

2.3

Subtraction of Real Numbers

What you should learn

GOAL 1 Subtract real numbers using the subtraction rule.

GOAL 2 Use subtraction of real numbers to solve **real-life** problems such as finding the differences in stock market prices in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems such as analyzing data on visitors to a historical site in **Ex. 81**.



Native American dance at Nez Perce National Historical Park

GOAL 1 SUBTRACTING REAL NUMBERS

Some addition expressions can be evaluated using subtraction.

ADDITION PROBLEM

$$5 + (-3) = 2$$

$$9 + (-6) = 3$$

EQUIVALENT SUBTRACTION PROBLEM

$$5 - 3 = 2$$

$$9 - 6 = 3$$

Adding the opposite of a number is equivalent to subtracting the number.

SUBTRACTION RULE

To subtract b from a , add the opposite of b to a .

$$a - b = a + (-b)$$

Example: $3 - 5 = 3 + (-5)$

The result is the difference of a and b .

EXAMPLE 1 Using the Subtraction Rule

Find the difference.

a. $-4 - 3$

b. $10 - 11$

c. $11 - 10$

d. $-\frac{3}{2} - \left(-\frac{1}{2}\right)$

SOLUTION

$$\begin{aligned} \text{a. } -4 - 3 &= -4 + (-3) \\ &= -7 \end{aligned}$$

Add the opposite of 3.

Use rules of addition.

$$\begin{aligned} \text{b. } 10 - 11 &= 10 + (-11) \\ &= -1 \end{aligned}$$

Add the opposite of 11.

Use rules of addition.

$$\begin{aligned} \text{c. } 11 - 10 &= 11 + (-10) \\ &= 1 \end{aligned}$$

Add the opposite of 10.

Use rules of addition.

$$\begin{aligned} \text{d. } -\frac{3}{2} - \left(-\frac{1}{2}\right) &= -\frac{3}{2} + \frac{1}{2} \\ &= -1 \end{aligned}$$

Add the opposite of $-\frac{1}{2}$.

Use rules of addition.

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The commutative property of addition, $a + b = b + a$, shows that the order in which two numbers are added does not affect the result. But as you may have noticed in parts (b) and (c) of Example 1, the order in which two numbers are subtracted does affect the result. Subtraction is *not* commutative.

Expressions containing more than one subtraction can also be evaluated by “adding the opposite.” To do this, use the left-to-right rule for order of operations.

STUDENT HELP

INTERNET **HOMEWORK HELP**
Visit our Web site
www.mcdougallittell.com
for extra examples.

EXAMPLE 2 Evaluating Expressions with More than One Subtraction

Evaluate the expression $3 - (-4) - 2 + 8$.

SOLUTION

$$\begin{aligned} 3 - (-4) - 2 + 8 &= 3 + 4 + (-2) + 8 && \text{Add the opposites of } -4 \text{ and } 2. \\ &= 7 + (-2) + 8 && \text{Add 3 and 4.} \\ &= 5 + 8 && \text{Add 7 and } -2. \\ &= 13 && \text{Add 5 and 8.} \end{aligned}$$

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When an expression is written as a sum, the parts that are added are the **terms** of the expression. For instance, you can write the expression $5 - x$ as the sum $5 + (-x)$. The terms are 5 and $-x$. You can use the subtraction rule to find the terms of an expression.

EXAMPLE 3 Finding the Terms of an Expression

Find the terms of $-9 - 2x$.

SOLUTION Use the subtraction rule.

$$-9 - 2x = -9 + (-2x) \quad \text{Rewrite the difference as a sum.}$$

► In this form, you can see that the terms of the expression are -9 and $-2x$.

STUDENT HELP

Look Back
For help with
functions, see p. 46.

EXAMPLE 4 Evaluating a Function

Evaluate the function $y = -5 - x$ for these values of x : -2 , -1 , 0 , and 1 .

Organize your results in a table and describe the pattern.

SOLUTION

Input	Function	Output
$x = -2$	$y = -5 - (-2)$	$y = -3$
$x = -1$	$y = -5 - (-1)$	$y = -4$
$x = 0$	$y = -5 - (0)$	$y = -5$
$x = 1$	$y = -5 - (1)$	$y = -6$

► From the table, you can see that each time x increases by 1, y decreases by 1.

FOCUS ON APPLICATIONS



STOCK MARKET

The Dow Jones utilities average is an average price of the shares of 15 large gas and electric companies. The average is used to show the general trend in the stock prices of these types of companies.

