3.7 Formulas and Functions

What *you should learn*

GOAL 1 Solve a formula for one of its variables.

GOAL 2 Rewrite an equation in function form.

Why *you should learn it*

▼ To solve **real-life** problems about rate, time, and distance as in *Pathfinder's* flight to Mars in **Example 4**.

REAL LIFE

GOAL 1 SOLVING FORMULAS

In this lesson you will work with equations that have more than one variable. A **formula** is an algebraic equation that relates two or more real-life quantities.

EXAMPLE 1 *Solving and Using an Area Formula*

Use the formula for the area of a rectangle, $A = lw$.

a. Find a formula for l in terms of A and w.

b. Use the new formula to find the length of a rectangle that has an area of 35 square feet and a width of 7 feet.

SOLUTION

a. Solve for length l.

$A = lw$ **Write original formula.**

$\frac{A}{w} = l$ **To isolate *l*, divide each side by *w*.**

b. Substitute the given values into the new formula.

$$l = \frac{A}{w} = \frac{35}{7} = 5$$

▶ The length of the rectangle is 5 feet.

EXAMPLE 2 *Solving a Temperature Conversion Formula*

Solve the temperature formula $C = \frac{5}{9}(F - 32)$ for F.

SOLUTION

$C = \frac{5}{9}(F - 32)$ **Write original formula.**

$\frac{9}{5} \cdot C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$ **Multiply each side by $\frac{9}{5}$.**

$\frac{9}{5}C = F - 32$ **Simplify.**

$\frac{9}{5}C + 32 = F - 32 + 32$ **Add 32 to each side.**

$\frac{9}{5}C + 32 = F$ **Simplify.**

EXAMPLE 3 *Solving and Using an Interest Formula*

a. The simple interest I on an investment of P dollars at an interest rate r for t years is given by $I = Prt$. Solve this formula for r.

b. Find the interest rate r for an investment of \$1500 that earned \$54 in simple interest in one year.

SOLUTION

a. $I = Prt$ — Write original formula.

$\frac{I}{Pt} = \frac{Prt}{Pt}$ — Divide each side by Pt.

$\frac{I}{Pt} = r$ — Simplify.

b. $r = \frac{I}{Pt} = \frac{54}{1500 \cdot 1} = \frac{54}{1500} = 0.036$, or 3.6%

EXAMPLE 4 *Solving and Using a Distance Formula*

PATHFINDER The *Pathfinder* was launched on December 4, 1996. During its 212-day flight to Mars, it traveled about 310 million miles. Estimate the time the *Pathfinder* would have taken to reach Mars traveling at the following speeds.

a. 30,000 miles per hour

b. 40,000 miles per hour

c. 60,000 miles per hour

d. 80,000 miles per hour

Which of the four speeds is the best estimate of *Pathfinder's* average speed?

FOCUS ON APPLICATIONS

REAL LIFE **PATHFINDER** Since its landing on July 4, 1997, the Mars Pathfinder has returned 2.6 billion bits of information, including more than 16,550 images. ▶ Source: NASA

SOLUTION

Use the formula $d = rt$ and solve for the time t.

$d = rt$ — Write original formula.

$\frac{d}{r} = \frac{rt}{r}$ — Divide each side by r.

$\frac{d}{r} = t$ — Simplify.

a. $t = \frac{d}{r} = \frac{310{,}000{,}000}{30{,}000} = 10{,}333 \text{ hours} \approx 431 \text{ days}$

b. $t = \frac{d}{r} = \frac{310{,}000{,}000}{40{,}000} = 7{,}750 \text{ hours} \approx 323 \text{ days}$

c. $t = \frac{d}{r} = \frac{310{,}000{,}000}{60{,}000} \approx 5{,}167 \text{ hours} \approx 215 \text{ days}$

d. $t = \frac{d}{r} = \frac{310{,}000{,}000}{80{,}000} = 3{,}875 \text{ hours} \approx 161 \text{ days}$

▶ The trip took 212 days, so *Pathfinder*'s average speed was about 60,000 mi/h.

GOAL 2 REWRITING EQUATIONS IN FUNCTION FORM

A two-variable equation is written in *function form* if one of its variables is isolated on one side of the equation. The isolated variable is the output and is a function of the input. For instance, the equation $P = 4s$ describes the perimeter P of a square as a function of its side length s.

EXAMPLE 5 *Rewriting an Equation in Function Form*

STUDENT HELP

Look Back For help with functions, see page 46.

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

Rewrite the equation $3x + y = 4$ so that y is a function of x.

SOLUTION

$3x + y = 4$	Write original equation.
$3x + y - 3x = 4 - 3x$	Subtract $3x$ from each side.
$y = 4 - 3x$	Simplify.

▶ The equation $y = 4 - 3x$ represents y as a function of x.

EXAMPLE 6 *Rewriting an Equation in Function Form*

a. Rewrite the equation $3x + y = 4$ so that x is a function of y.

b. Use the result to find x when $y = -2, -1, 0$, and 1.

