4.5 Direct Variation

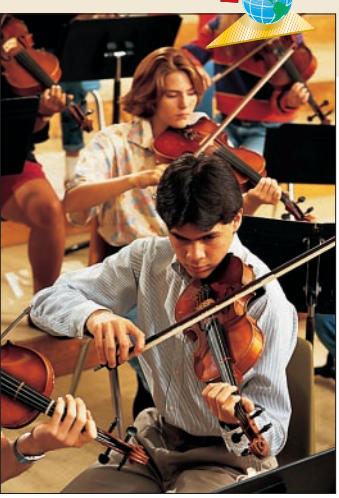
What you should learn

GOAL 1 Write linear equations that represent direct variation.

GOAL 2 Use a ratio to write an equation for direct variation, such as the ratio of tail length to body length in alligators in **Example 4**.

Why you should learn it

▼ To solve **real-life** problems such as lengths of several stringed instruments in **Exs. 36 and 37**.



GOAL 1 RECOGNIZING AND USING DIRECT VARIATION

Two variables x and y *vary directly* if there is a nonzero number k such that the following is true.

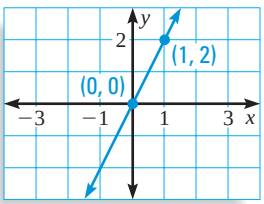
$$y = kx \quad \longleftarrow \text{Model for direct variation}$$

The number k is the **constant of variation**. Two quantities that vary directly are said to have **direct variation**.

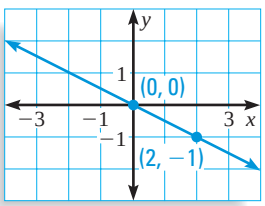
EXAMPLE 1 Graphs of Direct Variation Models

Find the constant of variation and the slope of each direct variation model.

a. $y = 2x$



b. $y = -\frac{1}{2}x$



SOLUTION

a. For the equation $y = 2x$, the constant of variation is $k = 2$.

To find the slope of the line, use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - 0}{1 - 0} = 2$$

b. For the equation $y = -\frac{1}{2}x$, the constant of variation is $k = -\frac{1}{2}$.

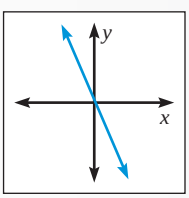
To find the slope of the line, use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

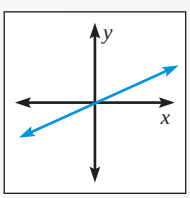
$$m = \frac{-1 - 0}{2 - 0} = -\frac{1}{2}$$

PROPERTIES OF GRAPHS OF DIRECT VARIATION MODELS

- The graph of $y = kx$ is a line through the origin.
- The slope of the graph of $y = kx$ is k.



$k < 0$



$k > 0$

EXAMPLE 2 Writing a Direct Variation Equation

The variables x and y vary directly. When $x = 5$, $y = 20$.

a. Write an equation that relates x and y.

b. Find the value of y when $x = 10$.

SOLUTION

a. Because x and y vary directly, the equation is of the form $y = kx$. You can solve for k as follows.

$y = kx$ — Write model for direct variation.

$20 = k(5)$ — Substitute 5 for x and 20 for y.

$4 = k$ — Divide each side by 5.

▶ An equation that relates x and y is $y = 4x$.

b. $y = 4(10)$ — Substitute 10 for x in $y = 4x$.

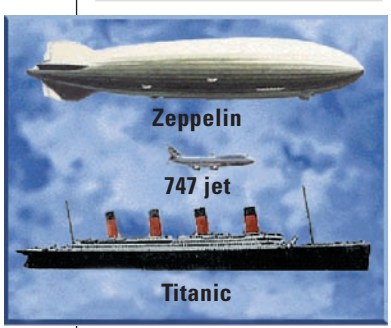
$y = 40$

▶ When $x = 10$, $y = 40$.

EXAMPLE 3 Writing a Direct Variation Model

ZEPPELIN In 1852 Henri Giffard built the first airship successfully used for transportation. It had a volume of 88,000 cubic feet and could support 5650 pounds. The *Graf Zeppelin II*, built in 1937, had a volume of 7,063,000 cubic feet, making it one of the two largest airships ever built. The weight an airship can support varies directly with its volume. How much weight could the *Graf Zeppelin II* support?

