find an equation of the line.*

GOAL 2 MODELING A REAL-LIFE SITUATION

FOCUS ON PEOPLE

REAL LIFE **LAMECK AGUTA** of Kenya won the Boston Marathon in 1997. The high temperature on that day was 52° and the low temperature was 42°.

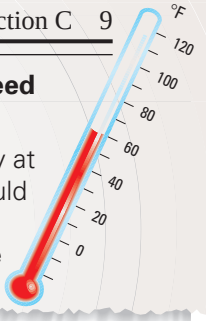
EXAMPLE 3 *Writing and Using a Linear Model*

MARATHON The information below was taken from an article that appeared in a newspaper.

Section C 9

Temperature Affects Running Speed

The best temperature for running is below 60°F. If a person ran optimally at 17.6 feet per second, he or she would slow by about 0.3 foot per second for every 5° increase in temperature above 60°F.



a. Use the information to write a linear model for optimal running pace.

b. Use the model to find the optimal running pace for a temperature of 80°F.

SOLUTION

a. Let T represent the temperature in degrees Fahrenheit. Let P represent the optimal pace in feet per second.

From the article, you know that the optimal running pace at 60° F is 17.6 feet per second so one point on the line is $(T_1, P_1) = (60, 17.6)$. Find the slope of the line.

$$m = \frac{\text{change in } P}{\text{change in } T} = \frac{-0.3}{5} = -0.06.$$

Use the point-slope form to write the model.

$P - P_1 = m(T - T_1)$ Write point-slope form.

$P - 17.6 = (-0.06)(T - 60)$ Substitute for m, T_1, and P_1.

$P - 17.6 = -0.06T + 3.6$ Use distributive property.

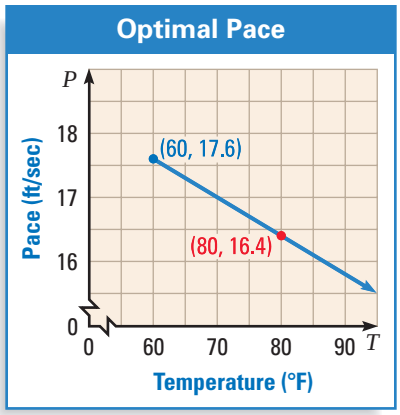
$P = -0.06T + 21.2$ Add 17.6 to each side.

b. Use the model to find the optimal pace at 80°F.

$$P = -0.06(80) + 21.2$$
$$= 16.4$$

▶ At 80°F the optimal running pace is 16.4 feet per second.

✓ **CHECK** A graph can help you check this result. You can see that as the temperature increases the optimal running pace decreases.



GUIDED PRACTICE

Vocabulary Check ✓

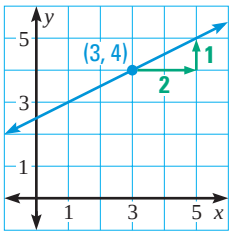
1. Name the following form of the equation of a line: $y - y_1 = m(x - x_1)$.

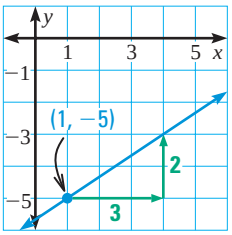
Concept Check ✓

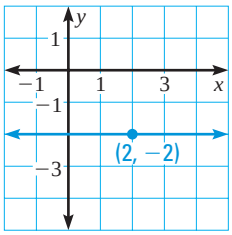
2. What are the values of m, x_1, and y_1 in the equation $y - (-1) = 3(x - 2)$?

Skill Check ✓

Write an equation of the line in point-slope form.

3. 

4. 

5. 

Write an equation of the line in point-slope form that passes through the given point and has the given slope.

6. $(2, -1)$, $m = 3$
7. $(3, 4)$, $m = \frac{1}{2}$
8. $(-5, -7)$, $m = -2$
9. $(-3, 10)$, $m = 4$
10. $(-2, -9)$, $m = 4$
11. $(7, -7)$, $m = -8$

Write an equation of the line that passes through the given points.

