3.3

Solving Multi-Step Equations

GOAL 1 USING TWO OR MORE TRANSFORMATIONS

Solving a linear equation may require two or more transformations. Here are some guidelines.

- Simplify one or both sides of the equation (if needed).
- Use inverse operations to isolate the variable.

EXAMPLE 1 Solving a Linear Equation

Solve $\frac{1}{3}x + 6 = -8$.

SOLUTION To isolate the variable, undo the addition and then the multiplication.

$$\frac{1}{3}x + 6 = -8 \qquad \text{Write original equation.}$$

$$\frac{1}{3}x + 6 - 6 = -8 - 6 \qquad \text{Subtract 6 from each side.}$$

$$\frac{1}{3}x = -14 \qquad \text{Simplify.}$$

$$3\left(\frac{1}{3}x\right) = 3(-14) \qquad \text{Multiply each side by 3.}$$

$$x = -42 \qquad \text{Simplify.}$$

EXAMPLE 2 Combining Like Terms First

Solve $7x - 3x - 8 = 24$.

SOLUTION

$$7x - 3x - 8 = 24 \qquad \text{Write original equation.}$$

$$4x - 8 = 24 \qquad \text{Combine like terms.}$$

$$4x - 8 + 8 = 24 + 8 \qquad \text{Add 8 to each side.}$$

$$4x = 32 \qquad \text{Simplify.}$$

$$\frac{4x}{4} = \frac{32}{4} \qquad \text{Divide each side by 4.}$$

$$x = 8 \qquad \text{Simplify.}$$

▶ The solution is 8. When you check the solution, substitute 8 for *each x* in the original equation.

EXAMPLE 3 *Using the Distributive Property*

Solve $5x + 3(x + 4) = 28$.

SOLUTION

Method 1 Show All Steps

$$5x + 3(x + 4) = 28$$
$$5x + 3x + 12 = 28$$
$$8x + 12 = 28$$
$$8x + 12 - 12 = 28 - 12$$
$$8x = 16$$
$$\frac{8x}{8} = \frac{16}{8}$$
$$x = 2$$

Method 2 Do Some Steps Mentally

$$5x + 3(x + 4) = 28$$
$$5x + 3x + 12 = 28$$
$$8x + 12 = 28$$
$$8x = 16$$
$$x = 2$$

EXAMPLE 4 *Distributing a Negative*

Solve $4x - 3(x - 2) = 21$.

SOLUTION

$$4x - 3(x - 2) = 21 \qquad \text{Write original equation.}$$
$$4x - 3x + 6 = 21 \qquad \text{Use distributive property.}$$
$$x + 6 = 21 \qquad \text{Combine like terms.}$$
$$x = 15 \qquad \text{Subtract 6 from each side.}$$

EXAMPLE 5 *Multiplying by a Reciprocal First*

Solve $66 = -\frac{6}{5}(x + 3)$.

SOLUTION

It is easier to solve this equation if you don't distribute $-\frac{6}{5}$.

$$66 = -\frac{6}{5}(x + 3) \qquad \text{Write original equation.}$$
$$\left(-\frac{5}{6}\right)(66) = \left(-\frac{5}{6}\right)\left(-\frac{6}{5}\right)(x + 3) \qquad \text{Multiply by reciprocal of } -\frac{6}{5}.$$
$$-55 = x + 3 \qquad \text{Simplify.}$$
$$-58 = x \qquad \text{Subtract 3 from each side.}$$

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Solving equations systematically is an example of deductive reasoning. Notice how each solution step is based on number properties or properties of equality.

GOAL 2 SOLVING REAL-LIFE PROBLEMS

EXAMPLE 6 *Using a Known Formula*

CHECKING VITAL SIGNS A body temperature of 95°F or lower may indicate the medical condition called hypothermia. What temperature in the Celsius scale may indicate hypothermia?

The Fahrenheit and Celsius scales are related by the equation $F = \frac{9}{5}C + 32$.

SOLUTION Use the formula to convert 95°F to Celsius.

