5.3 Writing Linear Equations Given Two Points
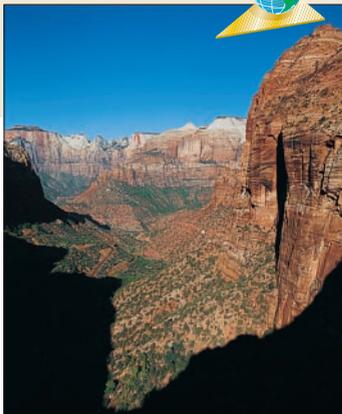
GOAL 1 USING TWO POINTS TO WRITE AN EQUATION

So far in this chapter, you have been writing equations of lines for which you were given the slope. In this lesson you will work with problems in which you must first find the slope. To do this, you can use the formula given in Chapter 4.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \longleftarrow \text{Slope of a line through } (x_1, y_1) \text{ and } (x_2, y_2)$$

EXAMPLE 1 Writing an Equation Given Two Points

Write an equation of the line that passes through the points $(1, 6)$ and $(3, -4)$.

SOLUTION

1 Find the slope of the line. Let $(x_1, y_1) = (1, 6)$ and $(x_2, y_2) = (3, -4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Write formula for slope.}$$

$$= \frac{-4 - 6}{3 - 1} \qquad \text{Substitute.}$$

$$= \frac{-10}{2} = -5 \qquad \text{Simplify.}$$

2 Find the y-intercept. Let $m = -5$, $x = 1$, and $y = 6$ and solve for b.

$$y = mx + b \qquad \text{Write slope-intercept form.}$$

$$6 = (-5)(1) + b \qquad \text{Substitute } -5 \text{ for } m, 1 \text{ for } x, \text{ and } 6 \text{ for } y.$$

$$6 = -5 + b \qquad \text{Simplify.}$$

$$11 = b \qquad \text{Solve for } b.$$

3 Write an equation of the line.

$$y = mx + b \qquad \text{Write slope-intercept form.}$$

$$y = -5x + 11 \qquad \text{Substitute } -5 \text{ for } m \text{ and } 11 \text{ for } b.$$

✓ **CHECK** You can use a graph to check your work. Plot the two given points, $(1, 6)$ and $(3, -4)$, and draw a line through them. Then check that the line has a y-intercept of 11 and a slope of -5.
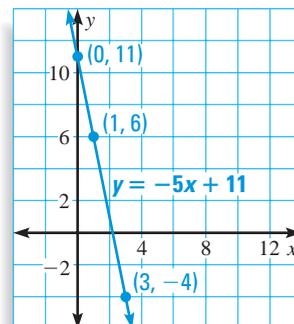
WRITING AN EQUATION OF A LINE GIVEN TWO POINTS

STEP ❶ **Find the slope.** Substitute the coordinates of the two given points into the formula for slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

STEP ❷ **Find the y-intercept.** Substitute the slope m and the coordinates of one of the points into the slope-intercept form, $y = mx + b$, and solve for the y-intercept b.

STEP ❸ **Write an equation of the line.** Substitute the slope m and the y-intercept b into the slope-intercept form, $y = mx + b$.
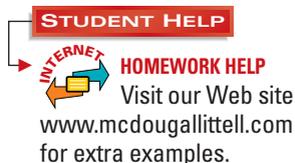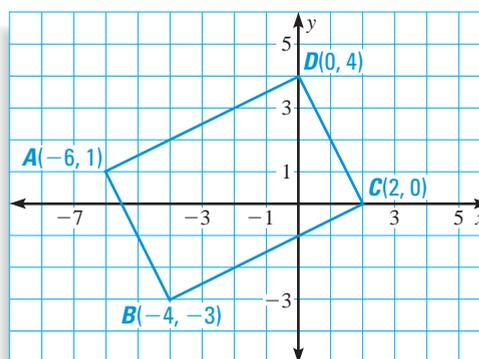

EXAMPLE 2   *Writing Equations of Perpendicular Lines*

GEOMETRY CONNECTION Two different nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other.

a. Show that $\overline{AB}$ and $\overline{BC}$ are perpendicular sides of figure *ABCD*.

b. Write equations for the lines containing $\overline{AB}$ and $\overline{BC}$.

