5.1 Writing Linear Equations in Slope-Intercept Form

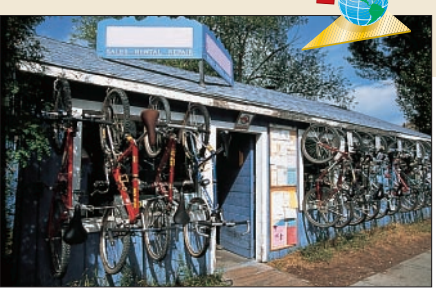
What you should learn

GOAL 1 Use the slope-intercept form to write an equation of a line.

GOAL 2 Model a **real-life** situation with a linear function, such as the population of California in **Example 3**.

Why you should learn it

▼ To model **real-life** situations, such as finding the cost of renting a bicycle in **Exs. 9–11**.



GOAL 1 USING SLOPE-INTERCEPT FORM

In this lesson you will learn to write an equation of a line using its slope and y-intercept. To do this, you need to use the **slope-intercept form** of the equation of a line.

$$y = mx + b \quad \longleftarrow \text{Slope-intercept form}$$

In the equation, m is the slope and b is the y-intercept. Recall that the y-intercept is the y-coordinate of the point where the line crosses the y-axis.

EXAMPLE 1 Writing an Equation of a Line

Write an equation of the line whose slope m is -2 and whose y-intercept b is 5.

SOLUTION

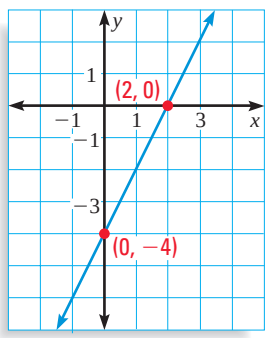
You are given the slope $m = -2$ and the y-intercept $b = 5$.

$y = mx + b$ Write slope-intercept form.

$y = -2x + 5$ Substitute -2 for m and 5 for b.

EXAMPLE 2 Writing an Equation of a Line from a Graph

Write an equation of the line shown in the graph.



SOLUTION

The line intersects the y-axis at $(0, -4)$, so the y-intercept b is -4. The line also passes through the point $(2, 0)$. Find the slope of the line:

$$m = \frac{\text{rise}}{\text{run}} = \frac{-4 - 0}{0 - 2} = \frac{-4}{-2} = 2$$

Knowing the slope and y-intercept, you can write an equation of the line.

$y = mx + b$ Write slope-intercept form.

$y = 2x + (-4)$ Substitute 2 for m and -4 for b.

$y = 2x - 4$ Simplify.

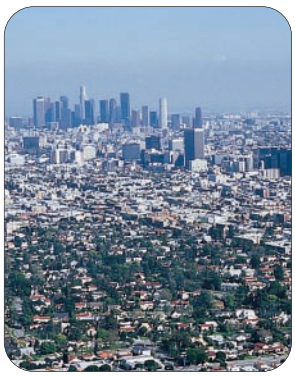
STUDENT HELP

Study Tip

In Example 2, you can also find the slope of the line by reading the graph.

$$\frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$$

FOCUS ON APPLICATIONS



REAL LIFE **POPULATION DATA** In 1990, California had a population of about 29.76 million. During the next 15 years, the state's population was expected to increase by an average of about 0.31 million people per year.

▶ Source: U.S. Bureau of the Census

GOAL 2 MODELING A REAL-LIFE SITUATION

In Lesson 4.8 you saw examples of linear functions.

$f(x) = mx + b$ ← **Linear function**

A *linear model* is a linear function that is used to model a real-life situation. In a linear model, m is the rate of change and b is the initial amount. In such models, sometimes line graphs can be used to approximate change that occurs in discrete steps.

EXAMPLE 3 *A Linear Model for Population*

POPULATION CHANGE Linear functions can approximate population change.

a. Write a linear equation to approximate the expected population of California in any year between 1990 and 2005. Use the information below the photo.

b. Use the equation to predict the population of California in 2005.

