4.3

Quick Graphs Using Intercepts

What you should learn

GOAL 1 Find the intercepts of the graph of a linear equation.

GOAL 2 Use intercepts to make a quick graph of a linear equation as in **Example 4**.

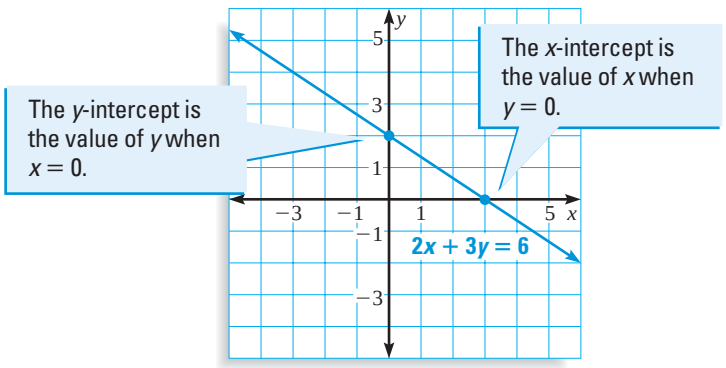
Why you should learn it

▼ To solve **real-life** problems, such as finding numbers of school play tickets to be sold to reach a fundraising goal in **Ex. 63**.

GOAL 1 FINDING THE INTERCEPTS OF A LINE

In Lesson 4.2 you graphed a linear equation by writing a table of values, plotting the points, and drawing a line through the points.

In this lesson, you will learn a quicker way to graph a linear equation. To do this, you need to realize that only two points are needed to determine a line. Two points that are usually convenient to use are points where a graph crosses the axes.



An ***x*-intercept** is the x-coordinate of a point where a graph crosses the x-axis. A ***y*-intercept** is the y-coordinate of a point where a graph crosses the y-axis.

EXAMPLE 1 *Finding Intercepts*

Find the x-intercept and the y-intercept of the graph of the equation $2x + 3y = 6$.

SOLUTION

To find the x-intercept of $2x + 3y = 6$, let $y = 0$.

$2x + 3y = 6$ **Write original equation.**

$2x + 3(0) = 6$ **Substitute 0 for *y*.**

$x = 3$ **Solve for *x*.**

▶ The x-intercept is 3. The line crosses the x-axis at the point $(3, 0)$.

To find the y-intercept of $2x + 3y = 6$, let $x = 0$.

$2x + 3y = 6$ **Write original equation.**

$2(0) + 3y = 6$ **Substitute 0 for *x*.**

$y = 2$ **Solve for *y*.**

▶ The y-intercept is 2. The line crosses the y-axis at the point $(0, 2)$.

GOAL 2 USING INTERCEPTS TO GRAPH EQUATIONS

EXAMPLE 2 *Making a Quick Graph*

STUDENT HELP

INTERNET HOMEWORK HELP
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for extra examples.

Graph the equation $3.5x + 7y = 14$.

SOLUTION

Find the intercepts.

$3.5x + 7y = 14$ — Write original equation.

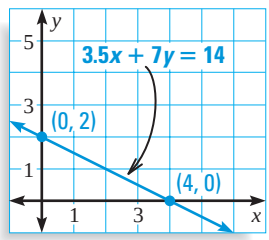
$3.5x + 7(0) = 14$ — Substitute 0 for y.

$x = \frac{14}{3.5} = 4$ — The x-intercept is 4.

$3.5x + 7y = 14$ — Write original equation.

$3.5(0) + 7y = 14$ — Substitute 0 for x.

$y = \frac{14}{7} = 2$ — The y-intercept is 2.



Draw a coordinate plane that includes the points (4, 0) and (0, 2).

Plot the points (4, 0) and (0, 2) and draw a line through them.

EXAMPLE 3 *Drawing Appropriate Scales*

Graph the equation $y = 4x + 40$.

