2.7

Division of Real Numbers

What you should learn

GOAL 1 Divide real numbers.

GOAL 2 Use division to simplify algebraic expressions.

Why you should learn it

▼ To solve **real-life** problems like finding the average velocity of a model rocket in **Exs. 68–70.**

GOAL 1 DIVIDING REAL NUMBERS

For every real number other than zero there exists a number called its *reciprocal*. The product of a number and its **reciprocal** is 1. For instance, the product of the number -3 and its reciprocal $-\frac{1}{3}$ is 1. This property is sometimes referred to as the *inverse property of multiplication*.

- The reciprocal of a is $\frac{1}{a}$. $(a \neq 0)$ **Example:** -8 and $-\frac{1}{8}$
- The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. $(a \neq 0, b \neq 0)$ **Example:** $-\frac{2}{5}$ and $-\frac{5}{2}$

Zero is the only real number that has no reciprocal. There is no number that when multiplied by zero gives a product of 1. Because zero does not have a reciprocal, you cannot divide by zero.

You can use a reciprocal to write a division expression as a product.

DIVISION RULE

To divide a number a by a nonzero number b, multiply a by the reciprocal of b.

$a \div b = a \cdot \frac{1}{b}$ **Example:** $-1 \div 3 = -1 \cdot \frac{1}{3} = -\frac{1}{3}$

The result is the quotient of a and b.

EXAMPLE 1 *Dividing Real Numbers*

Find the quotient.

a. $10 \div (-2)$ **b.** $-39 \div \left(-4\frac{1}{3}\right)$ **c.** $\dfrac{-\frac{1}{3}}{4}$ **d.** $\dfrac{1}{-\frac{3}{4}}$

SOLUTION

a. $10 \div (-2) = 10 \cdot \left(-\frac{1}{2}\right) = -5$

b. $-39 \div \left(-4\frac{1}{3}\right) = -39 \div \left(-\frac{13}{3}\right) = -39 \cdot \left(-\frac{3}{13}\right) = 9$

c. $\dfrac{-\frac{1}{3}}{4} = -\frac{1}{3} \div 4 = -\frac{1}{3} \cdot \frac{1}{4} = -\frac{1}{12}$

d. $\dfrac{1}{-\frac{3}{4}} = 1 \div \left(-\frac{3}{4}\right) = 1 \cdot \left(-\frac{4}{3}\right) = -\frac{4}{3}$

GOAL 2 WORKING WITH ALGEBRAIC EXPRESSIONS

In Example 1, notice that applying the division rule results in the following patterns for finding the sign of a quotient.

THE SIGN OF A QUOTIENT

- The quotient of two numbers with the same sign is positive.

  $-a \div (-b) = \frac{a}{b}$ **Example:** $-20 \div (-4) = 5$

- The quotient of two numbers with opposite signs is negative.

  $-a \div b = -\frac{a}{b}$ **Example:** $-20 \div 4 = -5$

EXAMPLE 2 *Using the Distributive Property to Simplify*

STUDENT HELP

INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for extra examples.

Simplify the expression $\frac{32x - 8}{4}$.

SOLUTION

$$\frac{32x - 8}{4} = (32x - 8) \div 4$$ Rewrite fraction as division expression.

$$= (32x - 8)\left(\frac{1}{4}\right)$$ Multiply by reciprocal.

$$= (32x)\left(\frac{1}{4}\right) - (8)\left(\frac{1}{4}\right)$$ Use distributive property.

$$= 8x - 2$$ Simplify.

The order of operations you learned in Chapter 1 also applies to real numbers and to algebraic expressions.

EXAMPLE 3 *Evaluating an Expression*

Evaluate the expression when $a = -2$ and $b = -3$.

a. $\frac{-2a}{a + b}$ **b.** $\frac{a^2 - 4}{b}$ **c.** $\frac{a}{3 + b}$

STUDENT HELP

Study Tip
Remember that 0 can be divided by any nonzero number. The result will always be zero.

Division *by* zero is undefined.