SOLUTION

a. **VERBAL MODEL**

$$\text{California Population} = \text{1990 Population} + \text{Increase per year} \cdot \text{Years since 1990}$$

LABELS

California Population = P (million people)

1990 Population = **29.76** (million people)

Increase per year = **0.31** (million people per year)

Years since 1990 = t (years)

ALGEBRAIC MODEL

$P = 29.76 + 0.31 \cdot t$ **Linear model**

UNIT ANALYSIS Check that *people* are the units in the algebraic model:

$$\textit{people} = \text{people} + \frac{\text{people}}{\cancel{\text{year}}} \cdot \cancel{\text{year}}$$

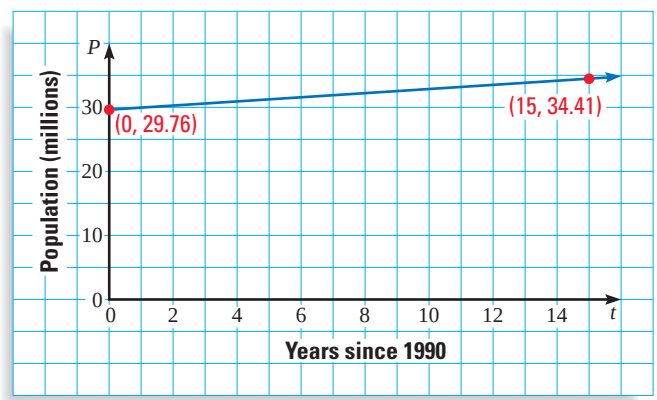
b. When $t = 15$, the year will be 2005. Substitute in your equation.

$P = 29.76 + 0.31(15)$

$= 29.76 + 4.65$

$= 34.41$

▶ In 2005 the population will be about 34.4 million people. The graph represents this prediction.



STUDENT HELP

Look Back
For help with graphing linear equations, see p. 219.

EXAMPLE 4 A Linear Model for Telephone Charges

You can approximate collect telephone call charges using a linear function.

a. Write a linear equation to approximate the cost of collect telephone calls. Use the information shown on the telephone bill to create your model.

b. How much would a 9-minute collect telephone call cost?

Date	Time	Area Number	1st Min.	Rate	Add'l Min	Amount
Jan 7	857PM	360 555-1829	$3.44	$0.24	4	$4.40
Jan 16	558PM	360 555-1829	$3.44	$0.24	1	$3.68
Jan 17	837PM	360 555-1829	$3.44	$0.24	17	$7.52
Jan 22	601PM	360 555-1829	$3.44	$0.24	9	$5.60
Feb 3	859PM	360 555-1829	$3.44	$0.24	10	$5.84

SOLUTION

PROBLEM SOLVING STRATEGY

a. **VERBAL MODEL**

$$\text{Total cost} = \text{Cost for first minute} + \text{Rate per additional minute} \cdot \text{Number of additional minutes}$$

LABELS

Total cost = C (dollars)

Cost for first minute = **3.44** (dollars)

Rate per additional minute = **0.24** (dollars per minute)

Number of additional minutes = t (minutes)

ALGEBRAIC MODEL

$$C = 3.44 + 0.24 \cdot t$$ Linear model

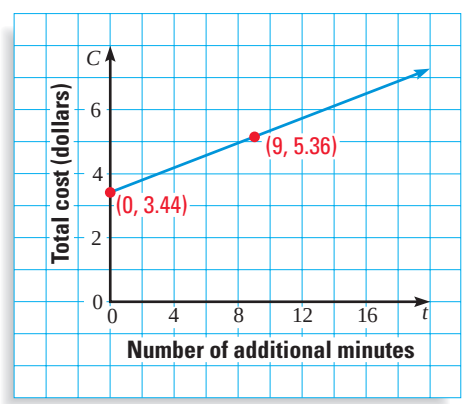
UNIT ANALYSIS Check that *dollars* are the units in the algebraic model:

$$\textit{dollars} = \text{dollars} + \frac{\text{dollars}}{\cancel{\text{min}}} \cdot \cancel{\text{min}}$$

b. A 9-minute call would involve 8 additional minutes. Substitute in your equation.

$$C = 3.44 + 0.24 \cdot t$$
$$= 3.44 + 0.24(8)$$
$$= 5.36$$

▶ A 9-minute call would cost $5.36. The graph represents this prediction.



