2.6

The Distributive Property

What you should learn

GOAL 1 Use the distributive property.

GOAL 2 Simplify expressions by combining like terms.

Why you should learn it

▼ To solve **real-life** problems such as finding how much you can spend on jeans in **Exs. 70 and 71**.

GOAL 1 USING THE DISTRIBUTIVE PROPERTY

To multiply 3(68) mentally, you could think of 3(68) as

$$3(60 + 8) = 3(60) + 3(8) = 180 + 24 = 204.$$

This is an example of the *distributive property*.

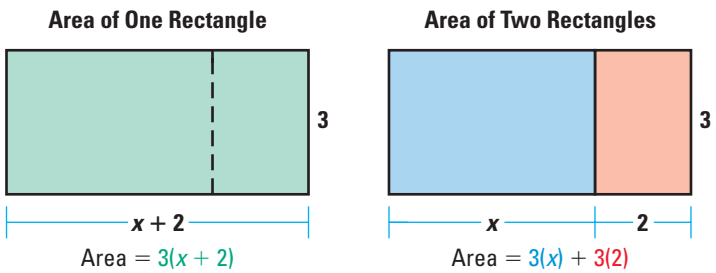
The distributive property is a very important algebraic property. Before discussing the property, study an example that suggests why the property is true.

EXAMPLE 1 *Using an Area Model*

Find the area of a rectangle whose width is 3 and whose length is $x + 2$.

SOLUTION

You can find the area in two ways.



▶ Because both ways produce the same area, the following statement is true.

$$3(x + 2) = 3(x) + 3(2)$$

Example 1 suggests the **distributive property**. In the equation above, the factor 3 is *distributed* to each term of the sum $(x + 2)$. There are four versions of the distributive property, as follows.

THE DISTRIBUTIVE PROPERTY

The product of a and $(b + c)$:

$a(b + c) = ab + ac$ **Example:** $5(x + 2) = 5x + 10$

$(b + c)a = ba + ca$ **Example:** $(x + 4)8 = 8x + 32$

The product of a and $(b - c)$:

$a(b - c) = ab - ac$ **Example:** $4(x - 7) = 4x - 28$

$(b - c)a = ba - ca$ **Example:** $(x - 5)9 = 9x - 45$

STUDENT HELP

Study Tip

In Example 2, the middle steps are usually done as mental math, without writing out the steps. For instance, part (a) can be written as $2(x + 5) = 2x + 10$.

EXAMPLE 2 Using the Distributive Property

a. $2(x + 5) = 2(x) + 2(5) = 2x + 10$

b. $(x - 4)x = (x)x - (4)x = x^2 - 4x$

c. $(1 + 2x)8 = (1)8 + (2x)8 = 8 + 16x$

d. $y(1 - y) = y(1) - y(y) = y - y^2$

Study the next problems carefully. Remember that a factor with a negative sign must multiply *each* term of an expression. Forgetting to distribute the negative sign to each term is a common error.

EXAMPLE 3 Using the Distributive Property

a. $-3(x + 4) = -3(x) + (-3)(4)$ — Distribute the -3.

$= -3x - 12$ — Simplify.

b. $(y + 5)(-4) = (y)(-4) + (5)(-4)$ — Distribute the -4.

$= -4y - 20$ — Simplify.

c. $-(6 - 3x) = (-1)(6 - 3x)$ — $-a = -1 \cdot a$

$= (-1)(6) - (-1)(3x)$ — Distribute the -1.

$= -6 + 3x$ — Simplify.

d. $(x - 1)(-9x) = (x)(-9x) - (1)(-9x)$ — Distribute the $-9x$.

$= -9x^2 + 9x$ — Simplify.

EXAMPLE 4 Mental Math Calculations

You are shopping for compact discs. You want to buy six compact discs for $11.95 each. Use the distributive property to calculate the total cost mentally.

SOLUTION

If you think of $11.95 as $12.00 − $.05, the mental math is easier.

$6(11.95) = 6(12 - 0.05)$ — Write 11.95 as a difference.

$= 6(12) - 6(0.05)$ — Use the distributive property.

$= 72 - 0.30$ — Find the products mentally.

$= 71.70$ — Find the difference mentally.

▶ The total cost of 6 compact discs at $11.95 each is $71.70.