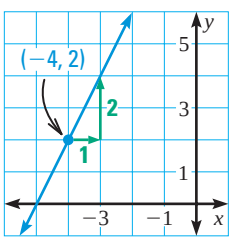
12. $(5, 12)$, $(6, -2)$
13. $(-4, -1)$, $(-9, 2)$
14. $(-17, -8)$, $(-7, -4)$
15. $(2, 12)$, $(7, 2)$
16. $(3, -12)$, $(8, 4)$
17. $(-4, -7)$, $(2, 2)$

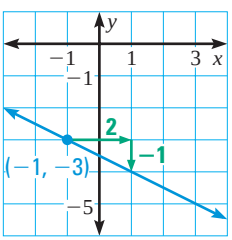
PRACTICE AND APPLICATIONS

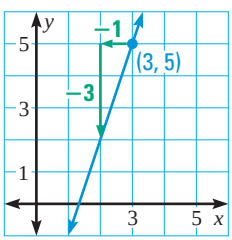
STUDENT HELP

Extra Practice to help you master skills is on p. 801.

USING A GRAPH **Write an equation of the line in point-slope form.**

18. 

19. 

20. 

POINT-SLOPE FORM **Write an equation in point-slope form of the line that passes through the given points.**

21. $(1, 5)$, $(-1, -5)$
22. $(-2, -5)$, $(7, -6)$
23. $(-9, 10)$, $(-4, -3)$
24. $(4, -5)$, $(-2, -7)$
25. $(-5, 10)$, $(-4, -2)$
26. $(-3, -2)$, $(-3, -8)$
27. $(-3, -9)$, $(-6, -8)$
28. $(-3, -7)$, $(-4, -8)$
29. $(1, -7)$, $(-1, -5)$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 18–20
Example 2: Exs. 21–29
Example 3: Exs. 57–61

WRITING EQUATIONS **Write an equation in point-slope form of the line that passes through the given point and has the given slope.**

30. $(-1, -3)$, $m = 4$
31. $(-6, 2)$, $m = -5$
32. $(-10, 0)$, $m = 2$
33. $(-8, -2)$, $m = 2$
34. $(-4, 3)$, $m = -6$
35. $(-3, 4)$, $m = 6$
36. $(12, 2)$, $m = -7$
37. $(8, -1)$, $m = 0$
38. $(5, -12)$, $m = -11$

COMPARING FORMS **Write the point-slope form of the equation of the line that passes through the point and has the given slope. Then rewrite the equation in slope-intercept form.**

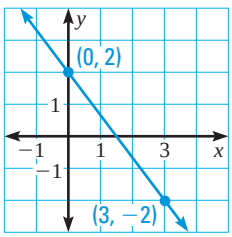
39. $(1, 4)$, $m = 2$ **40.** $(-2, 4)$, $m = 3$ **41.** $(6, 2)$, $m = \frac{1}{2}$

42. $(-3, -1)$, $m = 4$ **43.** $(5, -1)$, $m = -1$ **44.** $(-5, -5)$, $m = -2$

45. $(-1, 1)$, $m = -\frac{1}{8}$ **46.** $(4, -2)$, $m = \frac{1}{4}$ **47.** $(-9, -6)$, $m = -\frac{2}{3}$

WRITING EQUATIONS **Write an equation of the line.**

48.



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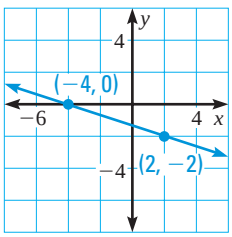

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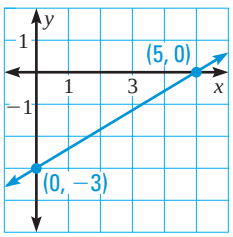


WRITING EQUATIONS GIVEN TWO POINTS **Write the point-slope form of the equation of the line that passes through the two points.**

51. $(1, 3)$, $(2, 5)$ **52.** $(-2, 3)$, $(2, -5)$ **53.** $(7, -10)$, $(15, -22)$

54. $(2, 6)$, $(5, 1)$ **55.** $(-3, -4)$, $(1, -1)$ **56.** $(4, -2)$, $(-9, 5)$

FIELD TRIP **In Exercises 57–59, you are going on a trip to the Natural History Museum. At 9:00 A.M., you leave for the museum, which is 120 miles away. At 10:15 A.M., you are 63 miles away from the museum.**

57. Write a linear equation that gives the distance d (in miles) from the museum in terms of the time t. Let t represent the number of minutes since 9:00 A.M.