SOLUTION

a.

$3x + y = 4$	Write original equation.
$3x + y - y = 4 - y$	Subtract y from each side.
$3x = 4 - y$	Simplify.
$\frac{3x}{3} = \frac{4 - y}{3}$	Divide each side by 3.
$x = \frac{4 - y}{3}$	Simplify.

▶ The equation $x = \frac{4 - y}{3}$ represents x as a function of y.

b. It is helpful to organize your work in columns.

INPUT	SUBSTITUTE	OUTPUT
$y = -2$	$x = \frac{4 - (-2)}{3}$	$x = 2$
$y = -1$	$x = \frac{4 - (-1)}{3}$	$x = \frac{5}{3}$
$y = 0$	$x = \frac{4 - (0)}{3}$	$x = \frac{4}{3}$
$y = 1$	$x = \frac{4 - (1)}{3}$	$x = 1$

GUIDED PRACTICE

Vocabulary Check ✓

1. Give an example of a formula. State what real-life quantity each variable represents.

Concept Check ✓

Tell whether the formula shows correctly the relationships among perimeter, length, and width of a rectangle.

2. $P = 2l + 2w$ **3.** $P = l + w \cdot 2$ **4.** $P = 2(l + w)$

5. $l = \frac{P - 2w}{2}$ **6.** $l = \frac{P}{2w} - 2$ **7.** $w = \frac{P}{2} - 2l$

8. Write a formula that describes the side length s of a square as a function of its perimeter P.

Skill Check ✓

9. Rewrite the equation $2x + 2y = 10$ so that y is a function of x.

10. Use the result of Exercise 9 to find y when $x = -2, -1, 0, 1,$ and 2.

PRACTICE AND APPLICATIONS

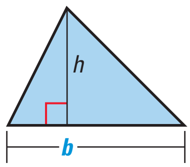
STUDENT HELP

Extra Practice to help you master skills is on p. 799.

SOLVING A FORMULA **Solve for the indicated variable.**

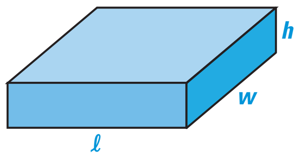
11. Area of a Triangle

Solve for b: $A = \frac{1}{2}bh$



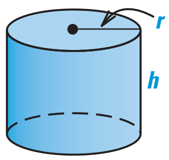
12. Volume of a Rectangular Prism

Solve for h: $V = \ell wh$



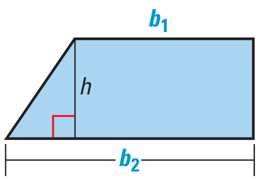
13. Volume of a Circular Cylinder

Solve for h: $V = \pi r^2 h$



14. Area of a Trapezoid

Solve for b_2: $A = \frac{1}{2}h(b_1 + b_2)$



REWRITING EQUATIONS **Rewrite the equation so that y is a function of x.**

15. $2x + y = 5$ **16.** $3x + 5y = 7$ **17.** $13 = 12x - 2y$

18. $2x = -3y + 10$ **19.** $9 - y = 1.5x$ **20.** $1 + 7y = 5x - 2$

21. $\frac{y}{5} - 7 = -2x$ **22.** $\frac{1}{4}y + 3 = -5x$ **23.** $-3x + 4y - 5 = -14$

24. $7x - 4x + 12 = 36 - 5y$ **25.** $\frac{1}{3}(y + 2) + 3x = 7x$

26. $5(y - 3x) = 8 - 2x$ **27.** $4x - 3(y - 2) = 15 + y$

28. $3(x - 2y) = -12(x + 2y)$ **29.** $\frac{1}{5}(25 - 5y) = 4x - 9y + 13$

STUDENT HELP

HOMEWORK HELP
Examples 1–4: Exs. 11–14, 34–38
Example 5: Exs. 15–29
Example 6: Exs. 30–33

SUBSTITUTION **Rewrite the equation so that x is a function of y. Then use the result to find x when $y = -2, -1, 0$, and 1.**

30. $2x + y = 5$

31. $3y - x = 12$

32. $4(5 - y) = 14x + 3$

33. $5y - 2(x - 7) = 20$

DISCOUNTS **In Exercises 34 and 35, use the relationship between the sale price S, the list price L, and the discount rate r.**

34. Solve for r in the formula $S = L - rL$.

35. Use the new formula to find the discount rate as a decimal and as a percent.

a. Sale price: \$80
List price: \$100

b. Sale price: \$72
List price: \$120

c. Sale price: \$12.50
List price: \$25

STUDENT HELP

Skills Review
For help with writing decimals as percents, see pages 784–785.

36. *Writing* You can solve the formula $d = rt$ for any of its variables. This means that you need to learn only one formula to solve a problem asking for distance, rate, or time. Explain.