SOLUTION

a. Find the slopes.

Slope of $\overline{AB}$: $m = \dfrac{1 - (-3)}{-6 - (-4)} = -2$

Slope of $\overline{BC}$: $m = \dfrac{0 - (-3)}{2 - (-4)} = \dfrac{1}{2}$

▶ $\overline{AB}$ and $\overline{BC}$ are perpendicular because $\dfrac{1}{2}$ is the negative reciprocal of -2.

b. Find the y-intercepts of the lines containing $\overline{AB}$ and $\overline{BC}$. Substitute the slopes from part (a) and the coordinates of one point into $y = mx + b$.

For $\overline{AB}$	**For $\overline{BC}$**
$y = mx + b$	$y = mx + b$
$-3 = (-2)(-4) + b$	$-3 = \left(\dfrac{1}{2}\right)(-4) + b$
$-11 = b$	$-1 = b$

Write an equation in slope-intercept form by substituting for m and b.

Equation of line containing $\overline{AB}$	**Equation of line containing $\overline{BC}$**
$y = mx + b$	$y = mx + b$
$y = -2x - 11$	$y = \dfrac{1}{2}x - 1$

EXAMPLE 3 *Writing and Using a Linear Model*

ARCHAEOLOGY While working at an archaeological dig, you find an upper leg bone (femur) that belonged to an adult human male. The bone is 43 centimeters long. In humans, femur length is linearly related to height. To estimate the height of the person, you measure the femur and height of two complete adult male skeletons found at the same excavation.

> **Person 1**: 40-centimeter femur, 162-centimeter height
>
> **Person 2**: 45-centimeter femur, 173-centimeter height

Estimate the height of the person whose femur was found.

SOLUTION

Write a linear equation to model the height of a person in terms of the femur length. Let x represent femur length (in centimeters) and let y represent the height of the person (in centimeters).

Find the slope of the line through the points (40, 162) and (45, 173).

$$m = \frac{173 - 162}{45 - 40}$$

$$= \frac{11}{5} = \mathbf{2.2}$$

Find the y-intercept.

$y = \mathbf{m}x + b$	**Write slope-intercept form.**
$173 = \mathbf{2.2}(45) + b$	**Substitute for *y*, *m*, and *x*.**
$\mathbf{74} = b$	**Solve for *b*.**

Write a linear equation.

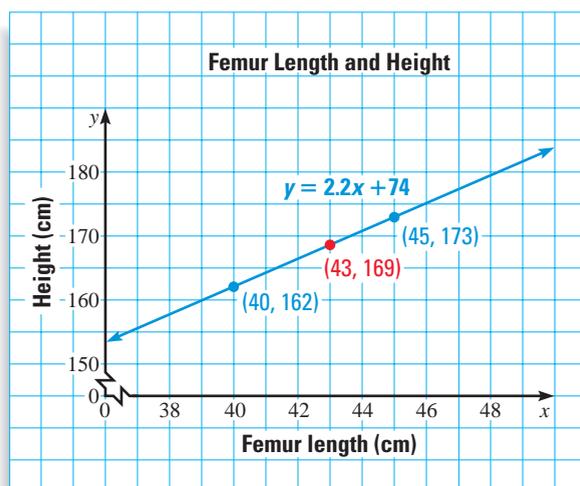|	
$y = \mathbf{m}x + \mathbf{b}$	**Write slope-intercept form.**
$y = \mathbf{2.2}x + \mathbf{74}$	**Substitute 2.2 for *m* and 74 for *b*.**

Estimate the height of the person whose femur is 43 centimeters by substituting 43 for x in the equation you wrote.

$$y = 2.2x + 74$$

$$y = 2.2(43) + 74$$

$$y = 168.6$$

▶ The person was about 169 centimeters tall.