58. Find the distance you are from the museum after you have traveled 2 hours.

59. According to your equation, when will you reach the museum?

TRAVEL **In Exercises 60 and 61, you are flying from Montreal to Miami in a plane. The plane leaves Montreal at 8:00 A.M. and at 9:15 A.M. is flying over Washington, DC.**

60. Use the map to write a linear equation that gives the distance y (in kilometers) remaining in the flight in terms of the number of minutes x since 8:00 A.M.

61. When will you reach Miami? (Assume that the plane is flying at a constant speed.)

MOUNTAIN CLIMBING **In Exercises 62 and 63, a mountain climber is scaling a 300-foot cliff at a constant rate. The climber starts at the bottom at 12:00 P.M. By 12:30 P.M., the climber has moved 62 feet up the cliff.**

62. Write an equation that gives the distance d (in feet) remaining in the climb in terms of the time t (in hours). What is the slope of the line?

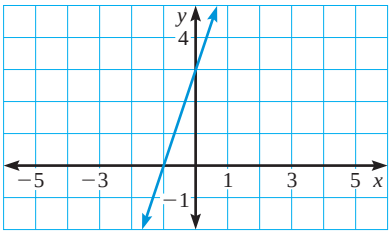
63. At what time will the mountain climber reach the top of the cliff?

Test Preparation

64. MULTIPLE CHOICE The time t (in hours) needed to produce x units of a product is modeled by $t = px + s$. If it takes 265 hours to produce 200 units and 390 hours to produce 300 units, what is the value of s?

Ⓐ 1.25 Ⓑ 15 Ⓒ 100 Ⓓ 125

65. MULTIPLE CHOICE What is the equation in point-slope form of the line in the graph?



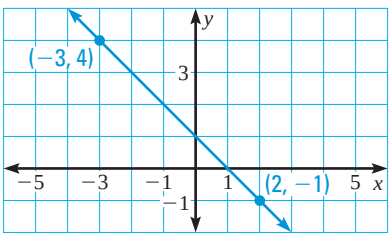
Ⓐ $y - 3 = 3(x - 0)$

Ⓑ $y = 3x + 3$

Ⓒ $y - (-1) = 3(x - 3)$

Ⓓ $y - 3 = 3[x - (-1)]$

66. MULTIPLE CHOICE What is the equation in point-slope form of the line perpendicular to the line in the graph and through $(2, -1)$?



Ⓐ $y = -x + 1$

Ⓑ $y - (-1) = -1(x - 2)$

Ⓒ $y - 4 = -1(x + 3)$

Ⓓ $y - (-1) = 1(x - 2)$

Challenge

Find the value of *k* so that the line through the given points has slope *m*.

67. $(2k, 3), (1, k); m = 2$

68. $(k + 1, k - 1), (k, -k); m = k + 1$

69. $(k, k + 1), (3, 2); m = 3$

70. $(k + 1, 3 + 2k), (k - 1, 1 - k); m = k$

MIXED REVIEW

CHECKING SOLUTIONS OF INEQUALITIES **Check whether the given number is a solution of the inequality.** (Review 1.4)

71. $2x < 24; 8$

72. $7y + 6 \geq 10; 3$

73. $16p - 9 \leq 71; 5$

74. $12a \leq a - 9; -2$

75. $4x \leq 28; 7$

76. $6c - 4 > 14; 3$

GRAPHING FUNCTIONS **Graph the function.** (Review 4.8 for 5.6)

77. $f(x) = -3x + 4$

78. $g(x) = -x - 7$

79. $h(x) = 2x - 8$

80. $f(x) = -5x - 7$

81. $f(x) = 6x + 7$

82. $f(x) = 3 - 7x$

83. $g(x) = -\frac{1}{2}x - 3$

84. $f(x) = 8x + \frac{2}{3}$

85. $h(x) = \frac{6}{5}x + 5$

86. TEEN SURVEY In a school of 500 teens, 40 have taken drum lessons. What are the odds that a randomly chosen teen has not taken drum lessons? (Review 2.8)

