4.8 Functions and Relations

What *you should learn*

GOAL 1 Identify when a relation is a function.

GOAL 2 Use function notation to represent **real-life** situations, such as modeling butterfly migration in **Example 4**.

Why *you should learn it*

▼ To solve **real-life** problems such as projecting the number of high school students and college students in **Exs. 50–52**.

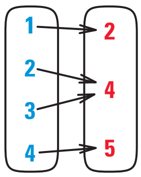
REAL LIFE

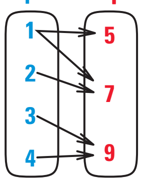
GOAL 1 IDENTIFYING FUNCTIONS

In defining functions, you learned that for every input there corresponds exactly one output, as in the case of linear functions defined by the rule $y = mx + b$. However, there are rules that associate more than one output with an input, as in $x = y^2$ where the input $x = 4$ corresponds to the outputs $y = 2$ and $y = -2$. A rule of this type, where an input can have more than one output, is a *relation*. A **relation** is defined to be any set of ordered pairs.

EXAMPLE 1 *Identifying Functions*

Decide whether the relation is a function. If so, give the domain and the range.

a. 

b. 

SOLUTION

a. The relation is a function. For each input there is exactly one output. The domain of the function is the set of input values 1, 2, 3, and 4. The range is the set of output values 2, 4, and 5.

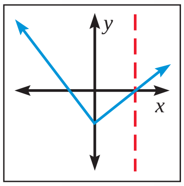
b. The relation is not a function because the input 1 has two outputs: 5 and 7.

....

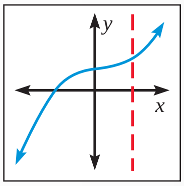
When you graph a relation, the input is given by the horizontal axis and the output is given by the vertical axis.

VERTICAL LINE TEST FOR FUNCTIONS

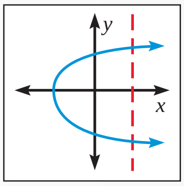
A relation is a function of the horizontal-axis variable if and only if no vertical line passes through two or more points on the graph.



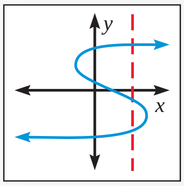
function



function



not a function



not a function

STUDENT HELP

Look Back
For help with domain and range, see p. 47.

GOAL 2 USING FUNCTION NOTATION

When a function is defined by an equation, it is convenient to give the function a name. In making a table of values for the equation $y = 3x + 2$, you apply the rule *multiply by three and add two* to transform an input x into the output y. By giving the rule *multiply by three and add two* the name f, you have a shorthand notation for applying this rule to different numbers and variables. The fact that this rule transforms the input 1 into the output 5 can now be written $f(1) = 5$.

In general, the symbol $f(x)$ replaces y and is read as "the value of f at x" or simply as "f of x." It does not mean f times x. $f(x)$ is called **function notation**.

EXAMPLE 2 *Evaluating a Function*

> **STUDENT HELP**
>
> **Study Tip**
> You don't have to use *f* to name a function. You can use other letters, such as *g* and *h*.

Evaluate the function for the given value of the variable.

a. $f(x) = 2x - 3$ when $x = -2$ **b.** $g(x) = -5x$ when $x = 0$

SOLUTION

a. $f(x) = 2x - 3$ **Write original function.**

$f(-2) = 2(-2) - 3$ **Substitute −2 for *x*.**

$= -7$ **Simplify.**

b. $g(x) = -5x$ **Write original function.**

$g(0) = -5(0)$ **Substitute 0 for *x*.**

$= 0$ **Simplify.**

Since function notation allows you to write $f(x)$ in place of y, the **graph of a function** f is the set of all points $(x, f(x))$, where x is in the domain of the function. To graph a linear function, you may want to rewrite the function using x-y notation.

EXAMPLE 3 *Graphing a Linear Function*

Graph $f(x) = -\frac{1}{2}x + 3$.