GUIDED PRACTICE

Vocabulary Check ✓

1. Compare writing linear equations given the slope and a point with writing linear equations given two points.

Concept Check ✓

2. Explain why you should find the slope m first when finding the equation of a line passing through two points.
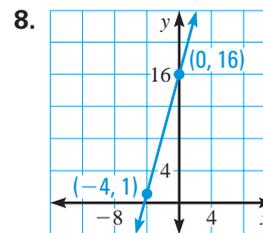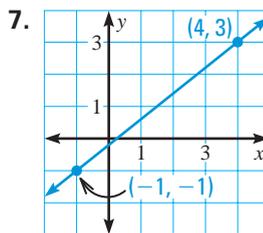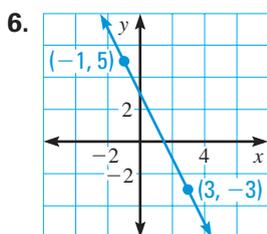
Skill Check ✓

Give the slope of a line perpendicular to the given line.

3. $y = -4x + 2$ **4.** $y = \frac{1}{2}x - 3$ **5.** $y = x - 3$

Write an equation in slope-intercept form of the line shown in the graph.

6.

7.

8.

Write an equation in slope-intercept form of the line that passes through the points.

9. $(-1, 1), (4, 5)$ **10.** $(3, -2), (-6, 4)$ **11.** $(-4, 3), (-1, -7)$

12. $(-2, 5), (-6, -8)$ **13.** $(-8, -4), (4, 2)$ **14.** $(-1, -3), (-8, -9)$

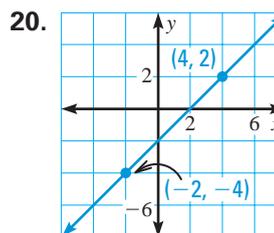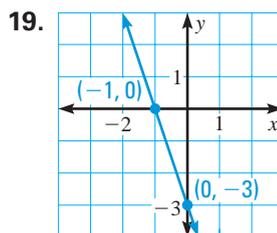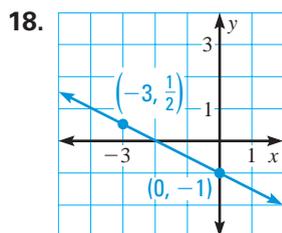**15.** $(5, 3), (4, -3)$ **16.** $(6, -10), (-3, -8)$ **17.** $(12, 2), (7, 2)$

PRACTICE AND APPLICATIONS

WRITING EQUATIONS **Write an equation in slope-intercept form of the line shown in the graph.**

18.

19.

20.

GRAPHING **Graph the points and draw a line through them. Write an equation in slope-intercept form of the line that passes through the points.**

21. $(-1, -2), (2, 6)$ **22.** $(1, 4), (5, -1)$ **23.** $(-1, -2), (3, -2)$

24. $(2, 0), (-2, 6)$ **25.** $(2, -3), (-3, 7)$ **26.** $(0, -5), (3, 4)$

27. $(6, -4), (-1, 2)$ **28.** $(-2, -1), (8, 8)$ **29.** $(1, 1), (7, 4)$

30. $(2, 4), (1, -2)$ **31.** $(5, -6), (5, -3)$ **32.** $(-3, -5), (1, 9)$

33. $(-5, 2), (6, 1)$ **34.** $(-6, 2), (-4, 11)$ **35.** $(-1, 10), (12, -4)$

SLOPE-INTERCEPT FORM Write an equation in slope-intercept form of the line that passes through the points.

36. $(-6, -5), (1, 4)$ **37.** $(2, 3), (4, 3)$ **38.** $(5, -10), (12, -7)$

39. $(14, -3), (-6, 9)$ **40.** $(-7, 9), (-3, 8)$ **41.** $(-8, 9), (10, -3)$

42. $\left(\frac{1}{4}, 2\right), \left(-5, \frac{2}{3}\right)$ **43.** $\left(\frac{1}{2}, -\frac{1}{2}\right), \left(\frac{1}{9}, \frac{3}{9}\right)$ **44.** $(-8.5, 6.75), (3.33, -9.75)$