SCUBA DIVING **In Exercises 37–39, use the following information.**
If a scuba diver starts at sea level, the pressure on the diver at a depth of d feet is given by the formula $P = 64d + 2112$, where P represents the total pressure in pounds per square foot. Suppose the current pressure on a diver is 4032 pounds per square foot.

37. Solve the formula for d.

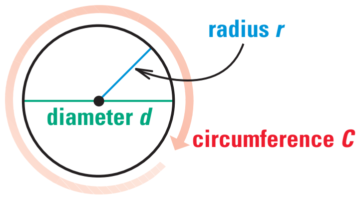
38. What is the diver's current depth?

39. In the original equation, is P a function of d or is d a function of P? Explain.

STUDENT HELP

INTERNET
KEYSTROKE HELP
Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

CIRCLE RELATIONSHIPS **In Exercises 40–44, use the information at the right. Use the *Table* feature on a graphing calculator (or spreadsheet software) as indicated.**



$C = 2\pi r$

40. In the formula, is the circumference a function of the radius or is the radius a function of the circumference?

41. If X is the radius, what expression gives the circumference? On your graphing calculator enter this expression as Y_1.

42. Solve the circumference formula for r. If Y_1 is the circumference, what expression involving Y_1 equals the radius? On your graphing calculator enter this expression as Y_2. To enter Y_1 into your expression, use the *Variables* menu.

43. **LOGICAL REASONING** Enter 10 different values for the radius of a circle in the X-column. (You may need to choose "Ask" on the Table Setup menu to do this.) View your table and then explain the relationship between the three columns.

44. **GIANT SEQUOIA** The *General Sherman* tree, a sequoia tree in Sequoia National Park, has the greatest volume of any tree in the world. The distance around the tree (the circumference) measures 102.6 feet. Show how to find the distance straight through the center of the tree (the diameter) without measuring it directly. Then find this value to the nearest tenth of a foot.

Test Preparation

45. MULTIPLE CHOICE Which of the choices shows how to rewrite the equation $2y - 3(y - 2x) = 8(x - 1) + 3$ so that y is a function of x?

Ⓐ $x = \frac{5 - y}{2}$ Ⓑ $x = \frac{-y + 5}{10}$

Ⓒ $y = -14x + 5$ Ⓓ $y = -2x + 5$

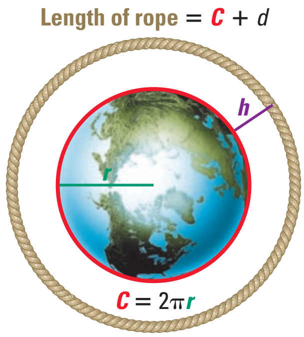
46. MULTIPLE CHOICE The formula $A = \frac{1}{2}h(b_1 + b_2)$ relates the area and the dimensions of a trapezoid. Which way can you rewrite this relationship?

Ⓐ $A = \frac{1}{2}hb_1 + b_2$ Ⓑ $h = \frac{2A}{b_1 + b_2}$

Ⓒ $h = \frac{A}{b_1 + b_2}$ Ⓓ $b_1 = 2A - b_2$

★ Challenge

ROPE AROUND THE WORLD In Exercises 47–49, imagine a rope wrapped around Earth at the equator (Earth's circumference *C*). Then think of adding *d* feet to the rope's length so it can now circle Earth at a distance *h* feet above the equator at all points.



47. Write an equation to model this situation.

48. Solve your equation in Exercise 47 for d.

49. How much rope do you need to insert if you want the rope to circle Earth 500 feet above the equator? Use 3.14 as an approximation for π.

EXTRA CHALLENGE
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MIXED REVIEW

CHECKING SOLUTIONS Check whether the given number is a solution of the inequality. (Review 1.4)

50. $y - 5 > 4; 9$ **51.** $4 + 7y > 12; 1$ **52.** $3x - 3 \geq x + 3; 5$

SIMPLIFYING EXPRESSIONS Simplify the variable expression. (Review 2.5)

53. $-(2)^3(b)$ **54.** $(-2)^3(b)$ **55.** $3(-x)(-x)(-x)$

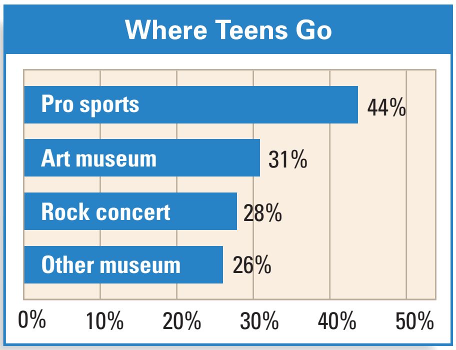
56. $(-4)(-c)(-c)(-c)$ **57.** $(-4)^2(t)(t)$ **58.** $(-5)^2(-y)(-y)$

TEEN ATTENDANCE Each bar shows the percent of teens that attend the activity in a 12-month period. (Review 1.6 for 3.8)

59. Which activity shown is attended by the most teens?

60. What percent attend rock concerts?

61. How many out of 100 attend art museums?



Source: *YOUTHviews*