3.5 Linear Equations and Problem Solving

What you should learn

GOAL 1 Draw a diagram to help you understand **real-life** problems, such as planning where bicyclists should meet in **Exercise 22**.

GOAL 2 Use tables and graphs to check your answers.

Why you should learn it

▼ To solve **real-life** problems, such as designing a high school yearbook in **Example 1**.

GOAL 1 DRAWING A DIAGRAM

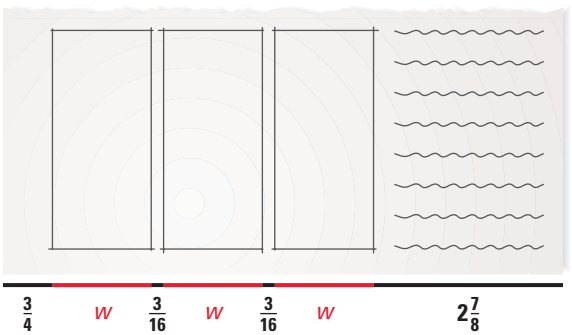
You have learned to use a verbal model to solve real-life problems. Verbal models can be used with other problem solving strategies, such as drawing a diagram.

EXAMPLE 1 *Visualizing a Problem*

YEARBOOK DESIGN A page of your school yearbook is $8\frac{1}{2}$ inches by 11 inches. The left margin is $\frac{3}{4}$ inch and the space to the right of the pictures is $2\frac{7}{8}$ inches. The space between pictures is $\frac{3}{16}$ inch. How wide can each picture be to fit three across the width of the page?

SOLUTION

DRAW A DIAGRAM The diagram shows that the page width is made up of the width of the left margin, the space to the right of the pictures, two spaces between pictures, and three picture widths.



VERBAL MODEL

$$\boxed{\text{Left margin}} + \boxed{\text{Space to right of pictures}} + 2 \cdot \boxed{\text{Space between pictures}} + 3 \cdot \boxed{\text{Picture width}} = \boxed{\text{Page width}}$$

LABELS

Left margin $= \frac{3}{4}$ (inches)

Space to right of pictures $= 2\frac{7}{8}$ (inches)

Space between pictures $= \frac{3}{16}$ (inches)

Picture width $= w$ (inches)

Page width $= 8\frac{1}{2}$ (inches)

ALGEBRAIC MODEL

$$\frac{3}{4} + 2\frac{7}{8} + 2 \cdot \frac{3}{16} + 3 \cdot w = 8\frac{1}{2}$$

▶ When you solve for w, you will find that each picture can be $1\frac{1}{2}$ inches wide.

GOAL 2 USING TABLES AND GRAPHS AS CHECKS

REAL LIFE Language Study

EXAMPLE 2 *Using a Table as a Check*

At East High School, 579 students take Spanish. This number has been increasing at a rate of about 30 students per year. The number of students taking French is 217 and has been decreasing at a rate of about 2 students per year. At these rates, when will there be three times as many students taking Spanish as taking French?

SOLUTION

First write an expression for the number of students taking each language after some number of years. Then set the expression for students taking Spanish equal to three times the expression for students taking French. Subtraction is used in the expression for students taking French, because there is a decrease.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Number taking Spanish now}} + \boxed{\text{Rate of increase for Spanish}} \cdot \boxed{\text{Number of years}} = 3 \cdot \left(\boxed{\text{Number taking French now}} - \boxed{\text{Rate of decrease for French}} \cdot \boxed{\text{Number of years}} \right)$$

LABELS

Number of students taking Spanish now = **579** (students)

Rate of *increase* for Spanish = **30** (students per year)

Number of years = x (years)

Number of students taking French now = **217** (students)

Rate of *decrease* for French = **2** (students per year)

ALGEBRAIC MODEL

$579 + 30 \cdot x = 3(217 - 2 \cdot x)$ Write algebraic model.

$579 + 30x = 651 - 6x$ Use distributive property.

$579 + 36x = 651$ Add $6x$ to each side.

$36x = 72$ Subtract 579 from each side.

$x = 2$ Divide each side by 36.

▶ At these rates, the number of students taking Spanish will be three times the number of students taking French in two years.