GOAL 2 SIMPLIFYING BY COMBINING LIKE TERMS

In a term that is the product of a number and a variable, the number is the **coefficient** of the variable.

$$-x + 3y^2$$

-1 is the coefficient of x. 3 is the coefficient of y^2.

Like terms are terms in an expression that have the same variable raised to the same power. In the expression below, $5x$ and $-3x$ are like terms, but $5x$ and $-x^2$ are *not* like terms. The **constant terms** -4 and 2 are also like terms.

$$-x^2 + 5x + (-4) + (-3x) + 2$$

The distributive property allows you to *combine like terms* that have variables by adding coefficients. An expression is **simplified** if it has no grouping symbols and if all the like terms have been combined.

EXAMPLE 5 *Simplifying by Combining Like Terms*

a. $8x + 3x = (8 + 3)x$ Use the distributive property.

$= 11x$ Add coefficients.

b. $4x^2 + 2 - x^2 = 4x^2 - x^2 + 2$ Group like terms.

$= 3x^2 + 2$ Combine like terms.

c. $3 - 2(4 + x) = 3 + (-2)(4 + x)$ Rewrite as an addition expression.

$= 3 + [(-2)(4) + (-2)(x)]$ Distribute the -2.

$= 3 + (-8) + (-2x)$ Multiply.

$= -5 + (-2x) = -5 - 2x$ Combine like terms and simplify.

FOCUS ON APPLICATIONS

REAL LIFE **BUSES** Some cities offer discount fares to students who use public transportation to get to school.

EXAMPLE 6 *Using the Distributive Property to Simplify a Function*

GETTING TO SCHOOL It takes you 45 minutes to get to school. You spend t minutes walking to the bus stop, and the rest of the time riding the bus. You walk 0.06 mi/min and the bus travels 0.5 mi/min. The total distance you travel is given by the function $D = 0.06t + 0.5(45 - t)$. Simplify this function.

SOLUTION

$D = 0.06t + 0.5(45 - t)$ Write original function.

$= 0.06t + 22.5 - 0.5t$ Use the distributive property.

$= 0.06t - 0.5t + 22.5$ Group like terms.

$= -0.44t + 22.5$ Combine like terms.

▶ The total distance you travel can be given by $D = -0.44t + 22.5$.

GUIDED PRACTICE

Vocabulary Check ✓

1. In the expression $4x^2 - 7y - 8$, what is the coefficient of the x^2-term? What is the coefficient of the y-term?

2. What are the like terms in the expression $-6 - 3x^2 + 3y + 7 - 4y + 9x^2$?

Concept Check ✓

ERROR ANALYSIS **Describe the error. Then simplify the expression correctly.**

3. $-2(6.5 - 2.1) = 13 - 4.2$
$= 8.8$

4. $5 - 4(3 + x) = 5 - 12 + x$
$= -7 + x$

Skill Check ✓

Use the distributive property to rewrite the expression without parentheses.

5. $5(w - 8)$
6. $(y + 19)7$
7. $(12 - x)y$
8. $-4(u + 2)$
9. $(b - 6)\left(-\frac{5}{6}\right)$
10. $-\frac{2}{3}(t - 24)$

Use the distributive property and mental math to simplify the expression.

11. $6(1.15) = 6(1 + 0.15)$
$= \underline{?}$

12. $9(1.95) = 9(\underline{?} - \underline{?})$
$= \underline{?}$

Simplify the expression by combining like terms if possible.

13. $9x + 2$
14. $6x^2 - 4x^2$
15. $-3y - 2x$
16. $3x^2 + 4x + 8 - 7x^2$
17. $8t^2 - 2t + 5t - 4$
18. $-6w - 12 - 3w + 2x^2$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 798.

LOGICAL REASONING **Decide whether the statement is *true* or *false*. If false, rewrite the right-hand side of the equation so the statement is true.**

19. $3(2 + 7) \stackrel{?}{=} 3(2) + 7$
20. $(3 + 8)4 \stackrel{?}{=} 3(4) + 8(4)$
21. $7(15 - 6) \stackrel{?}{=} 7(15) - 7(6)$
22. $(9 - 2)13 \stackrel{?}{=} 9 - 2(13)$
23. $\frac{2}{9}\left(\frac{1}{3} - \frac{4}{9}\right) \stackrel{?}{=} \frac{2}{9}\left(\frac{1}{3}\right) - \frac{2}{9}\left(\frac{4}{9}\right)$
24. $-3.5(6.1 + 8.2) \stackrel{?}{=} -3.5(6.1) - 3.5(8.2)$