GUIDED PRACTICE

Vocabulary Check ✓

1. Is 5 the *x-intercept* or the *y-intercept* of the line $y = 3x + 5$? Explain.

Concept Check ✓

2. Describe the rate of change in the context of Example 3 on page 274.

Write an equation of the line in slope-intercept form.

Skill Check ✓

3. The slope is 1; the *y*-intercept is 0.

4. The slope is -7; the *y*-intercept is $-\frac{2}{3}$.

5. The slope is -1; the *y*-intercept is 3.

6. The slope is -2; the *y*-intercept is 0.

7. The slope is -3; the *y*-intercept is $\frac{1}{2}$.

8. The slope is 4; the *y*-intercept is -6.

RENTING A BIKE **Suppose that bike rentals cost $4 plus $1.50 per hour.**

9. Write an equation to model the total cost *y* of renting a bike for *x* hours.

10. Graph the equation. Label the *y*-intercept.

11. Use the equation to find the cost of renting a bike for 12 hours.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 801.

WRITING EQUATIONS **Write an equation of the line in slope-intercept form.**

12. The slope is 3; the *y*-intercept is -2.

13. The slope is 1; the *y*-intercept is 2.

14. The slope is 0; the *y*-intercept is 4.

15. The slope is 2; the *y*-intercept is -1.

16. The slope is $\frac{3}{2}$; the *y*-intercept is 3.

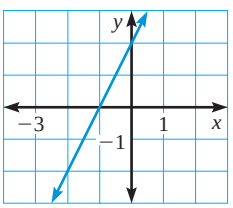
17. The slope is $-\frac{1}{4}$; the *y*-intercept is 1.

18. The slope is -6; the *y*-intercept is $\frac{3}{4}$.

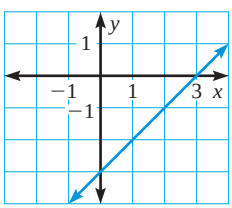
19. The slope is -3; the *y*-intercept is $-\frac{1}{2}$.

GRAPHICAL REASONING **Write an equation of the line shown in the graph.**

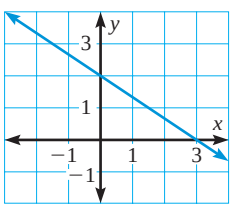
20.



21.

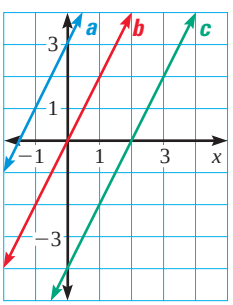


22.

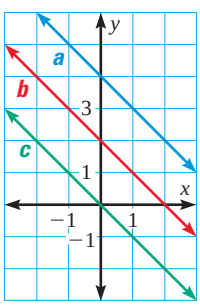


PARALLEL LINES **Write an equation of each line in slope-intercept form.**

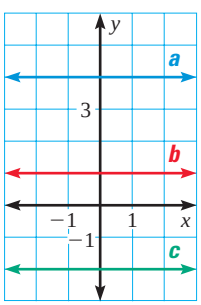
23.



24.



25.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 12–19
Example 2: Exs. 20–25
Example 3: Exs. 26, 27
Example 4: Exs. 28, 29

SOUTH CAROLINA POPULATION **In Exercises 26 and 27, use the following information. In 1990 the population of South Carolina was approximately 3,486,000. During the next five years, the population increased by approximately 37,400 people per year.**

INTERNET **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

26. Write an equation to model the population P of South Carolina in terms of t, the number of years since 1990.

27. Estimate the population of South Carolina in 1996.

RENTING A MOVING VAN **In Exercises 28 and 29, a rental company charges a flat fee of $30 and an additional $.25 per mile to rent a moving van.**

28. Write an equation to model the total charge y (in dollars) in terms of x, the number of miles driven.

29. Copy and complete the table using the equation you found in Exercise 28.

Miles (x)	25	50	75	100
Cost (y)	?	?	?	?