✓**CHECK** Use the information in the original problem statement to make a table.

For example, the number of students taking Spanish in Year 1 is 579 + 30 = 609.

Year	0 (now)	1	2
Number taking Spanish	579	609	639
Number taking French	217	215	213
3 · Number taking French	651	645	639

equal

STUDENT HELP

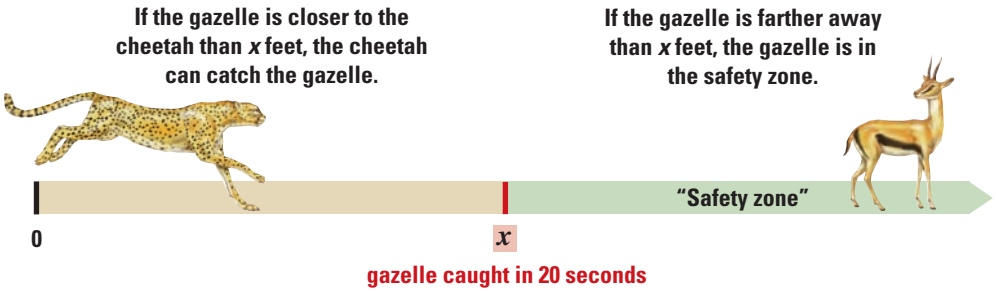
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 *Using a Graph as a Check*

GAZELLE AND CHEETAH A gazelle can run 73 feet per second for several minutes. A cheetah can run faster (88 feet per second) but can only sustain its top speed for about 20 seconds before it is worn out. How far away from the cheetah does the gazelle need to stay for it to be safe?

SOLUTION

You can use a diagram to picture the situation. If the gazelle is too far away for the cheetah to catch it within 20 seconds, the gazelle is probably safe.



To find the "safety zone," you can first find the starting distance for which the cheetah reaches the gazelle in 20 seconds.

To find the distance each animal runs in 20 seconds, use the formula $d = rt$.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Distance gazelle runs in 20 sec	+	Gazelle's starting distance from cheetah	=	Distance cheetah runs in 20 sec

LABELS

Gazelle's distance = **73 · 20** (feet)

Gazelle's starting distance from cheetah = x (feet)

Cheetah's distance = **88 · 20** (feet)

ALGEBRAIC MODEL

$73 \cdot 20 + x = 88 \cdot 20$

FOCUS ON APPLICATIONS

REAL LIFE **GAZELLES** seem to have an instinct for the distance they must keep from predators. They usually will not run from a prowling cheetah until it enters their "safety zone."

APPLICATION LINK
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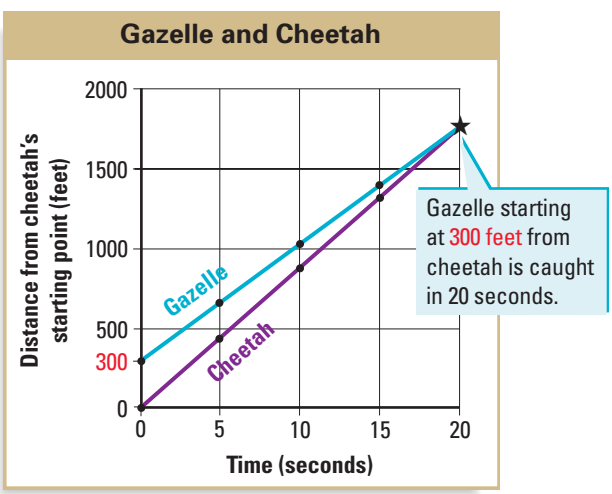
You find that $x = 300$, so the gazelle needs to stay more than 300 feet away from the cheetah to be safe.

CHECK You can use a table or a graph to check your answer. A graph helps you see the relationships. To make a graph, first make a table and then plot the points. You can just find and mark the points for every 5 seconds.



GUIDED PRACTICE

Concept Check ✓

1. **YEARBOOK DESIGN** In Example 1, the top and bottom margins are each $\frac{7}{8}$ inch. There are 5 rows of pictures, with $\frac{3}{16}$ inch of vertical space between pictures. Draw a diagram and label all the heights on your diagram. Then write and solve an equation to find the height of each picture.