45. PERPENDICULAR LINES Which of the lines are perpendicular? Explain.

**line p:** $y = \frac{1}{5}x + 2$ **line q:** $y = 5x - \frac{1}{2}$ **line r:** $y = -5x + 3$

46. Write an equation of a line through $(0, 2)$ that is perpendicular to $y = -4x + 6$.

47. Write an equation of a line through $(4, 5)$ that is perpendicular to $y = \frac{1}{2}x + 3$.
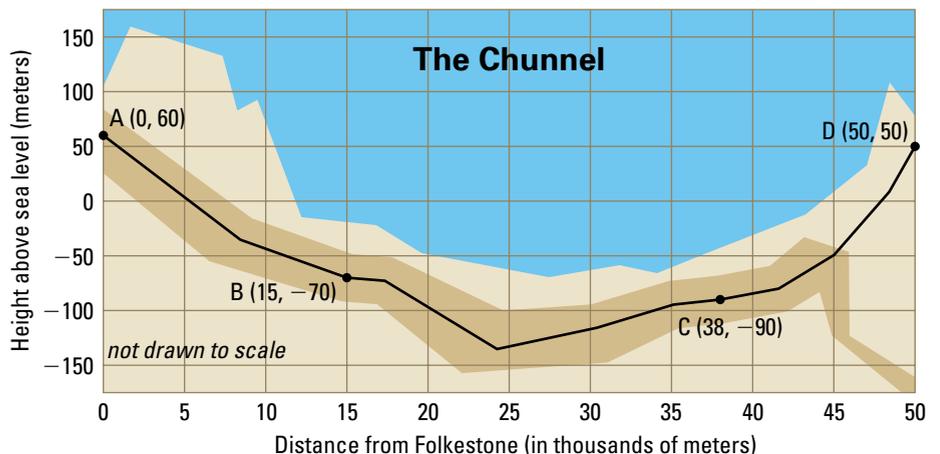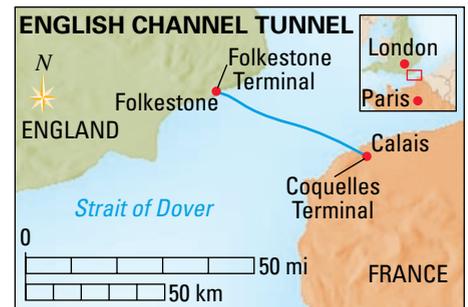
GEOMETRY **CONNECTION** In Exercises 48–50, use the graph.

48. Find the perpendicular sides of trapezoid *WXYZ*. How do you know mathematically that these sides are perpendicular?

49. Write equations of the lines containing the perpendicular sides.

50. Write equations of the lines containing the two parallel sides. How do you know these sides are parallel?
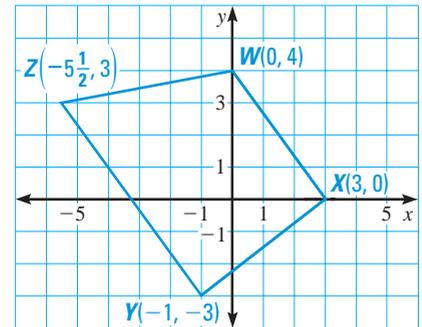
CHUNNEL In Exercises 51 and 52, use the diagram of the *Chunnel*, a railroad tunnel built under the English channel. It is one of the most ambitious engineering feats of the twentieth century.

51. Write an equation of the line from point *A* to point *B*. What is the slope of the line?

52. Write an equation of the line from point *C* to point *D*. What is the slope of the line? Is the Chunnel steeper on the French side or on the English side?

In Exercises 53–55, use the following information.
At sea level, the speed of sound in air is linearly related to the air temperature.
If it is 35°C, sound will travel at a rate of 352 meters per second. If it is 15°C,
sound will travel at a rate of 340 meters per second.

53. Write a linear equation that models speed of sound s in terms of air
temperature T.

54. How fast will sound travel at sea level if it is 25°C outside?

55. If sound travels at a rate of 346 meters per second at sea level, what is the
temperature?

ECHOES **In Exercises 56 and 57, use the following information.**
You yell across a canyon. Your sound travels at a rate of 343 meters per second.

56. Use your equation from Exercise 53 to determine the air temperature.

57. After you shout, it takes 4 seconds to hear your echo. How far are you from
the canyon wall?

**Test
Preparation**

58. MULTIPLE CHOICE What is the equation of the line that passes through the
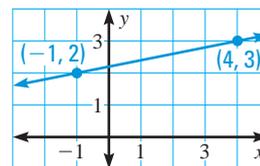points $(7, 4)$ and $(-5, -2)$?