$F = \frac{9}{5}C + 32$ **Write known formula.**

$95 = \frac{9}{5}C + 32$ **Substitute 95° for F.**

$63 = \frac{9}{5}C$ **Subtract 32 from each side.**

$35 = C$ **Multiply by $\frac{5}{9}$, the reciprocal of $\frac{9}{5}$.**

▶ A temperature of 35°C or lower may indicate hypothermia.

EXAMPLE 7 *Using a Verbal Model*

SCIENCE CONNECTION The temperature within Earth's crust increases about 30° Celsius for each kilometer of depth beneath the surface. If the temperature at Earth's surface is 24°C, at what depth would you expect the temperature to be 114°C?

Crust
0–40 km
Mantle
Outer Core
Inner Core
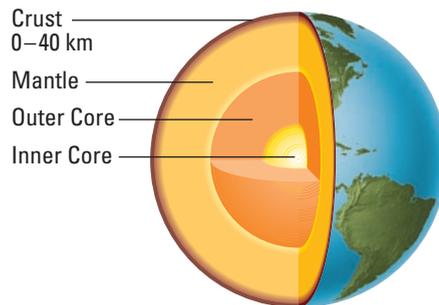
SOLUTION

PROBLEM
SOLVING
STRATEGY

VERBAL MODEL

Temperature inside Earth	=	Temperature at Earth's surface	+	Rate of temperature increase	·	Depth below surface

LABELS
Temperature inside Earth = **114**   (degrees Celsius)
Temperature at Earth's surface = **24**   (degrees Celsius)
Rate of temperature increase = **30**   (degrees Celsius per kilometer)
Depth below surface = **d**   (kilometers)

ALGEBRAIC MODEL $114 = 24 + 30 \cdot d$

▶ The solution is $d = 3$. The temperature will be 114°C at 3 kilometers deep.

3.3 *Solving Multi-Step Equations* **147**

GUIDED PRACTICE

Concept Check ✓

1. LOGICAL REASONING Copy the solution steps shown. Then, in the right-hand column, describe the transformation that was used in each step. (Lists of types of transformations are shown on pages 132 and 138.)

2. For which equations in Exercises 6–8 would you use the distributive property first when solving? Explain.

Solution Steps	Explanation
$\frac{5x}{2} + 3 = 6$	Original Equation
$\frac{5x}{2} = 3$	____?____
$5x = 6$	____?____
$x = \frac{6}{5}$	____?____

Skill Check ✓ **In Exercises 3–8, solve the equation. Show how to check your solution.**

3. $4x + 3 = 11$

4. $\frac{1}{2}x - 9 = 11$

5. $3x - x + 15 = 41$

6. $5(x - 7) = 90$

7. $\frac{3}{4}(x + 6) = 12$

8. $6x - 4(-3x + 2) = 10$

9. 🌍 **SUMMER JOB** You have a summer job running errands for a local business. You earn $5 per day, plus $2 for each errand. Write and solve an equation to find how many errands you need to run to earn $17 in one day.

PRACTICE AND APPLICATIONS

CHECKING SOLUTIONS Check whether the given number is a solution of the equation.

10. $9x - 5x - 19 = 21; -10$

11. $\frac{3}{4}x + 1 = -8; -12$

12. $6x - 4(9 - x) = 106; 7$

13. $7x - 15 = -1; -2$

14. $\frac{1}{2}x - 7 = -4; 6$

15. $\frac{x}{4} - 7 = 13; 24$

SOLVING EQUATIONS Solve the equation.

16. $2x + 7 = 15$

17. $3x - 1 = 8$

18. $\frac{x}{3} - 5 = -1$

19. $\frac{x}{2} + 13 = 20$

20. $30 = 16 + \frac{1}{5}x$

21. $6 = 14 - 2x$

22. $7 + \frac{2}{3}x = -1$

23. $3 - \frac{3}{4}x = -6$

24. $22 = 18 - \frac{1}{4}x$

25. $8x - 3x = 10$

26. $-7x + 4x = 9$

27. $x + 5x - 5 = 1$

28. $3x - 7 + x = 5$

29. $3(x - 2) = 18$

30. $12(2 - x) = 6$

31. $\frac{9}{2}(x + 3) = 27$

32. $-\frac{4}{9}(2x - 4) = 48$

33. $17 = 2(3x + 1) - x$

34. $\frac{4x}{3} + 3 = 23$

35. $-10 = 4 - \frac{7x}{4}$

36. $-10 = \frac{1}{2}x + x$

SOLVING EQUATIONS Solve the equation.