SOLUTION

a. $\frac{-2a}{a + b} = \frac{-2(-2)}{-2 + (-3)} = \frac{4}{-5} = -\frac{4}{5}$

b. $\frac{a^2 - 4}{b} = \frac{(-2)^2 - 4}{-3} = \frac{4 - 4}{-3} = \frac{0}{-3} = 0$

c. $\frac{a}{3 + b} = \frac{-2}{3 + (-3)} = \frac{-2}{0}$ (Undefined)

FOCUS ON APPLICATIONS

REAL LIFE **HOT-AIR BALLOONING**
A hot-air balloon pilot fires the burners of the balloon as it nears the ground, to slow the descent rate and land the balloon gently.

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

EXAMPLE 4 *Finding a Velocity*

HOT-AIR BALLOONING You are descending in a hot-air balloon. You descend 500 feet in 40 seconds. What is your velocity?

SOLUTION

VERBAL MODEL

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

LABELS

Velocity $= v$ (ft/sec)

Displacement $= -500$ (feet)

Time $= 40$ (seconds)

ALGEBRAIC MODEL

$$v = \frac{-500}{40}$$

$$= -12.5$$

▶ Your velocity, -12.5 ft/sec, is negative because you are descending.

. . . . . .

When a function is defined by an equation, its domain is restricted to real numbers for which the function can be evaluated. Division by zero is undefined, so input values that make you divide by zero must be excluded from the domain.

For instance, the function $y = \frac{5}{x - 2}$ does not have $x = 2$ in its domain.

EXAMPLE 5 *Finding the Domain of a Function*

Find the domain of the function $y = \frac{-x}{1 - x}$.

STUDENT HELP

Look Back
For help with domain and range, see p. 47.

SOLUTION

To find the domain of $y = \frac{-x}{1 - x}$, you must exclude any values for which the denominator, $1 - x$, is equal to zero. To do this, you need to solve the equation $1 - x = 0$. Use mental math.

You can ask: What number subtracted from 1 equals 0?
The answer is 1, because $1 - 1 = 0$.

▶ Since $1 - x = 0$ when $x = 1$, you can see that $x = 1$ is not in the domain of the function, because you cannot divide by zero. All other real numbers are in the domain because there are no other values of x that will make the denominator zero. The domain is all real numbers *except* $x = 1$.

✓**CHECK** When $x = 1$, $y = \frac{-x}{1 - x}$

$$= \frac{-(1)}{1 - (1)}$$

$$= \frac{-1}{0} \longleftarrow \text{Undefined}$$

GUIDED PRACTICE

Vocabulary Check ✓

1. The product of a number and ? is 1.

Concept Check ✓

2. Is dividing by a number the same as multiplying by the opposite of the number? Explain your reasoning.

3. Is the reciprocal of a negative number *sometimes, always*, or *never* positive?

4. A friend tells you that $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$. Is your friend correct? Use examples or counterexamples to support your answer.

5. Describe the error shown at the right.

$$6 \div -\frac{2}{3} = 6 \cdot \frac{3}{2} = 9$$

Skill Check ✓

Find the reciprocal of the number.

6. 34
7. -8
8. $-\frac{3}{4}$
9. $-2\frac{1}{5}$

Find the quotient.

10. $7 \div \frac{1}{2}$
11. $10 \div \left(-\frac{1}{5}\right)$
12. $-12 \div 3$
13. $\frac{16}{\frac{-2}{9}}$
14. Find the domain of the function $y = \frac{x+1}{x+2}$.
15. **STOCK** You own 18 shares of stock in a computer company. The total value of the shares changes by $-\$3.06$. By how much does the value of each share of stock change?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 798.