FOCUS ON APPLICATIONS



REAL LIFE **ZEPPELIN SIZE** The enormous *Graf Zeppelin II* was 804 ft long. The *Titanic* was similar in length (882.5 ft) but traveled at $\frac{1}{3}$ of the speed.

INTERNET **APPLICATION LINK** www.mcdougallittell.com

SOLUTION

Use the data about the first airship. Find a model that relates the volume V (in cubic feet) to the weight w (in pounds) that an airship can support.

$w = kV$ — Write model for direct variation.

$5650 = k(88{,}000)$ — Substitute 5650 for w and 88,000 for V.

$\frac{5650}{88{,}000} = k$ — Divide each side by 88,000.

$0.064 \approx k$ — Solve for k.

A direct variation model for the weight an airship can support is $w = 0.064V$.

Use the model to find the weight the *Graf Zeppelin II* could support.

$w = 0.064V$ — Write the model for direct variation.

$w = 0.064(7{,}063{,}000)$ — Substitute 7,063,000 for V.

$w = 452{,}032$ — Simplify.

▶ The *Graf Zeppelin II* could support about 452,000 pounds.

GOAL 2 USING A RATIO TO MODEL DIRECT VARIATION

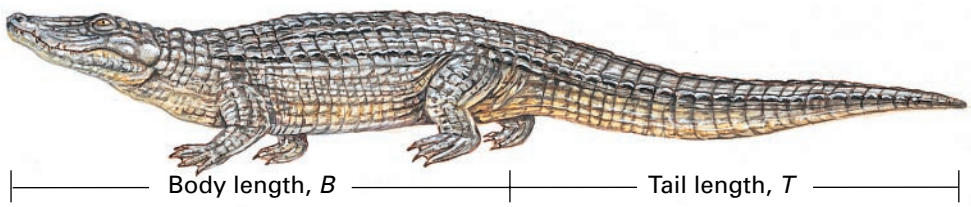
The model $y = kx$ for direct variation can be rewritten as follows.

$$k = \frac{y}{x}$$ ← Ratio form of direct variation model

The ratio form tells you that if x and y have *direct variation*, then the ratio of y to x is the same for all values of x and y. Sometimes real-life data can be approximated by a direct variation model, even though the data may not fit the model exactly.

EXAMPLE 4 *Using a Ratio Based on Data to Write a Model*

ANIMAL STUDIES The tail and body lengths (in feet) of 8 alligators are shown below. The ages range from 2 years to over 50 years. ▶ Source: St. Augustine Alligator Farm



Tail, T	1.41	2.04	2.77	2.77	3.99	4.67	4.69	5.68
Body, B	1.50	2.41	3.08	3.23	4.28	5.04	5.02	6.38

a. Write a model that relates the tail length T to the body length B.

b. Estimate the body length of an alligator whose tail length is 4.5 feet.

SOLUTION

a. Begin by finding the ratio of tail length to body length for each alligator.

Tail, T	1.41	2.04	2.77	2.77	3.99	4.67	4.69	5.68
Body, B	1.50	2.41	3.08	3.23	4.28	5.04	5.02	6.38
Ratio	0.94	0.85	0.90	0.86	0.93	0.93	0.93	0.89

The ratio is about 0.9 for each alligator, so use a direct variation model.

$k = \frac{T}{B}$ Write ratio model for direct variation.

$0.9 = \frac{T}{B}$ Substitute 0.9 for k.

$0.9B = T$ Multiply each side by B.

b. Use the model $0.9B = T$ to estimate the body length of the alligator.

$0.9B = 4.5$ Substitute 4.5 for T.

$B = 5$ Divide each side by 0.9.

▶ You estimate that the alligator's body is about 5 feet long.

FOCUS ON CAREERS

ZOOLOGISTS are biologists who study animals in natural or controlled surroundings.

CAREER LINK
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GUIDED PRACTICE

Vocabulary Check ✓

1. Explain what it means for x and y to vary directly.

Concept Check ✓

2. In a direct variation equation, how are the constant of variation and the slope related?

Skill Check ✓

Graph the equation. State whether the two quantities have direct variation. If they have direct variation, find the constant of variation and the slope of the direct variation model.