   (A) $y = \frac{1}{2}x - \frac{1}{2}$ (B) $y = -\frac{1}{2}x + \frac{1}{2}$

   (C) $y = -\frac{1}{2}x - \frac{1}{2}$ (D) $y = \frac{1}{2}x + \frac{1}{2}$

59. MULTIPLE CHOICE What is the equation of the line
shown in the graph?

   (A) $y = 5x + 11$ (B) $y = 5x - 17$

   (C) $y = \frac{11}{5}x + \frac{1}{5}$ (D) $y = \frac{1}{5}x + \frac{11}{5}$

60. MULTIPLE CHOICE Choose which lines are perpendicular.

Line p passes through $(4, 0)$ and $(6, 4)$

Line q passes through $(0, 4)$ and $(6, 4)$

Line r passes through $(0, 4)$ and $(0, 0)$

   (A) line p and line q (B) line p and line r

   (C) line q and line r (D) none of these

★ **Challenge**

CRITICAL THINKING **In Exercises 61–64, use a graph to determine whether
the given three points seem to lie on the same line. If they do, prove
algebraically that they lie on the same line and write an equation of the line.**

61. $(-3, -1), (0, 1), (12, 9)$ **62.** $(4, -2), (-1, 2), (-8, 9)$

63. $(-2, -1), (3, 2), (7, 5)$ **64.** $(3, -3), (-1, 13), (1, 5)$

65. What is an equation of the line that passes through the

points $\left(3\frac{1}{2}, 4\right)$ and $\left(-5, -\frac{9}{2}\right)$?

MIXED REVIEW

SIMPLIFYING EXPRESSIONS Simplify the expression. (Review 2.7)

66. $\dfrac{6x + 12y}{24}$

67. $\dfrac{48a - 56b}{16}$

68. $\dfrac{14x}{-28y - 1}$

SOLVING EQUATIONS Solve the equation. (Review 3.4)

69. $4x - 11 = -31$

70. $5x - 7 + x = 19$

71. $18 = 4 - \dfrac{2x}{5}$

72. $2x - 6 = 20$

73. $\dfrac{1}{2}a + 8\dfrac{1}{2}a = 3$

74. $7y = 9y - 8$

PLOTTING POINTS Plot the points in the table. Determine whether the slope of the line given by the points is *positive* or *negative*. (Review 4.1, 4.4 for 5.4)
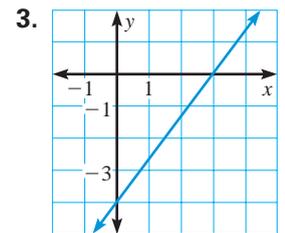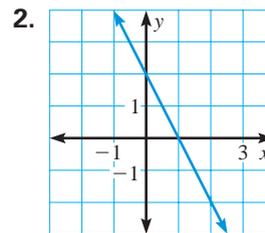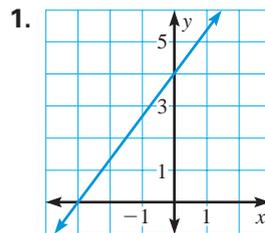
75.

x	−1	0	1	2	3	4
y	−7	−4	−1	2	5	8

76.

x	−2	−1	0	1	2	3
y	7	5	3	1	−1	−3

QUIZ 1

Write an equation of the line. (Lesson 5.1)

1.

2.

3.

Write an equation of the line that passes through the point and has the given slope. Write the equation in slope-intercept form. (Lesson 5.2)

4. $(-5, 4)$, $m = 2$

5. $(2, 1)$, $m = -3$

6. $(-3, -6)$, $m = 1$

7. Find an equation of the line that is perpendicular to the line $y = -x + 6$ and passes through $(-2, -4)$. (Lesson 5.3)

Write an equation in slope-intercept form of the line that passes through the points. (Lesson 5.3)

8. $(8, -3)$, $(5, -2)$

9. $(-2, 6)$, $(-5, -7)$

10. $(4, 4)$, $(-7, 4)$

🌎 **CANOE RENTAL** In Exercises 11–13, suppose your family rents a canoe for a deposit of $10 plus $28 per day. (Lesson 5.1)

11. Write an equation to model the total cost y of renting a canoe for x days.

12. Graph the equation. Label the y-intercept.

13. Use the equation to find the cost of renting a canoe for 3 days.