FINDING QUOTIENTS **Find the quotient.**

16. $-51 \div (-17)$
17. $45 \div (-9)$
18. $-24 \div 4$
19. $49 \div (-7)$
20. $64 \div (-8)$
21. $-99 \div 9$
22. $-35 \div (-70)$
23. $18 \div (-54)$
24. $-90 \div \left(-\frac{2}{3}\right)$
25. $56 \div \left(-2\frac{4}{7}\right)$
26. $\frac{-26}{-\frac{1}{2}}$
27. $\frac{36}{-\frac{5}{6}}$
28. $60 \div (-10)$
29. $-12.6 \div 1.8$
30. $-18 \div \frac{3}{8}$
31. $-87 \div \left(-\frac{3}{5}\right)$

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

32. $42y \div \frac{1}{7}$
33. $6t \div \left(-\frac{1}{2}\right)$
34. $58z \div \left(-\frac{2}{5}\right)$
35. $-\frac{x}{12} \div 3$
36. $\frac{d}{4} \div 6$
37. $\frac{3y}{4} \div \frac{1}{2}$
38. $-\frac{2b}{7} \div \frac{7}{9}$
39. $33x \div \frac{3}{11}$
40. $49x \div 3\frac{1}{2}$
41. $8x^2 \div \left(-\frac{4}{5}\right)$
42. $68x \div \left(-\frac{17}{9}\right)$
43. $-54x^2 \div \frac{-9}{5}$
44. $\frac{42t}{-14z} \div \frac{-6}{7t}$
45. $8 \cdot \frac{x}{8}$
46. $3 \cdot \left(-\frac{y}{3}\right)$
47. $-7 \cdot \left(-\frac{2w}{-7}\right)$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–47
Example 2: Exs. 48–51
Example 3: Exs. 52–57
Example 4: Exs. 62–64
Example 5: Exs. 58–61

DISTRIBUTIVE PROPERTY **Simplify the expression.**

48. $\frac{18x - 9}{3}$ **49.** $\frac{22x + 10}{-2}$ **50.** $\frac{-56 + x}{-8}$ **51.** $\frac{45 - 5x}{5}$

EVALUATING EXPRESSIONS **Evaluate the expression for the given value(s) of the variable(s).**

52. $\frac{x - 5}{6}$ when $x = 30$ **53.** $\frac{3r - 7}{11}$ when $r = 17$

54. $\frac{3a - b}{a}$ when $a = \frac{1}{3}$ and $b = -3$ **55.** $\frac{15x^2 + 10}{y}$ when $x = -3$ and $y = \frac{2}{3}$

56. $\frac{28 - 4x}{y}$ when $x = 2$ and $y = \frac{1}{2}$ **57.** $\frac{3a - 4b}{ab}$ when $a = -\frac{1}{3}$ and $b = \frac{1}{4}$

FINDING THE DOMAIN **Find the domain of the function.**

58. $y = \frac{1}{3x}$ **59.** $y = \frac{3}{2 - x}$ **60.** $y = \frac{1}{x + 2}$ **61.** $y = \frac{4}{x^2}$

62. **COOKING** *Tom Kar Gai* is a soup from Thailand. You need $13\frac{3}{4}$ ounces of chicken broth to make one serving. If you have $82\frac{1}{2}$ ounces of chicken broth, how many servings can you make?

63. **SCUBA DIVING** You are scuba diving in the ocean. You dive down 22.5 feet in 9 seconds. What is your average velocity?

64. **BASKETBALL** Kareem Abdul-Jabbar scored 38,387 points and grabbed 17,440 rebounds in 1560 National Basketball Association games. How many points did he average per game? How many rebounds did he average per game? (Round your answers to the nearest tenth.) ▶ Source: NBA

FINDING A PATTERN **In Exercises 65–67, use the table below.**

Fraction	$\frac{x}{1}$	$\frac{x}{0.1}$	$\frac{x}{0.01}$	$\frac{x}{0.001}$	$\frac{x}{0.0001}$	$\frac{x}{0.00001}$
Product	$1x$	$10x$	?	?	?	?