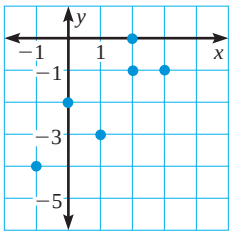
87. PROBABILITY CONNECTION A jar contains 10 blue marbles, 7 red marbles, and 4 yellow marbles. Without looking, you reach into the jar and choose one marble. What is the probability that the marble is red? (Review 2.8)

QUIZ 2

Self-Test for Lessons 5.4 and 5.5

Copy the graph and draw a best-fitting line for the scatter plot. Find an equation of the line. State whether *x* and *y* have a *positive correlation* or a *negative correlation*. (Lesson 5.4)

1.



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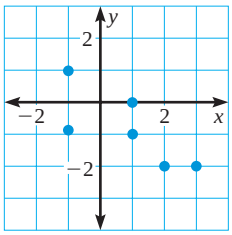

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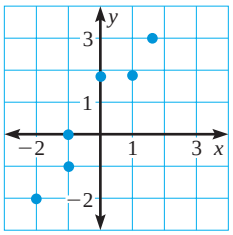


Write an equation in point-slope form of the line that passes through the given point and has the given slope. (Lesson 5.5)

4. $(1, -4)$, $m = -1$

5. $(2, 5)$, $m = 2$

6. $(-3, -2)$, $m = -2$

7. $(-3, -3)$, $m = 0$

8. $(-6, 6)$, $m = \frac{1}{2}$

9. $(3, 8)$, $m = 7$

Write an equation in point-slope form of the line that passes through the given points. (Lesson 5.5)

10. $(0, 0)$, $(-3, -1)$

11. $(-5, -2)$, $(-6, -2)$

12. $(-5, -3)$, $(1, 6)$

MATH & History — Archaeological Dating

APPLICATION LINK
www.mcdougallittell.com

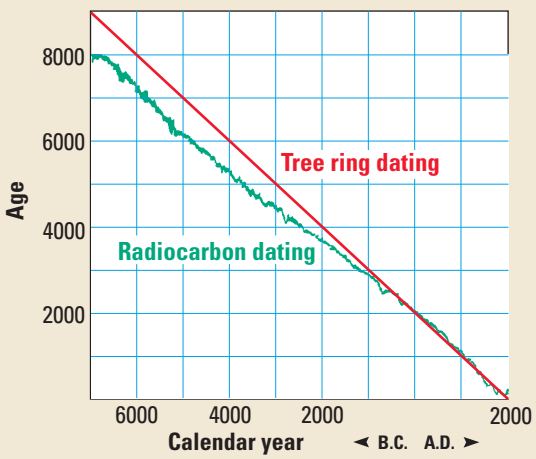
THEN

BY THE MIDDLE OF THE TWENTIETH CENTURY, radiocarbon dating was being used to find the ages of objects. During the 1950s, increasing differences emerged between the radiocarbon dates and the historical dates of civilizations.

NOW

TODAY, archaeologists calibrate tree ring dates with radiocarbon dates to accurately determine the age of an object.

1. Radiocarbon dates calibrate well with tree ring dates after approximately what year?
2. Does radiocarbon dating produce calendar years that are earlier or later than those based on tree ring dating?



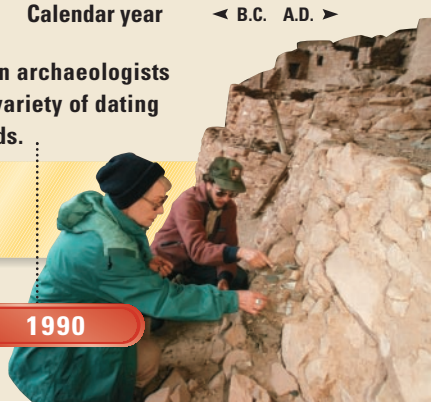
1929

An excavation unearths pottery at Mesa Verde.

1930

A. E. Douglass uses tree rings to date Mesa Verde sites.

Modern archaeologists use a variety of dating methods.



1990