37. $5m - (4m - 1) = -12$

38. $55x - 3(9x + 12) = -64$

39. $22x + 2(3x + 5) = 66$

40. $9x - 5(3x - 12) = 30$

ERROR ANALYSIS In Exercises 41–43, find and correct the error.

41.

$2(x - 3) = 5$
$2x - 3 = 5$
$2x = 8$
$x = 4$

42.

$5 - 3x = 10$
$2x = 10$
$x = 5$

43.

$\frac{1}{4}x - 2 = 7$
$x - 2 = 28$
$x = 30$

44. LOGICAL REASONING Solve the equation $-6x + 3(4x - 1) = 9$. Organize your work into two columns. In the left-hand column show the solution steps. In the right-hand column explain the transformation you used in each step.

45. *Writing* Solve the equation $\frac{1}{9}x + 1 = 4$ by following each sequence of steps described in parts (a) and (b). Which method do you prefer? Explain.

a. Multiply and then subtract.   **b.** Subtract and then multiply.

CHOOSING A METHOD In Exercises 46–49, solve the equation two different ways as in Exercise 45. Explain which method you prefer and why.

46. $\frac{1}{8}x - 5 = 3$ **47.** $\frac{x}{3} + 6 = -2$ **48.** $-4x - 2 = 4$ **49.** $\frac{2}{3}x + 1 = \frac{1}{3}$

50. CONSECUTIVE INTEGERS *Consecutive integers* are integers that follow each other in order (for example, 5, 6, and 7). You want to find three consecutive integers whose sum is 84.

a. Why does the equation $n + (n + 1) + (n + 2) = 84$ model the situation?

b. Solve the equation in part (a). Then find the three consecutive integers.

51. FIND THE NUMBERS The sum of three numbers is 123. The second number is 9 less than two times the first number. The third number is 6 more than three times the first number. Find the three numbers.

52. ⊕ FARMING PROJECT You have a 90-pound calf you are raising for a 4-H project. You expect the calf to gain 65 pounds per month. In how many months will the animal weigh 1000 pounds?

53. ⊕ HOURS OF LABOR The bill (parts and labor) for the repair of a car was $458. The cost of parts was $339. The cost of labor was $34 per hour. Write and solve an equation to find the number of hours of labor.

54. GEOMETRY CONNECTION   In any triangle, the sum of the measures of the angles is 180°. In triangle ABC, $\angle A$ is four times as large as $\angle B$. Angle C measures 20° less than $\angle B$. Find the measure of each angle.

55. ⊕ STUDENT THEATER Your school's drama club charges $4 per student for admission to *Our Town*. The club borrowed $400 from parents to pay for costumes, props, and the set. After paying back the parents, the drama club has $100. How many students attended the play?

56. 🌐 **HOLIDAY PAY** You earn $9 per hour. On major holidays, such as Thanksgiving, you earn twice as much per hour. You earned a total of $405 for the week including Thanksgiving. Write and solve an equation to find how many hours you worked on Thanksgiving if you worked 35 hours during the rest of the week.