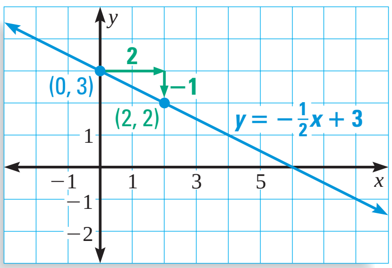
SOLUTION

Rewrite the function as $y = -\frac{1}{2}x + 3$.

The y-intercept is 3, so plot (0, 3).

The slope is $-\frac{1}{2}$.

Draw a slope triangle to locate a second point on the line.



EXAMPLE 4 *Writing and Using a Linear Function*

BUTTERFLIES Use the diagram about monarch butterfly migration.

a. Write a linear function that models the distance traveled by a migrating butterfly.

b. Use the model to estimate the distance traveled after 30 days of migration.

c. Graph your model and label the point that represents the distance traveled after 30 days.

Monarch butterflies migrate from the northern United States to Mexico.

SOLUTION

a. You can see in the photo caption below that the butterflies travel about 2000 miles in 40 days. Their average speed is 50 miles per day. To create a linear model, we approximate the actual speed with this constant.

PROBLEM SOLVING STRATEGY

VERBAL MODEL Distance traveled = Average speed • Time

LABELS

Time = t (days)

Average speed = **50** (miles per day)

Distance traveled = $f(t)$ (miles)

ALGEBRAIC MODEL $f(t) = 50\,t$

b. To estimate the distance traveled after 30 days, substitute 30 for t.

$f(t) = 50t$ **Write linear function.**

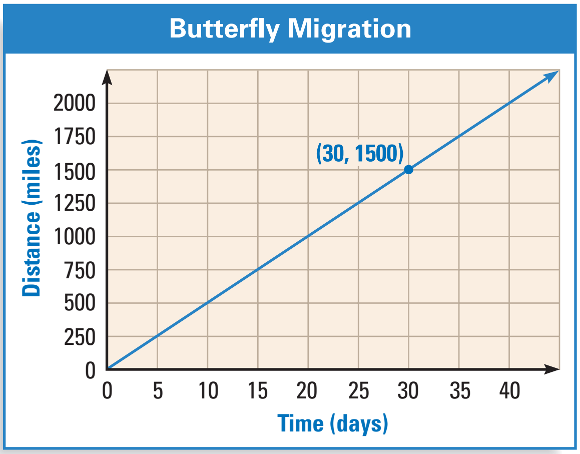
$f(30) = 50 \cdot 30$ **Substitute 30 for *t*.**

$= 1500$ **Simplify.**

The distance traveled after 30 days is 1500 miles.

UNIT ANALYSIS You can use unit analysis to check that *miles* are the units of the solution.

$$\frac{\text{miles}}{\cancel{\text{day}}} \cdot \cancel{\text{days}} = \text{miles}$$



c. From the graph, you can see that distance traveled is a function of time.

FOCUS ON APPLICATIONS

REAL LIFE MONARCH BUTTERFLY Thousands of monarch butterflies migrate in one large flock. The 2000-mile trip takes about 40 days.

GUIDED PRACTICE

Vocabulary Check ✓

1. Is every function a relation? Is every relation a function? Explain.

Concept Check ✓

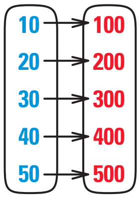
2. Describe a line that cannot be the graph of a linear function.

Skill Check ✓

Evaluate the function $f(x) = 3x - 10$ for the given value of x.

3. $x = 0$ **4.** $x = 20$ **5.** $x = -2$ **6.** $x = \frac{2}{3}$

In Exercises 7–9, decide whether the relation is a function. If it is a function, give the domain and the range.

7.