DISTRIBUTIVE PROPERTY **Use the distributive property to rewrite the expression without parentheses.**

25. $3(x + 4)$
26. $(w + 6)4$
27. $5(y - 2)$
28. $(7 - m)4$
29. $-(y - 9)$
30. $-3(r + 8)$
31. $-4(t - 8)$
32. $(x + 6)(-2)$
33. $x(x + 1)$
34. $(3 - y)y$
35. $-r(r - 9)$
36. $-s(7 + s)$
37. $2(3x - 1)$
38. $(4 + 3y)5$
39. $(2x - 4)(-3)$
40. $-9(a + 6)$
41. $4x(x + 8)$
42. $-2t(12 - t)$
43. $(3y - 2)5y$
44. $-2x(x - 8)$
45. $-9(-t - 3)$
46. $(6 - 3w)(-w^2)$
47. $5\left(\frac{1}{2}x - \frac{2}{3}\right)$
48. $-y(-y^2 + y)$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 81, 82
Example 2: Exs. 19–48
Example 3: Exs. 19–48
Example 4: Exs. 83–85
Example 5: Exs. 49–69
Example 6: Exs. 72–78

SIMPLIFYING EXPRESSIONS **Simplify the expression by combining like terms.**

49. $15x + (-4x)$ **50.** $-12y + 5y$ **51.** $-8b - 9b$

52. $5 - x + 2$ **53.** $-3 + y + 7$ **54.** $4 + a + a$

55. $1.3t - 2.1t$ **56.** $\frac{7}{9}w + \left(-\frac{2}{3}\right)w$ **57.** $107a - 208a$

58. $3x^2 + 2x^2 - 7$ **59.** $9x^3 - 4x^3 - 2$ **60.** $8b + 5 - 3b$

COMBINING LIKE TERMS **Apply the distributive property. Then simplify by combining like terms.**

61. $(3y + 1)(-2) + y$ **62.** $4(2 - a) - a$ **63.** $12s + (7 - s)2$

64. $(5 - 2x)(-x) + x^2$ **65.** $7x - 3x(x + 1)$ **66.** $-4(y + 2) - 6y$

67. $3t(t - 5) + 6t^2$ **68.** $-x^3 + 2x(x - x^2)$ **69.** $4w^2 - w(2w - 3)$

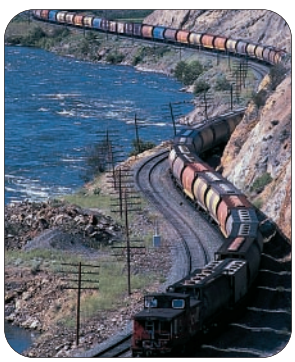
BUYING JEANS **You have \$58, and you want to buy a pair of jeans and a \$20 T-shirt. There is a 6% sales tax. If *x* represents the cost of the jeans, then the following inequality is a model that shows how much you can spend on the jeans.**

$$x + 20 + 0.06(x + 20) \leq 58$$

70. Simplify the left side of the inequality.

71. If the jeans cost \$35, can you buy both the T-shirt and the jeans?

FOCUS ON APPLICATIONS



REAL LIFE FREIGHT TRAINS A 150-car freight train is so heavy that it takes 1.5 miles to come to a complete stop if traveling 50 miles per hour.

FREIGHT TRAINS **A train with 150 freight cars is used to haul two types of grain. Each freight car can haul 97.3 tons of barley or 114 tons of corn. Let *n* represent the number of freight cars containing corn.**

72. Which function correctly represents the total weight the train can haul?

A. $W = 97.3(150 - n) + 114n$ **B.** $W = 97.3n + 114(150 - n)$

73. If 90 freight cars contain corn, what is the total weight the train is hauling?

74. If 72 freight cars contain barley, what is the total weight the train is hauling?

INVESTING MONEY **You receive \$5000. You decide to invest the money in a one-year bond paying 2% interest and in a one-year certificate of deposit paying 6% interest.**

75. Let m represent the amount of money invested in the one-year bond. Write a function that represents the total amount of money T that you have after one year. Simplify the function.

76. If you invest \$2000 in the one-year bond, how much money do you have after one year? What if you invest \$3000 in the one-year bond?