3. $y = x$ **4.** $y = 4x$ **5.** $y = \frac{1}{2}x$

6. $y = 2x$ **7.** $y = x - 4$ **8.** $y - 0.1x = 0$

SALARY In Exercises 9–11, you work a different number of hours each day. The table shows your total pay *p* and the number of hours *h* you worked.

Total pay, p	$18	$42	$48	$30
Hours worked, h	3	7	8	5
Ratio	?	?	?	?

9. Copy and complete the table by finding the ratio of your total pay each day to the number of hours you worked that day.

10. Write a model that relates the variables p and h.

11. If you work 6 hours on the fifth day, what will your total pay be?

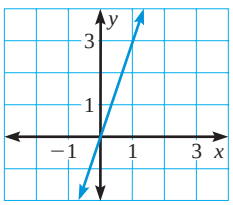
PRACTICE AND APPLICATIONS

STUDENT HELP

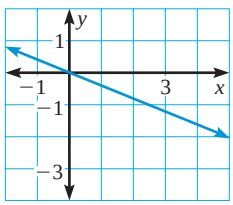
Extra Practice to help you master skills is on p. 800.

DIRECT VARIATION MODEL Find the constant of variation and the slope.

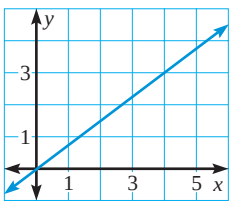
12. $y = 3x$



13. $y = -\frac{2}{5}x$



14. $y = 0.75x$



CONSTANT OF VARIATION Graph the equation. Find the constant of variation and the slope of the direct variation model.

15. $y = -3x$ **16.** $y = -5x$ **17.** $y = 0.4x$

18. $y = -x$ **19.** $y = \frac{5}{4}x$ **20.** $y = -\frac{1}{5}x$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 12–20
Example 2: Exs. 23–31
Example 3: Exs. 21–22, 32
Example 4: Exs. 36–37

RECOGNIZING DIRECT VARIATION In Exercises 21 and 22, state whether the two quantities have direct variation.

21. BICYCLING You ride your bike at an average speed of 14 miles per hour. The number of miles m you ride during h hours is modeled by $m = 14h$.

22. GEOMETRY CONNECTION The circumference C of a circle and its diameter d are related by the equation $C = \pi d$.

FOCUS ON PEOPLE

VALENTINA V. TERESHKOVA
In 1963 the Soviet astronaut Valentina Tereshkova was the first woman to travel in space.

FINDING EQUATIONS **In Exercises 23–31, the variables x and y vary directly. Use the given values to write an equation that relates x and y.**

23. $x = 4, y = 12$ **24.** $x = 7, y = 35$ **25.** $x = 18, y = 4$

26. $x = 22, y = 11$ **27.** $x = 5.5, y = 1.1$ **28.** $x = 16.5, y = 3.3$

29. $x = -1, y = -1$ **30.** $x = 7\frac{1}{5}, y = -9$ **31.** $x = -9, y = 3$

32. **SCIENCE CONNECTION** The volume V of blood pumped from your heart each minute varies directly with your pulse rate p. Each time your heart beats, it pumps approximately 0.06 liter of blood.

a. Find an equation that relates V and p.

b. **COLLECTING DATA** Take your pulse and find out how much blood your heart pumps per minute.

33. **ASTRONAUTS** Weight varies directly with gravity. With his equipment Buzz Aldrin weighed 360 pounds on Earth but only 60 pounds on the moon. If Valentina V. Tereshkova had landed on the moon with her equipment and weighed 54 pounds, how much would she have weighed on Earth with equipment?

SWIMMING POOLS **In Exercises 34 and 35, use the following information about several sizes of pools. The amount of chlorine needed in a pool varies with the amount of water in the pool.**

Amount of water (gallons)	6250	5000	12,500	15,000
Chlorine (pounds)	0.75	0.60	1.50	1.80

STUDENT HELP

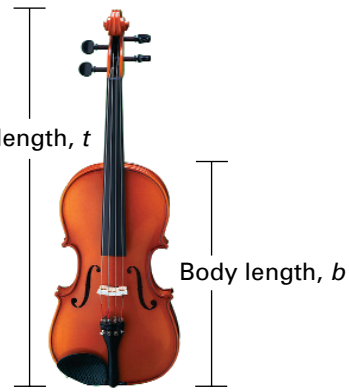
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with scatter plots in Exs. 34–35.