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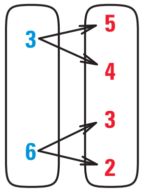

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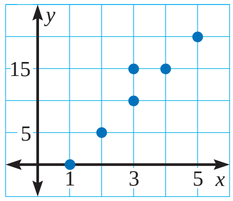


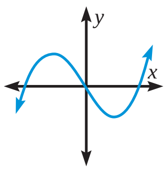
10. **CAR TRAVEL** Your average speed during a trip is 40 miles per hour. Write a linear function that models the distance you travel $d(t)$ as a function of t, the time spent traveling.

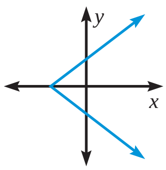
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 800.

GRAPHICAL REASONING **Decide whether the graph represents y as a function of x. Explain your reasoning.**

11.


12.


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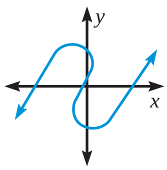
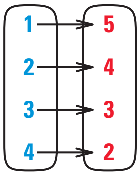
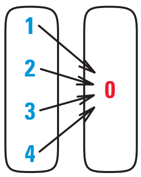


RELATIONS AND FUNCTIONS **In Exercises 14–19, decide whether the relation is a function. If it is a function, give the domain and the range.**

14.


15.


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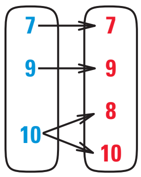


17.

Input	Output
0	2
1	4
2	6
3	8

18.

Input	Output
0	1
2	2
4	3
2	4

19.

Input	Output
1	1
3	2
5	3
7	1

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 11–19
Example 2: Exs. 20–28
Example 3: Exs. 29–40
Example 4: Exs. 50–52

EVALUATING FUNCTIONS **Evaluate the function when $x = 2$, $x = 0$, and $x = -3$.**

20. $f(x) = 10x + 1$ **21.** $g(x) = 8x - 2$ **22.** $h(x) = 3x + 6$

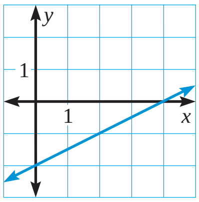
23. $g(x) = 1.25x$ **24.** $h(x) = 0.75x + 8$ **25.** $f(x) = 0.33x - 2$

26. $h(x) = \frac{3}{4}x - 4$ **27.** $g(x) = \frac{2}{5}x + 7$ **28.** $f(x) = \frac{2}{7}x + 4$

GRAPHICAL REASONING **Match the function with its graph.**

A. $f(x) = 3x - 2$ **B.** $f(x) = 2x + 2$ **C.** $f(x) = \frac{1}{2}x - 2$

29.



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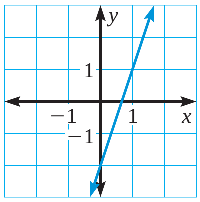
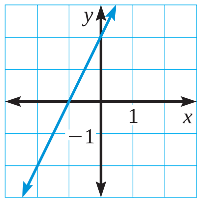


31.



GRAPHING FUNCTIONS **In Exercises 32–40, graph the function.**

32. $f(x) = -2x + 5$ **33.** $g(x) = -x + 4$ **34.** $h(x) = 5x - 6$

35. $g(x) = 2x - 3$ **36.** $h(x) = -\frac{1}{3}x + 2$ **37.** $f(x) = -\frac{1}{2}x + 1$

38. $f(x) = 4x + 1$ **39.** $h(x) = 5$ **40.** $g(x) = -3x - 2$

41. *Writing* Describe how to find the slope of the graph of linear function f given that $f(1) = 2$ and $f(3) = -1$.

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 42–49.