UNIT ANALYSIS **In Exercises 30–32, write a linear equation to model the situation. Use unit analysis to check your model.**

30. You borrow $40 from your sister. To repay the loan, you pay her $5 a week.

31. Your uncle weighed 180 pounds. He has lost 2 pounds a month for 8 months.

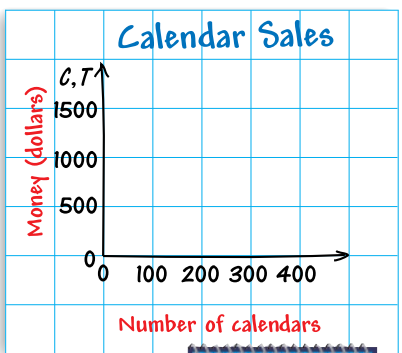
32. You have walked 5 miles on a hiking trail. You continue to walk at the rate of 2 miles per hour for 6 hours.

33. **TRAVELING HOME** You are traveling home in a bus whose speed is 50 miles per hour. At noon you are 200 miles from home. Write an equation that models your distance y from home in terms of t, the number of hours since noon. Why does the line given by this equation have a negative slope?

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 34–37.

FUNDRAISING **In Exercises 34–37, you are designing a calendar as a fund-raising project for your Biology Club. The cost of printing is $500, plus $2.50 per calendar.**

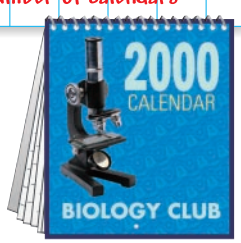


34. Write an equation to model the total cost C of printing x calendars.

35. You sell each calendar for $5.00. Write an equation to model the total income T for selling x calendars.

36. Graph both equations in the same coordinate plane. Estimate the point at which the two lines intersect. Explain the significance of this point of intersection in the context of the real-life problem.

37. **ANALYZING DATA** You estimate the club can sell 200 calendars. Write an analysis of how effective this fundraising project will be for the club.



Test Preparation

FLORIDA POPULATION **In Exercises 38 and 39, use the following information.** The U.S. Bureau of the Census predicted that the population of Florida would be about 17.4 million in 2010 and then would increase by about 0.22 million per year until 2025.

38. MULTIPLE CHOICE Choose the linear model that predicts the population P of Florida (in millions) in terms of t, the number of years since 2010.

(A) $P = 17.4t + 0.22$ (B) $P = -0.22t + 17.4$

(C) $P = 0.22t + 17.4$ (D) $P = -17.4t + 0.22$

39. MULTIPLE CHOICE According to the correct model, the population of Florida in 2011 will be about ? million.

(A) 17.18 (B) 3.8 (C) 17.62 (D) 39.4

Challenge

40. CLASS SIZE In January, your ceramics class begins with 12 students. In every month after January, three new students join and one student drops out.

a. Write a linear equation to model the situation.

b. Graph the model.

c. Predict the size of your ceramics class in May and June.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression when $x = -3$ and $y = 6$.** (Review 2.7)

41. $\frac{3x}{x + y}$ **42.** $\frac{x}{x + 2}$ **43.** $\frac{y^2 + x}{x}$

SCHOOL NEWSPAPER **You are designing a newspaper page with three photos. The page is $13\frac{1}{4}$ inches wide with 1 inch margins on both sides. You need to allow $\frac{3}{4}$ inch between photographs. How wide should you make the photos if they are of equal size?** (Review 3.5)

44. Sketch a diagram of the newspaper page.

45. Write an equation to model the problem. Use the diagram to help.

46. Solve the equation and answer the question.

WINDSURFING **Match the description with the linear model $y = 10$ or the linear model $y = 10x$. Graph the model.** (Review 4.2)

47. You rent a sailboard for $10 per hour.

48. You rent a life jacket for a flat fee of $10.

GRAPHING LINEAR EQUATIONS **Find the slope and the y-intercept of the graph of the equation. Then graph the equation.** (Review 4.6 for 5.2)

49. $y + 2x = 2$ **50.** $3x - y = -5$ **51.** $2y - 3x = 6$

52. $4x + 2y = 6$ **53.** $4y + 12x = 16$ **54.** $25x - 5y = 30$

55. $x + 3y = 15$ **56.** $x + 6y = 12$ **57.** $x - y = 10$