2. Show how unit analysis can help you check the verbal model for Example 3.

Skill Check ✓

DISTANCE PROBLEM In Exercises 3–5, use the following problem. You and your friend are each driving 379 miles from Los Angeles to San Francisco. Your friend leaves first, driving 52 miles per hour. She is 32 miles from Los Angeles when you leave, driving 60 miles per hour. How far do each of you drive before you are side by side? Use the following verbal model.

Rate your friend is driving	•	Time after you start driving	+	Friend's distance when you leave	=	Rate you are driving	•	Time after you start driving

3. Assign labels to each part of the verbal model. Use the labels to translate the verbal model into an algebraic model.

4. Solve the equation. Interpret your solution. How far do each of you drive before you are side by side?

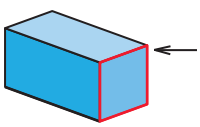
5. Make a table or a graph to check your solution.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 799.

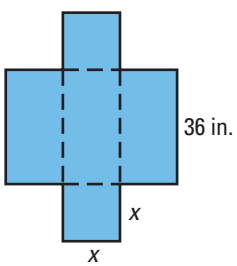
PACKAGE SIZE In Exercises 6–8, use this U.S. postal regulation: a rectangular package can have a combined length and *girth* of 108 inches. Suppose a package that is 36 inches long and as wide as it is high just meets the regulation.



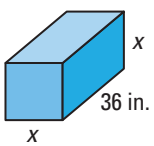
Girth is the distance around the end of the package.

6. **USING A DIAGRAM** Which diagram do you find most helpful for understanding the relationships in the problem? Explain.

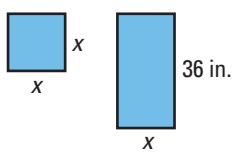
A.



B.



C.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 6–7, 14–16
Example 2: Exs. 8–12, 17–21
Example 3: Exs. 9–13, 17–21

7. **CHOOSING A MODEL** Choose the equation you would use to find the width of the package. Then find its width.

A. $x + 36 = 108$ **B.** $2x + 36 = 108$ **C.** $4x + 36 = 108$

8. **MAKING A TABLE** Make a table showing possible dimensions, girth, and combined length and girth for a "package that is 36 inches long and as wide as it is high." Which package in your table just meets the postal regulation?

UNIT ANALYSIS **In Exercises 9–12, find the resulting unit of measure.**

9. (hours per day) • (days)

10. (feet) ÷ (minute)

11. (inches) ÷ (inches per foot)

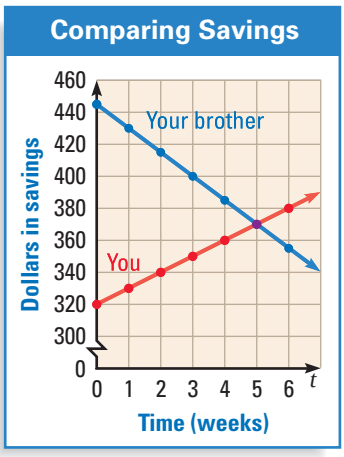
12. (miles per hour) • (hours per minute)

13. A VISUAL CHECK Use the graph to check the answer to the problem below. Is the solution correct? Explain.

You have \$320 and save \$10 each week. Your brother has \$445 and spends his income, plus \$15 of his savings each week. When will you and your brother have the same amount in savings?

$$320 + 10t = 445 - 15t$$
$$320 = 445 - 5t$$
$$-125 = -5t$$
$$25 = t$$

in 25 weeks



COVER DESIGN **In Exercises 14–16, you want the cover of a sports media guide to show two photos across its width. The cover is $6\frac{1}{2}$ inches wide, and the left and right margins are each $\frac{3}{4}$ inch. The space between the photos is $\frac{1}{2}$ inch. How wide should you make the photos?**

14. Draw a diagram of the cover.

15. Write an equation to model the problem. Use your diagram to help.

16. Solve the equation and answer the question.

17. LANGUAGE CLASSES At Barton High School, 45 students are taking Japanese. This number has been increasing at a rate of 3 students per year. The number of students taking German is 108 and has been decreasing at a rate of 4 students per year. At these rates, when will the number of students taking Japanese equal the number taking German? Write and solve an equation to answer the question. Check your answer with a table or a graph.