SOLUTION

Find the intercepts, by substituting 0 for y and then 0 for x.

$y = 4x + 40$ | $y = 4x + 40$

$0 = 4x + 40$ | $y = 4(0) + 40$

$-40 = 4x$ | $y = 40$

$-10 = x$ | The y-intercept is 40.

The x-intercept is -10.

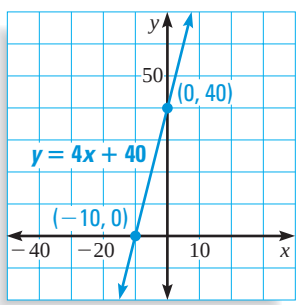
STUDENT HELP

Study Tip
When you make a quick graph, find the intercepts *before* you draw the coordinate plane. This will help you find an appropriate scale on each axis.

Draw a coordinate plane that includes the points $(-10, 0)$ and $(0, 40)$. With these values, it is reasonable to use tick marks at 10-unit intervals.

You may want to draw axes with at least two tick marks to the left of -10 and to the right of 0 on the x-axis and two tick marks below 0 and above 40 on the y-axis.

Plot the points $(-10, 0)$ and $(0, 40)$ and draw a line through them.



EXAMPLE 4 *Writing and Using a Linear Model*

ZOO FUNDRAISING You are organizing the annual spaghetti dinner to raise funds for a zoo. Your goal is to sell $1500 worth of tickets. Assuming 200 adults and 100 students will attend the dinner, how much should you charge for an adult ticket and for a student ticket?

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Number of adults}} \cdot \boxed{\text{Adult ticket price}} + \boxed{\text{Number of students}} \cdot \boxed{\text{Student ticket price}} = \boxed{\text{Total sales}}$$

LABELS

Number of adults = **200** (people)

Adult ticket price = x (dollars per person)

Number of students = **100** (people)

Student ticket price = y (dollars per person)

Total sales = **1500** (dollars)

ALGEBRAIC MODEL

$$200x + 100y = 1500 \quad \text{Write linear model.}$$

$$2x + y = 15 \quad \text{Divide by 100 to simplify model.}$$

FOCUS ON APPLICATIONS

REAL LIFE ZOO EXPENSES The American Zoo and Aquarium Institute estimates that it costs a zoo about $22,000 per year to house, feed, and care for a lion and about $18,500 for a polar bear.

INTERNET APPLICATION LINK
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This equation has many solutions. To get a better idea of the possible prices, make a quick graph.

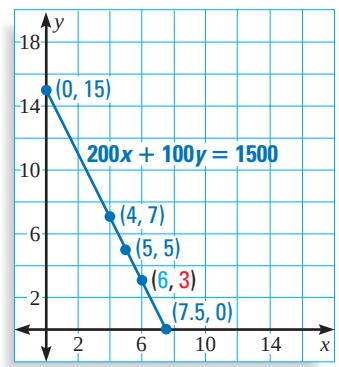
Find the intercepts.

$2x + y = 15$ | $2x + 0 = 15$ | $x = 7.5$ ← *x*-intercept

$2x + y = 15$ | $2(0) + y = 15$ | $y = 15$ ← *y*-intercept

Draw a coordinate plane that includes the points (7.5, 0) and (0, 15).

Plot the points (7.5, 0) and (0, 15) and draw a line through them. From the graph, you can determine several possible prices to charge.



Possible Prices to Raise $1500	
Adult	Student
$0.00	$15.00
$4.00	$7.00
$5.00	$5.00
$6.00	**$3.00**
$7.50	$0.00

▶ One reasonable price to charge is $6 for adults and $3 for students.

GUIDED PRACTICE

Vocabulary Check ✓

1. Decide whether 2 is the *x-intercept* or the *y-intercept* of the line $y = 2x + 2$. Explain your choice.

Concept Check ✓

2. How many points are needed to determine a line?

3. Describe a line that has no x-intercept.

Skill Check ✓

Find the *x*-intercept of the graph of the equation.