57. 🌐 **FIREFIGHTING** The formula $d = \frac{n}{2} + 26$ relates nozzle pressure n (in pounds per square inch) and the maximum distance the water reaches d (in feet) for a fire hose with a certain size nozzle. How much pressure is needed if such a hose is held 50 feet from a fire? ▶ Source: *Fire Department Hydraulics*
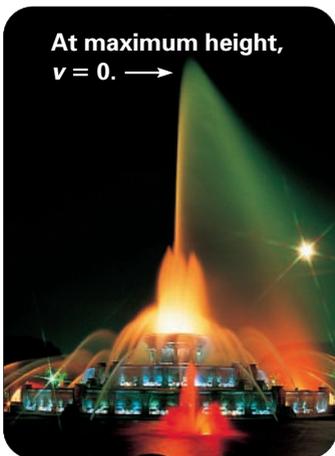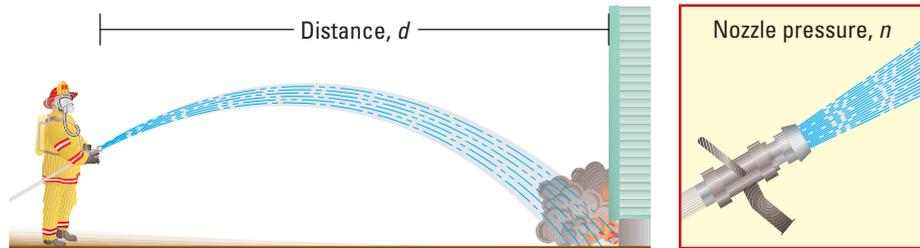
58. 🌐 **HEIGHT OF A FOUNTAIN** Neglecting air resistance, the upward velocity of the water in the stream of a particular fountain is given by the formula $v = -32t + 28$, where t is the number of seconds after the water leaves the fountain. While going upward, the water slows down until, at the top of the stream, the water has a velocity of 0 feet per second. How long does it take a droplet of water to reach the maximum height?

59. 🌐 **PHOTOCOPYING** An office needs 20 copies of a 420-page report. One photocopier can copy 1800 pages per hour. Another copier is faster and can copy 2400 pages per hour. To find the time it would take the two copiers together to complete the project, you can use this verbal model. Assign labels to the model to form an equation. Then solve the equation.

$$\boxed{\begin{array}{c}\text{Rate of first}\\\text{machine}\end{array}} \cdot \boxed{\begin{array}{c}\text{Time to}\\\text{complete}\end{array}} + \boxed{\begin{array}{c}\text{Rate of second}\\\text{machine}\end{array}} \cdot \boxed{\begin{array}{c}\text{Time to}\\\text{complete}\end{array}} = \boxed{\begin{array}{c}\text{Total}\\\text{pages}\end{array}}$$

60. **CRITICAL THINKING** Explain how you can use unit analysis to check that the verbal model in Exercise 59 is correct.

61. 🌐 **DATA ENTRY** A publishing company needs to enter 910 pages it received from an author into its word processing system, so the writing can be edited and formatted. One person can type 15 pages per hour. Another person can type 20 pages per hour. Write and solve an equation to find how long it will take the two people working together to enter all of the pages.

REPRESENTING COIN PROBLEMS In Exercises 62 and 63, use the following information. A person has quarters, dimes, and nickels with a total value of 500 cents ($5.00). The number of nickels is twice the number of quarters. The number of dimes is four less than the number of quarters.

62. Explain why the expression $5(2q)$ represents the value of the nickels if q represents the number of quarters. How can you simplify the expression?

63. Write and solve an equation to find the number of each type of coin.

64. **COIN PROBLEM** There are 4 times as many nickels as dimes in a coin bank. The coins have a total value of 600 cents ($6.00). Find the number of nickels.

65. MULTI-STEP PROBLEM Two student volunteers are stuffing envelopes for a local food pantry. The mailing will be sent to 560 possible contributors. Luis can stuff 160 envelopes per hour and Mei can stuff 120 envelopes per hour.

    a. Working alone, what fraction of the job can Luis complete in one hour? in t hours? Write the fraction in lowest terms.

    b. Working alone, what fraction of the job can Mei complete in t hours?

    c. Write an expression for the fraction of the job that Luis and Mei can complete in t hours if they work together.

    d. CRITICAL THINKING To find how long it will take Luis and Mei to complete the job if they work together, you can set the expression you wrote in part (c) equal to 1 and solve for t. Explain why this will work.

    e. How long will it take Luis and Mei to complete the job if they work together? Check your solution.

66. ODD INTEGERS Consecutive odd integers are odd integers listed in order such as 5, 7, and 9. Find three consecutive odd integers whose sum is 111. Are there three consecutive odd integers whose sum is 1111? Explain.