FINDING SLOPE **Find the slope of the graph of the linear function *f*.**

42. $f(2) = -3,\ f(-2) = 5$ **43.** $f(0) = 4,\ f(4) = 0$

44. $f(-3) = -9,\ f(3) = 9$ **45.** $f(6) = -1,\ f(3) = 8$

46. $f(9) = -1,\ f(-1) = 2$ **47.** $f(-2) = -1,\ f(2) = 6$

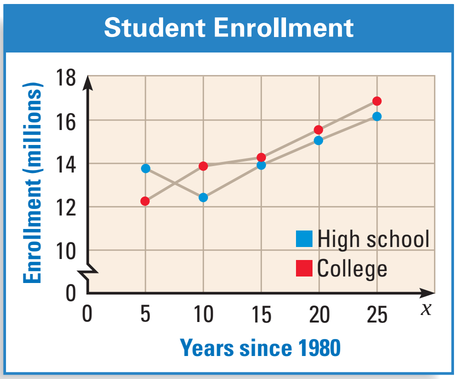
48. $f(2) = 2,\ f(3) = 3$ **49.** $f(-1) = 2,\ f(3) = 2$

HIGH SCHOOL AND COLLEGE STUDENTS **In Exercises 50–52, use the graph below. It shows the projected number of high school and college students in the United States for different years. Let *x* be the number of years since 1980.**

50. Is the projected high school enrollment a function of the year? Is the projected college enrollment a function of the year? Explain.

51. Let $f(x)$ represent the projected number of high school students in year x. Estimate $f(2005)$.

52. Let $g(x)$ represent the projected number of college students in year x. Estimate $g(2005)$.



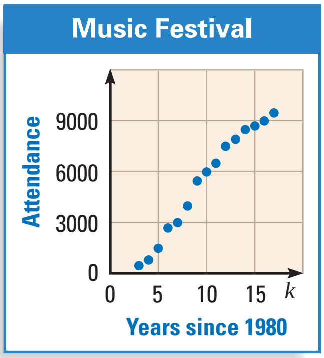
INTERNET **DATA UPDATE** of National Center for Education Statistics data at www.mcdougallittell.com

FOCUS ON APPLICATIONS

REAL LIFE **ZYDECO MUSIC** blends elements from a variety of cultures. The accordion has German origins. The rub board was invented by Louisiana natives whose roots go back to France and Canada.

53. **ZYDECO MUSIC** The graph shows the number of people who attended the Southwest Louisiana Zydeco Music Festival for different years, where k is the number of years since 1980. Is the number of people who attended the festival a function of the year? Explain.

▶ Source: Louisiana Zydeco Music Festival



54. **MASTERS TOURNAMENT** The table shows information about the top 7 winners of the 1997 Masters Tournament in Augusta, Georgia. Graph the relation. Is the money earned a function of the score? Explain. ▶ Source: Golfweb

Score	270	282	283	284	285	285	286
Prize ($)	486,000	291,600	183,600	129,600	102,600	102,600	78,570

55. **SCIENCE CONNECTION** It takes 4.25 years for starlight to travel 25 trillion miles. Let t be the number of years and let $f(t)$ be trillions of miles traveled. Write a linear function $f(t)$ that expresses the distance traveled as a function of time.

Test Preparation

QUANTITATIVE COMPARISON **In Exercises 56–58, choose the statement that is true about the given quantities.**

(A) The number in column A is greater.

(B) The number in column B is greater.

(C) The two numbers are equal.

(D) The relationship cannot be determined from the given information.

	Column A	Column B
56.	$f(x) = -7x + 4$ when $x = -4$	$f(x) = -7x + 4$ when $x = -5$
57.	$f(x) = \frac{1}{2}x - 3$ when $x = 7$	$f(x) = \frac{1}{2}x - 5$ when $x = 11$
58.	$g(x) = 6x - \frac{1}{7}$ when $x = \frac{2}{7}$	$h(t) = 6t - \frac{1}{7}$ when $t = \frac{2}{7}$

★ Challenge

RESTRICTING DOMAINS **In Exercises 59–62, use the equations $y = \frac{1}{x}$ and $y = \frac{1}{x+3}$.**