4. $y = 2x + 20$ **5.** $y = 0.1x + 0.3$ **6.** $y = x - \frac{1}{4}$

In Exercises 7–12, find the *x*-intercept and the *y*-intercept of the graph of the equation. Graph the equation.

7. $y = x + 2$ **8.** $y - 2x = 3$ **9.** $2x - y = 4$

10. $3y = -6x + 3$ **11.** $5y = 5x + 15$ **12.** $x - y = 1$

13. **FUNDRAISING** If your goal for the fundraising dinner in Example 4 is \$2000, find reasonable prices for adult tickets and student tickets.

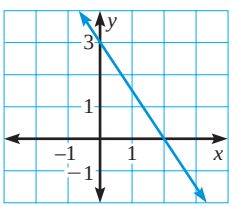
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 800.

USING GRAPHS TO FIND INTERCEPTS **Use the graph to find the *x*-intercept and the *y*-intercept of the line.**

14.



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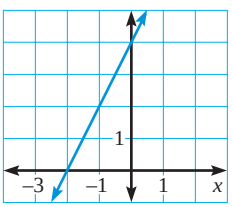

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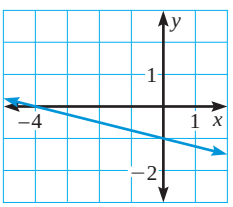


FINDING *X*-INTERCEPTS **Find the *x*-intercept of the graph of the equation.**

17. $x + 3y = 5$ **18.** $x - 2y = 6$ **19.** $2x + 2y = -10$

20. $3x + 4y = 12$ **21.** $5x - y = 45$ **22.** $-x + 3y = 27$

23. $-7x - 3y = 42$ **24.** $2x + 6y = -24$ **25.** $-12x - 20y = 60$

FINDING *Y*-INTERCEPTS **Find the *y*-intercept of the graph of the equation.**

26. $y = -2x + 5$ **27.** $y = 3x - 4$ **28.** $y = 8x + 27$

29. $y = 7x - 15$ **30.** $4x - 5y = -35$ **31.** $6x - 9y = 72$

32. $3x + 12y = -84$ **33.** $-x + 1.7y = 5.1$ **34.** $2x - 6y = -18$

USING INTERCEPTS **Graph the line that has the given intercepts.**

35. x-intercept: -2
y-intercept: 5

36. x-intercept: 4
y-intercept: 6

37. x-intercept: -7
y-intercept: -3

38. x-intercept: -3
y-intercept: -7

39. x-intercept: -12
y-intercept: -8

40. x-intercept: -7
y-intercept: 15

STUDENT HELP

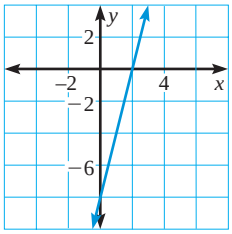
HOMEWORK HELP

Example 1: Exs. 17–34
Example 2: Exs. 35–55
Example 3: Exs. 44–55
Example 4: Exs. 60–63

MATCHING GRAPHS AND EQUATIONS **Match the equation with its graph.**

A. $8x - 2y = 16$ **B.** $8x - 2y = 8$ **C.** $8x - 2y = -8$

41.



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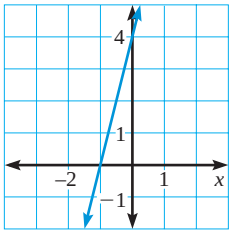

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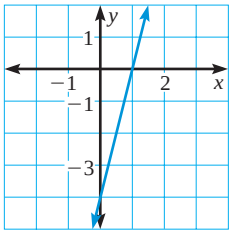


GRAPHING LINES **Find the *x*-intercept and the *y*-intercept of the line. Graph the equation. Label the points where the line crosses the axes.**