MOVING **The van that you are using to move can hold 16 moving boxes. Each box can hold 60 pounds of books or 15 pounds of clothes.**

77. Let b represent the number of boxes filled with books. Write a function that represents the total weight w of the boxes in the van.

78. Use the function you wrote in Exercise 77 to make an input-output table that shows the total weight of the boxes for each combination of boxes of books and clothes.

STUDENT HELP

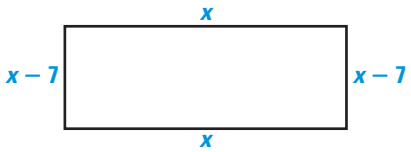
INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 75–76.

STUDENT HELP

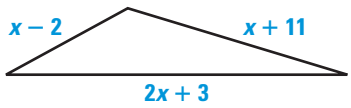
Skills Review
For help with perimeter and area, see p. 790.

GEOMETRY CONNECTION **Write and simplify an expression for the perimeter of the figure.**

79.

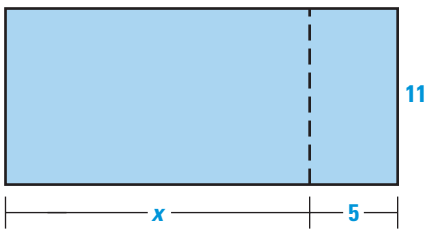


80.

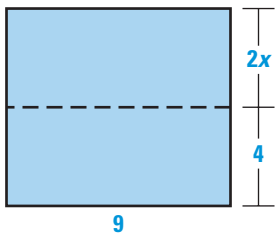


GEOMETRY CONNECTION **Write an expression modeling the area of the large rectangle as the product of its length and width. Then write another expression modeling this area as the sum of the areas of the two smaller rectangles. Simplify each expression.**

81.



82.



LEAVING A TIP **In Exercises 83–85, use the following information.** You and a friend decide to leave a 15% tip for restaurant service. You compute the tip, T, as $T = 0.15C$, where C represents the cost of the meal. Your friend claims that an easier way to mentally compute the tip is to calculate 10% of the cost of the meal plus one half of 10% of the cost of the meal.

83. Write an equation that represents your friend's method of computing the tip.

84. Simplify the equation. What property did you use to simplify the equation?

85. Will both methods give the same results? Explain.

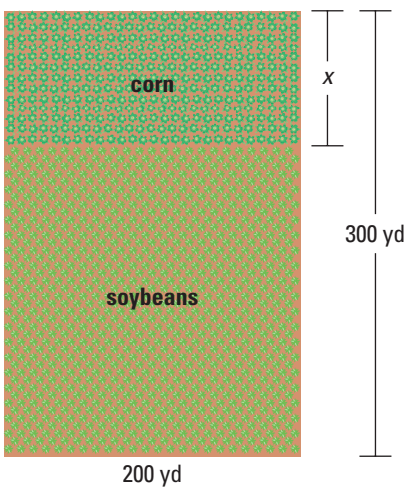
PLANTING PLAN **In Exercises 86–89, use the sketch below. You are a farmer planting corn and soybeans in a rectangular field measuring 200 yards by 300 yards. It costs $.06 per square yard to plant corn and $.05 per square yard to plant soybeans.**

86. Let x represent the width of the field you plant with corn. Write a function that represents the cost of planting the entire field. Simplify.

87. If the width of the field you plant with corn is 75 yards, what is the cost of planting the entire field?

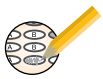
88. If the width of the field you plant with soybeans is 125 yards, what is the cost of planting the entire field?

89. You have $3400 to plant the entire field. Do you have enough money to plant a field of soybeans that is 90 yards wide?



90. *Writing* Write a problem like Exercise 81 that deals with the area of a rectangle. Explain how you can use the distributive property to model the total area of a rectangle as the sum of two rectangles.

Test Preparation



91. MULTI-STEP PROBLEM A customer of your flower shop wants to send flowers to 23 people. Each person will receive an \$11.99 "sunshine basket" or a \$16.99 "meadow bouquet."

a. Let s represent the number of people who will receive a sunshine basket. Which function can you use to find C, the total cost of sending flowers to all 23 people, depending on how many of each arrangement is sent?

Ⓐ $C = 16.99(23 - s) + 11.99s$ Ⓑ $C = 11.99s + 16.99(23)$

b. If 8 people receive a sunshine basket, what is the total cost of the flowers?

c. If 13 people receive a meadow bouquet, what is the total cost of the flowers?

d. CRITICAL THINKING If your customer can spend only \$300, what is the greatest number of people that can receive a meadow bouquet?