59. Use a graphing calculator or computer to graph both equations.

60. Explain why each equation is not a function for all real values of x.

61. How can you restrict the domain of each equation so that it is a function?

62. Describe what happens to the graph of $y = \frac{1}{x}$ near the value(s) of x for which the graph is not a function.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

MATRICES **Find the sum or the difference of the matrices.** (Review 2.4)

63. $\begin{bmatrix} 4 & -8 \\ 7 & 0 \end{bmatrix} - \begin{bmatrix} -5 & 3 \\ -5 & -7 \end{bmatrix}$

64. $\begin{bmatrix} -6.5 & -4.2 \\ 0 & 3.7 \end{bmatrix} + \begin{bmatrix} 2.4 & -5.1 \\ 4.3 & -3 \end{bmatrix}$

65. $\begin{bmatrix} 6.2 & -12 \\ -2.5 & -4.4 \\ 3.4 & -5.8 \end{bmatrix} - \begin{bmatrix} -3.6 & 5.9 \\ 9.8 & -4.3 \\ -9 & 7.4 \end{bmatrix}$

66. $\begin{bmatrix} 9 & 1 & 6 \\ -4 & -7 & 1 \\ -5 & 0 & -1 \end{bmatrix} + \begin{bmatrix} -6 & 3 & -5 \\ -2 & 4 & -4 \\ 0 & 5 & 1 \end{bmatrix}$

SOLVING EQUATIONS **Solve the equation if possible.** (Review 3.4)

67. $4x + 8 = 24$
68. $3n = 5n - 12$
69. $9 - 5z = -8z$
70. $-5y + 6 = 4y + 3$
71. $3b + 8 = 9b - 7$
72. $-7q - 13 = 4 - 7q$

GRAPHING LINES **In Exercises 73–78, write the equation in slope-intercept form. Then graph the equation.** (Review 4.6 for 5.1)

73. $2x - y + 3 = 0$
74. $x + 2y - 6 = 0$
75. $y - 2x = -7$
76. $5x - y = 4$
77. $x - 2y + 4 = 2$
78. $4y + 12 = 0$

79. **CHARITY WALK** You start 5 kilometers from the finish line and walk $1\frac{1}{2}$ kilometers per hour for 2 hours, where d is your distance from the finish line. Graph the situation. (Review 4.6 for 5.1)

QUIZ 3

Self-Test for Lessons 4.7 and 4.8

Solve the equation graphically. Check your solution algebraically. (Lesson 4.7)

1. $4x + 3 = -5$
2. $6x - 12 = -9$
3. $8x - 7 = x$
4. $-5x - 4 = 3x$
5. $\frac{1}{3}x + 5 = -\frac{2}{3}x - 8$
6. $\frac{3}{4}x + 2 = -\frac{3}{4}x - 6$

Evaluate the function when $x = 3$, $x = 0$, and $x = -4$. (Lesson 4.8)

7. $h(x) = 5x - 9$
8. $g(x) = -4x + 3$
9. $f(x) = 1.75x - 2$
10. $h(x) = -1.4x$
11. $f(x) = \frac{1}{4}x + 9$
12. $g(x) = \frac{4}{7}x + \frac{2}{7}$

In Exercises 13–18, graph the function. (Lesson 4.8)

13. $f(x) = -5x$
14. $h(x) = 4x - 7$
15. $f(x) = 3x - 2$
16. $h(x) = \frac{2}{5}x - 1$
17. $f(x) = \frac{1}{4}x + \frac{1}{2}$
18. $g(x) = -6x + 5$

19. **LUNCH BOX BUSINESS** You have a small business making and delivering box lunches. You calculate your average weekly cost y of producing x lunches using the function $y = 2.1x + 75$. Last week your cost was \$600. How many lunches did you make last week? Solve algebraically and graphically. (Lesson 4.7)