44. $y = x + 2$ **45.** $y = x - 3$ **46.** $y = 4x + 8$

47. $y = -6 + 3x$ **48.** $y = 5x + 15$ **49.** $2x + 4y = 16$

50. $-4x + 3y = 24$ **51.** $x - 7y = 21$ **52.** $6x - y = 36$

53. $2x + 9y = -36$ **54.** $4x + 5y = 20$ **55.** $0.5y = -2x + 8$

LOGICAL REASONING **In Exercises 56–59, tell whether the statement is *true* or *false*. Justify your answer.**

56. The y-intercept of the graph of $3x + 5y = 30$ is 10.

57. The x-intercept of the graph of $3x + 5y = 30$ is 10.

58. The point $(3, 5)$ is on the graph of $3x + 5y = 30$.

59. The graph of the equation $x = 4$ is a horizontal line.

FOCUS ON APPLICATIONS

REAL LIFE MARATHON
In 1896 the Olympic Games introduced the "marathon" race to honor a Greek soldier. The legend says a soldier ran 26.2 miles from Marathon to Athens in 3 hours to announce the victory over the Persians.

SCHOOL PLAY **In Exercises 60–63, use the following information.**
Your school drama club is putting on a play next month. By selling tickets for the play, the club hopes to raise \$600 for the drama fund for new costumes, scripts, and scenery for future plays. Let x represent the number of adult tickets they sell at \$8 each, and let y represent the number of student tickets they sell at \$5 each.

60. Graph the linear function $8x + 5y = 600$.

61. What is the x-intercept? What does it represent in this situation?

62. What is the y-intercept? What does it represent in this situation?

63. What are three possible numbers of adult and student tickets to sell that will make the drama club reach its goal?

MARATHON **In Exercises 64–66, you are running in a marathon. You either run 8 miles per hour or walk 4 miles per hour.**

64. Write an equation to show the relationship between time run and time walked during the 26.2-mile course.

65. Graph the equation from Exercise 64. What are some possible running and walking times if you complete the 26.2-mile course?

66. If you walk for a total of 1 hour during the course, how long will you have spent running when you cross the finish line of the marathon?

MOVIE PRICES **In Exercises 67 and 68, a theater charges $4 per person before 6:00 P.M. and $7 per person after 6:00 P.M. The total ticket sales for Saturday were $11,228.**

67. Make a graph showing the possible number of people who attended the theater before and after 6:00 P.M.

68. Suppose no one attended the theater before 6:00 P.M. How many people attended the theater after 6:00 P.M.? Explain how you know.

Test Preparation

69. **MULTI-STEP PROBLEM** The number of people who worked for the railroads in the United States each year from 1989 to 1995 can be modeled by the equation $y = -6.61x + 229$, where x represents the number of years since 1989 and y represents the number of railroad employees (in thousands).

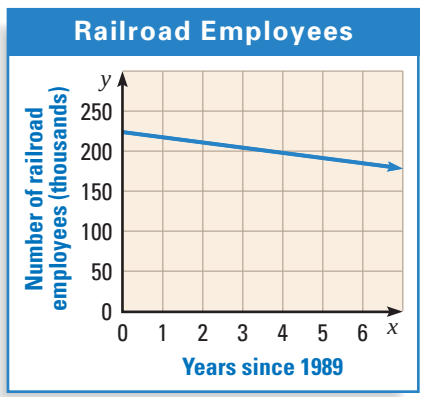
INTERNET **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

a. Find the y-intercept of the line. What does it represent?

b. Find the x-intercept of the line. What does it represent?

c. About how many people worked for the railroads in 1995?

d. *Writing* Do you think the line in the graph will continue to be a good model for the next 50 years? Explain.



Challenge

70. **CRITICAL THINKING** Consider the equation $6x + 8y = k$. What numbers could replace k so that the x-intercept and the y-intercept are both integers? Explain.