34. Use a graphing calculator or a computer to make a scatter plot of the data. Sketch the graph.

35. Does the scatter plot you made in Exercise 34 show direct variation?

VIOLIN FAMILY **In Exercises 36 and 37, use the following information.**
The violin family includes the bass, the cello, the viola, and the violin. The size of each instrument determines its tone. The shortest produces the highest tone, while the longest produces the deepest (lowest) tone.

Violin family	Bass	Cello	Viola	Violin
Total length t (inches)	72	47	26	23
Body length b (inches)	44	30	16	14



36. Write a direct variation model that you can use to relate the body length of a member of the violin family to its total length.

37. Another instrument in the violin family has a body length of 28 inches. Use your model from Exercise 36 to estimate the total length of the instrument. Is its tone higher or lower than a viola's tone?

Test Preparation

38. MULTIPLE CHOICE The variables x and y vary directly. When $x = 4$, $y = 24$. Which equation correctly relates x and y?

(A) $x = 4y$ (B) $y = 4x$ (C) $x = 6y$ (D) $y = 6x$

39. MULTIPLE CHOICE Which equation models the ratio form of direct variation?

(A) $\frac{-4}{3} = \frac{y}{x}$ (B) $-3y = 4x - 1$ (C) $\frac{4}{-3} = x + y$ (D) $y = -\frac{4}{3}x + 6$

40. MULTIPLE CHOICE Find the constant of variation of the direct variation model $3x = y$.

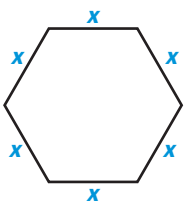
(A) 3 (B) $\frac{1}{3}$ (C) y (D) x

41. MULTIPLE CHOICE You buy some oranges for \$.80 per pound. Which direct variation model relates the total cost x to the number of pounds of oranges y?

(A) $y = 8x$ (B) $x = 0.8y$ (C) $y = 0.8x$ (D) $x = 8y$

Challenge

42. GEOMETRY CONNECTION Write an equation for the perimeter of the regular hexagon at the right. Does your equation model direct variation?



EXTRA CHALLENGE
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43. LOGICAL REASONING If a varies directly with b, then does b vary directly with a? Explain your reasoning.

MIXED REVIEW

SOLVING EQUATIONS **Solve the equation.** (Review 3.4)

44. $7z + 30 = -5$ **45.** $4b = 26 - 9b$ **46.** $2(w - 2) = 2$

47. $9x + 65 = -4x$ **48.** $55 - 5y = 9y + 27$ **49.** $7c - 3 = 4(c - 3)$

FUNCTIONS **In Exercises 50 and 51, rewrite the equation so that x is a function of y.** (Review 3.7)

50. $15 = 7(x - y) + 3x$ **51.** $3x + 12 = 5(x + y)$

52. HOURLY WAGE You get paid \$152.25 for working 21 hours. Find your hourly rate of pay. (Review 3.8)

53. ORDERED PAIRS Plot and label the points $R(2, 4)$, $S(0, -1)$, $T(3, 6)$, and $U(-1, -2)$ in a coordinate plane. (Review 4.1)

CHECKING SOLUTIONS **Decide whether the given point lies on the line. Justify your answer both algebraically and graphically.** (Review 4.2)

54. $x - y = 10$; $(5, -5)$ **55.** $3x - 6y = -2$; $(-4, -2)$

56. $5x + 6y = -1$; $(1, -1)$ **57.** $-4x - 3y = -8$; $(-4, 2)$

FINDING INTERCEPTS **Find the x-intercept and the y-intercept of the graph of the equation.** (Review 4.3 for 4.6)

58. $2x + y = 6$ **59.** $x - 1.1y = 10$ **60.** $x - 6y = 4$

61. $x + 5y = 10$ **62.** $y = x - 5$ **63.** $y = \frac{1}{3}x - \frac{7}{3}$