★ Challenge

92. LOGICAL REASONING You are tutoring a friend in algebra. After learning the distributive property, your friend attempts to apply this property to multiplication and gets $2(xy) = 2x \cdot 2y$.

Write a convincing argument to show your friend that this is incorrect.

93. LOGICAL REASONING Your friend does not understand how the product of a and $(b + c)$ is given by both $ab + ac$ and $ba + ca$.

EXTRA CHALLENGE
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Use the properties of multiplication to write a convincing argument to show your friend that both statements are correct.

MIXED REVIEW

RECIPROCALS **Find the reciprocal.** (Skills Review, p. 782)

94. $\frac{2}{7}$ **95.** $\frac{6}{11}$ **96.** 7 **97.** 1

98. $\frac{1}{121}$ **99.** 435 **100.** $4\frac{1}{2}$ **101.** $3\frac{3}{8}$

EXPRESSIONS WITH FRACTION BARS **Evaluate the expression.** (Review 1.3)

102. $\frac{10 \cdot 8}{4^2 + 4}$ **103.** $\frac{6^2 - 12}{3^2 + 15}$ **104.** $\frac{75 - 5^2}{11 + (3 \cdot 4)}$

105. $\frac{(3 \cdot 7) + 9}{2^3 + 5 - 3}$ **106.** $\frac{(2 + 5)^2}{3^2 - 2}$ **107.** $\frac{6 + 7^2}{3^3 - 9 - 7}$

EVALUATING EXPRESSIONS **Evaluate the expression.** (Review 2.2, 2.3)

108. $6 - (-8) - 11$ **109.** $4 - 8 - 3$ **110.** $6 + (-13) + (-5)$

111. $-7 + 9 - 8$ **112.** $20 + (-16) + (-3)$ **113.** $12.4 - 9.7 - (-6.1)$

QUIZ 2

Self-Test for Lessons 2.4–2.6

1. Find the sum and difference of the matrices. (Lesson 2.4)

$$\begin{bmatrix} 2 & -6 \\ -5 & 5 \\ -7 & 0 \end{bmatrix}, \begin{bmatrix} 3 & -7 \\ 1 & 4 \\ -2 & -8 \end{bmatrix}$$

Find the product. (Lesson 2.5)

2. $(-7)(9)$ **3.** $(3)(-7)$ **4.** $(-2)(-7)$ **5.** $(35)(-80)$

6. $(-15)\left(\frac{1}{5}\right)$ **7.** $(-1.8)(-6)$ **8.** $(11)(-5)(-2)$ **9.** $(-10)(-3)(9)$

Simplify the variable expression. (Lessons 2.5, 2.6)

10. $(-t)(-7.1)$ **11.** $(13)(-x)$ **12.** $(-5)(-b)(2b)(-b)$

13. $(-4)(-x)^3(x)\left(-\frac{1}{8}\right)$ **14.** $-28x + (-15x)$ **15.** $11y + (-9y)$

16. $-17t + 9t$ **17.** $-3(8 - b)$ **18.** $(-4y - 2)(-5)$

19. **SKYSCRAPERS** The heights (in feet and in meters) of the three tallest buildings in the United States are shown in the table. Organize the data in a matrix. (Lesson 2.4)

Building	Height (ft)	Height (m)
Sears Tower	1454	443
Empire State Building	1250	381
Aon Center	1136	346

▶ Source: Council on Tall Buildings and Urban Habitat

MATH & History — History of Negative Numbers

APPLICATION LINK www.mcdougallittell.com

THEN

THE 7TH CENTURY Hindu mathematician Brahmagupta explained operations with negative numbers. As late as the 16th century, many mathematicians still considered the idea of a number with a value less than zero absurd.

Hindu math manuscript

1. As early as 200 B.C., negative numbers were used in China. About how many years was it between the time the Chinese displayed a concept of negative numbers and when they were used in 628 A.D. in India?

NOW

TODAY, negative numbers are used in many ways, including financial statements, altitude measurements, temperature readings, golf scores, and atomic charges.

200 B.C. — The Chinese used red rods for positive quantities and black rods for negative quantities.

628 A.D. — Brahmagupta used negative numbers.

1700's — Algebraic use of the negative number became widespread.

NOW — A digital thermometer shows a negative value.