MIXED REVIEW

EVALUATING DIFFERENCES **Find the difference.** (Review 2.3 for 4.4)

71. $5 - 9$ **72.** $17 - 6$ **73.** $-8 - 9$ **74.** $|8| - 12.6$

75. $-\frac{2}{3} - \left(-\frac{7}{3}\right)$ **76.** $13.8 - 6.9$ **77.** $7 - |-1|$ **78.** $-4.1 - (-5.1)$

EVALUATING QUOTIENTS **In Exercises 79–86, find the quotient.** (Review 2.7 for 4.4)

79. $54 \div 9$ **80.** $-72 \div 8$ **81.** $12 \div \left(-\frac{1}{5}\right)$ **82.** $3 \div \frac{1}{4}$

83. $26 \div (-13)$ **84.** $-1 \div 8$ **85.** $-20 \div \left(-2\frac{1}{2}\right)$ **86.** $\frac{1}{8} \div \frac{1}{2}$

87. **SCHOOL BAKE SALE** You have one hour to make cookies for your school bake sale. You spend 20 minutes mixing the dough. It then takes 12 minutes to bake each tray of cookies. If you bake one tray at a time, which model can you use to find how many trays you can bake? (Review 3.3 and 3.6)

A. $x(20 + 12) = 60$ **B.** $12x + 20 = 60$

QUIZ 1

Self-Test for Lessons 4.1–4.3

Plot and label the ordered pairs in a coordinate plane. (Lesson 4.1)

1. $A(-4, 1), B(0, 2), C(-3, 0)$

2. $A(-1, -5), B(0, -7), C(1, 6)$

3. $A(-4, -6), B(1, -3), C(-1, 1)$

4. $A(2, -6), B(5, 0), C(0, -4)$

Find three different ordered pairs that are solutions of the equation. Graph the equation. (Lesson 4.2)

5. $y = 2x - 6$

6. $y = 4x - 1$

7. $y = 2(-3x + 1)$

8. $x = 3$

9. $y = -3(x - 4)$

10. $y = -5$

Find the *x*-intercept and the *y*-intercept of the line. Graph the line. Label the points where the line crosses the axes. (Lesson 4.3)

11. $y = 4 - x$

12. $y = -5 + 2x$

13. $y = 3x + 12$

14. $3x + 3y = 27$

15. $-6x + y = -3$

16. $y = 10x + 50$

MATH & History

Graphing Mathematical Relationships

INTERNET APPLICATION LINK
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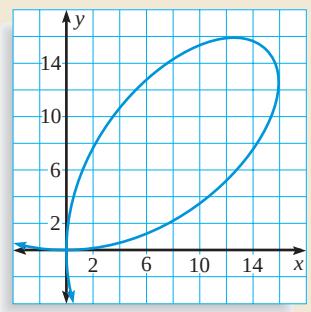
THEN

400 YEARS AGO, this chapter would not have been in your algebra book. In 1637 René Descartes published a method for drawing algebraic equations on a coordinate grid—introducing a new connection between algebra and geometry that linked the two branches more closely than ever before.

Descartes was interested in studying curves that are more complicated than the lines you have seen in this chapter.



Descartes was interested in equations like $x^3 + y^3 = 3axy$ when $a = 10$, which is graphed at the right. Use algebra to show whether the point is on the graph of the equation.

1. $(15, 15)$ **2.** $(0, 0)$ **3.** $(-1, -1)$ **4.** $(3, 0.3)$

NOW

TODAY, using a coordinate plane to graph an equation is one of the first things you learn in Algebra 1. Computers that can process large amounts of data have enabled mathematicians to study mathematical systems whose outputs are too complicated to graph by hand.

1596–1650

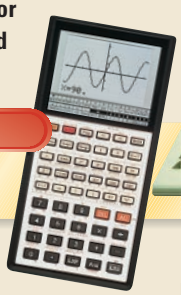
René Descartes

1637

Publication of Descartes's text: *La Géométrie*

The first graphing calculator produced

1985



Now

Computers used to graph mathematical equations