MIXED REVIEW

WRITING POWERS Write the expression in exponential form. **(Review 1.2)**

67. four squared   **68.** b cubed   **69.** $(a)(a)(a)(a)(a)(a)$

70. 10   **71.** $2 \cdot 2 \cdot 2 \cdot 2$ **72.** $3x \cdot 3x \cdot 3x \cdot 3x \cdot 3x$

EVALUATING EXPRESSIONS Evaluate the expression. **(Review 1.3, 2.7)**

73. $5 + 8 - 3$ **74.** $3^2 \cdot 4 + 8$ **75.** $16.9 - 1.5(1.8 + 0.2)$

76. $5 \cdot (12 - 4) + 7$ **77.** $-6 \div 3 - 4 \cdot 5$ **78.** $10 - [4.3 + 2(6.4 \div 8)]$

79. $\dfrac{-5 \cdot 4}{3 - 7^2 + 6}$ **80.** $2 - 8 \div \dfrac{-2}{3}$ **81.** $\dfrac{(3 - 6)^2 + 6}{-5}$

82. BICYCLING If you ride a bicycle 5 miles per hour, how many miles will you ride in 45 minutes? **(Review 1.1)**

PROFIT In Exercises 83–85, use the following information. You open a snack stand at a fair. The income and expenses (in dollars) for selling each type of food are shown in the matrices. **(Review 2.4)**

Day 1	Income	Expenses
Hamburgers	72	14
Hot dogs	85	18
Tacos	46	19

Day 2	Income	Expenses
Hamburgers	62	10
Hot dogs	52	11
Tacos	72	26

83. What were your total income and expenses for selling each type of food for the two days of the fair?

84. Which type of food had the largest profit?

85. Which type of food had the smallest profit?

Solve the equation. **(Lessons 3.1, 3.2, and 3.3)**

1. $8 - y = -9$ **2.** $x + \dfrac{1}{2} = 5$ **3.** $\left|-14\right| + z = 12$

4. $8b = 5$ **5.** $\dfrac{3}{4}q = 24$ **6.** $\dfrac{n}{-8} = -\dfrac{3}{8}$

7. $\dfrac{x}{5} + 10 = \dfrac{4}{5}$ **8.** $\dfrac{1}{4}(y + 8) = 5$ **9.** $25x - 4(4x + 6) = -69$

10. 🌎 **HISTORY TEST** You take a history test that has 100 regular points and 8 bonus points. You receive a total score of 91, which includes 4 bonus points. Write and solve an equation to find the score you would have had without the bonus points. **(Lesson 3.1)**

11. 🌎 **TEMPERATURE** The temperature is 72°F. What is the temperature in degrees Celsius? $\left(\text{Use the equation } F = \dfrac{9}{5}C + 32.\right)$ **(Lesson 3.3)**

## MATH & *History*   **History of Stairs**

THEN

IN 1675, François Blondel, director of France's Royal Academy of Architecture, decided upon a formula for building stairs to accommodate the human gait. He came up with $2R + T = 24$ inches (25.5 in today's inches) where $R =$ riser height and $T =$ tread width.

NOW

IN THE 1990S, a group of building code officials adopted the "7-11" rule which states that a riser should be no taller than 7 inches and no shorter than 4 inches, with a minimum tread width of 11 inches.

In Exercises 1 and 2 you will compare a traditional staircase with an 8.25-inch riser and a 9-inch tread and a staircase built using the 7-11 rule that has a 7-inch riser and an 11-inch tread. Each of the staircases has 12 steps.

1. Sketch and label an individual stair for each staircase.

2. Sketch and label each staircase. Which staircase is steeper? Which design will require more stairs if both need to reach the same height?
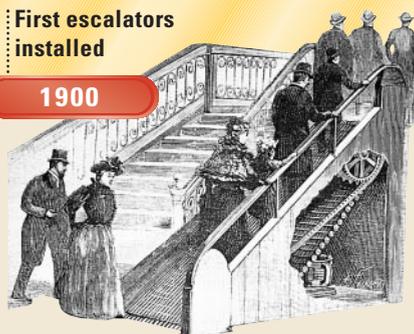
TREAD

RISER
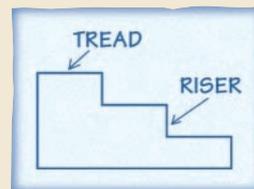
900–1200

The pyramid *El Castillo* in Chichen Itza, Mexico

First escalators installed

1900

Now

Modern escalator in New York City