65. Copy and complete the table by expressing the fraction as a product that has no decimal factor.

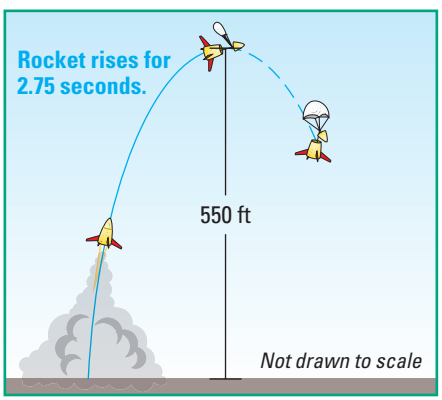
66. Describe the pattern in the products.

67. Use the pattern to simplify the expression $\frac{x}{0.0000001}$.

FOCUS ON PEOPLE

REAL LIFE **KAREEM ABDUL-JABBAR** holds the NBA career record for most points scored. He was a member of 6 NBA championship teams.

MODEL ROCKET **You launch a model rocket that rises 550 feet in 2.75 seconds. It then opens a parachute and falls at a rate of 11 feet per second.**



68. What is the rocket's average velocity going up?

69. What is the rocket's velocity coming down?

70. In how many seconds after the launch will the rocket reach the ground?

71. *Writing* If a and b are positive and $a < b$, is it true that $\frac{1}{b} < \frac{1}{a}$? Explain and show an example or a counterexample.

Test Preparation

72. MULTIPLE CHOICE Which of the following expressions is equivalent to the expression $\frac{6x - 4}{-\frac{2}{3}}$?

Ⓐ $2(3x - 2) \cdot \frac{3}{2}$ Ⓑ $(6x - 4) \cdot \left(-\frac{2}{3}\right)$ Ⓒ $-9x + 6$ Ⓓ $9x - 6$

73. MULTIPLE CHOICE Which of the following statements is false?

Ⓐ The reciprocal of any negative number is a negative number.

Ⓑ Dividing by a number is the same as multiplying by the reciprocal of the number.

Ⓒ The reciprocal of any positive number is a positive number.

Ⓓ The reciprocal of any number is greater than zero and less than 1.

Challenge

74. EXTENSION: CLOSURE A set of numbers is *closed* under an operation if applying the operation to any two numbers in the set results in another number in the set. For instance, positive integers are closed under addition because the sum of any two positive integers is a positive integer. Decide whether the set is closed under the given operation.

a. positive integers; subtraction

b. integers; addition and subtraction

c. integers; multiplication

d. integers; division

EXTRA CHALLENGE
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MIXED REVIEW

CONVERTING FRACTIONS **Write the fraction as a decimal. Round to the nearest hundredth if necessary.** (Skills Review, pp. 784–785)

75. $\frac{3}{4}$ **76.** $\frac{3}{5}$ **77.** $\frac{1}{10}$ **78.** $\frac{7}{25}$

79. $\frac{1}{6}$ **80.** $\frac{2}{7}$ **81.** $\frac{30}{40}$ **82.** $\frac{5}{11}$

EXPONENTIAL EXPRESSIONS **Evaluate the expression for the given value of the variable.** (Review 1.2, 2.5)

83. $3x^2$ when $x = 7$ **84.** $4(b^3)$ when $b = \frac{1}{2}$ **85.** $2(y^3)$ when $y = -5$

86. $5y^3$ when $y = 4$ **87.** $(6w)^4$ when $w = 2$ **88.** $12d^2$ when $d = 9$

89. $5x^2$ when $x = 0.3$ **90.** $32x^7$ when $x = -1$ **91.** $(7t)^3$ when $t = -\frac{3}{7}$

NUMERICAL EXPRESSIONS **Evaluate the expression.** (Review 1.3, 2.5)

92. $12 - 9 + 7$ **93.** $42 \div 6 + 8$ **94.** $2(11 - 7) \div 3$

95. $8 + (91 \div 13) \cdot \frac{4}{7}$ **96.** $\frac{3}{4} \cdot 8 - 6$ **97.** $23 - [(12 \div 3)^2 + 8]$

98. $11 \cdot (-5) + 20$ **99.** $-8 \cdot (-9) - 80$ **100.** $-16 + (-6) \cdot (-8)$