FOCUS ON APPLICATIONS

REAL LIFE **ELEPHANTS** need a supply of fresh water, some shade, and plentiful food such as grasses and leaves. To find these, elephants may make yearly migrations of a few hundred miles.

BIOLOGY CONNECTION **In Exercises 18–21, use the following information.**
A migrating elephant herd started moving at a rate of 6 miles per hour. One elephant stood still and was left behind. Then this stray elephant sensed danger and began running at a rate of 10 miles per hour to reach the herd. The stray caught up in 5 minutes.

18. How long (in hours) did the stray run to catch up? How far did it run?

19. Find the distance that the herd traveled while the stray ran to catch up. Then write an expression for the total distance the herd traveled. Let x represent the distance (in miles) that the herd traveled while the stray elephant stood still.

20. Use the distances you found in Exercises 18 and 19. Write and solve an equation to find how far the herd traveled while the stray stood still.

21. Make a table or a graph to check your answer.

Test Preparation

22. MULTI-STEP PROBLEM Two friends are 60 miles apart. They decide to ride their bicycles to meet each other. Sally starts from the college and heads east, riding at a rate of 21 miles per hour. At the same time Teresa starts from the river and heads west, riding at a rate of 15 miles per hour.

a. Draw a diagram showing the college and the river 60 miles apart and the two cyclists riding toward each other.

b. How far does each cyclist ride in t hours?

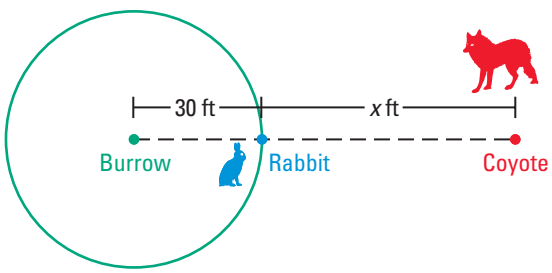
c. When the two cyclists meet, what must be true about the distances they have ridden? Write and solve an equation to find when they meet.

d. CHOOSING A DATA DISPLAY Would a park that is 26 miles west of the river be a good meeting place? Explain. Use a diagram, a table, or a graph to support your answer.

Challenge

RABBIT AND COYOTE

In Exercises 23–26, use this information. A rabbit is 30 feet from its burrow. It can run 25 feet per second. A coyote that can run 50 feet per second spots the rabbit and starts running toward it.



23. Write expressions for the time it will take the rabbit to get to its burrow and for the time it will take the coyote to get to the burrow.

24. Write and solve an equation that relates the two expressions in Exercise 23.

25. CRITICAL THINKING Interpret your results from Exercise 24. Include a description of what happens for different values of x.

EXTRA CHALLENGE
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26. Use a table or a graph to check your conclusions.

MIXED REVIEW

FRACTIONS, DECIMALS, AND PERCENTS **Write as a decimal rounded to the nearest hundredth. Then write as a percent.** (Skills Review, pages 784–785)

27. $\frac{3}{11}$ **28.** $\frac{11}{12}$ **29.** $\frac{25}{31}$ **30.** $\frac{100}{201}$

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 2.5 and 2.7)

31. $\frac{-6}{q} - 2r$ when $q = 2$ and $r = 11$ **32.** $\frac{3}{5}x - y$ when $x = -25$ and $y = -10$

33. $\frac{x + y}{12}$ when $x = -24$ and $y = 6$ **34.** $\frac{ab}{3a - 10}$ when $a = 8$ and $b = 7$

SOLVING EQUATIONS **Solve the equation.** (Review 3.3 and 3.4 for 3.6)

35. $18 + \frac{x}{3} = 9$ **36.** $8 = -\frac{2}{3}(2x - 6)$ **37.** $4(2 - n) = 1$

38. $-3y + 14 = -5y$ **39.** $-4a - 3 = 6a + 2$ **40.** $5x - (6 - x) = 2(x - 7)$