GOAL 2 USING SUBTRACTION IN REAL LIFE

EXAMPLE 5 Subtracting Real Numbers

STOCK MARKET You are writing a report about the Dow Jones utilities average. The daily closing averages for one week are given in the table. Find the change in the closing average since the previous day to complete the table.

Date	Feb. 9	Feb. 10	Feb. 11	Feb. 12	Feb. 13
Closing average	266.13	268.08	267.11	269.37	268.02
Change	—	?	?	?	?

► Source: *Wall Street Journal*

SOLUTION

Subtract each day's closing average from the closing average for the previous day.

DATE	CLOSING AVERAGE	CHANGE
Feb. 9	266.13	—
Feb. 10	268.08	$268.08 - 266.13 = +1.95$
Feb. 11	267.11	$267.11 - 268.08 = -0.97$
Feb. 12	269.37	$269.37 - 267.11 = +2.26$
Feb. 13	268.02	$268.02 - 269.37 = -1.35$

EXAMPLE 6 Using a Calculator



Use the solution from Example 5 to answer the following questions.

- Write the changes you found in order.
- Find the difference of the greatest number and the least number.

SOLUTION

- Written in order from least to greatest, the numbers are as follows.
 $-1.35, -0.97, 1.95, 2.26$
- The difference of the greatest number and the least number is
 $2.26 - (-1.35)$.

With a calculator, you can find the difference.

KEYSTROKES

DISPLAY

2.26 $-$ 1.35 $+/-$ $=$ 3.61

- The greatest number is 3.61 more than the least number.

STUDENT HELP



KEYSTROKE HELP

Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

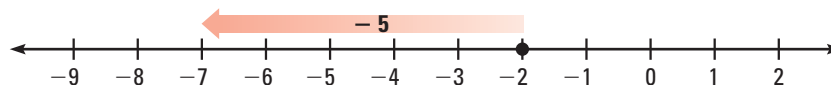
GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. Is $7x$ a term of the expression $4y^2 - 7x - 9$? Explain.

2. Use the number line to complete: $-2 - 5 = \underline{\quad}$.



3. Explain the steps you would take to evaluate the expression $5 - 7 - (-4)$.

Skill Check ✓

Use the subtraction rule to rewrite the subtraction expression as an equivalent addition expression. Then evaluate the expression.

4. $4 - 5$

5. $3 - (-8)$

6. $0 - 7$

7. $2 - (-3) - 6$

8. $-2.4 - 3$

9. $-3.6 - (-6)$

10. $\frac{1}{2} - \frac{1}{4}$

11. $\frac{2}{3} - \left(-\frac{1}{6}\right) - \frac{1}{3}$

Find the terms of the expression.

12. $12 - 5x$

13. $-7y^2 + 12y - 6$

14. $23 - 5w - 8y$

15. Evaluate the function $y = -x - 3$ for these values of x : $-2, -1, 0, 1$, and 2 .

Organize your results in a table. Describe the pattern.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 798.

DIFFERENCES Find the difference.

16. $4 - 9$

17. $6 - 13$

18. $8 - (-5)$

19. $-2 - (-7)$

20. $-11 - 8$

21. $24 - 39$

22. $137 - 355$

23. $-65 - (-59)$

24. $12.5 - 9.8$

25. $-3.2 - 1.7$

26. $5.4 - (-3.8)$

27. $-6.6 - (-16.1)$

28. $\frac{4}{3} - \frac{7}{3}$

29. $\frac{3}{4} - \left(-\frac{9}{4}\right)$

30. $-\frac{5}{8} - \frac{3}{4}$

31. $-\frac{9}{10} - \frac{1}{4}$

32. $6 - |-2|$

33. $15 - |-6|$

34. $|5| - 7.9$

35. $34.1 - |-57.2|$

EVALUATING EXPRESSIONS Evaluate the expression.

36. $2 - (-4) - 7$

37. $8 - 11 - (-6)$

38. $4 + (-3) - (-5)$

39. $3 - (-8) + (-9)$

40. $14 + (-7) - 12$

41. $-7 + 42 - 63$

42. $-8 - (-12) + 3$

43. $6 - 1 + 10 - (-8)$

44. $14 - 8 + 17 - (-23)$

45. $2.3 + (-9.1) - 1.2$

46. $1.3 + (-1.3) - 4.2$

47. $-8.5 - 3.9 + (-16.2)$

48. $8.4 - 5.2 - (-4.7)$

49. $-\frac{4}{9} - \frac{2}{3} + \left(-\frac{5}{6}\right)$

50. $\frac{7}{12} - \left(-\frac{3}{4}\right) + \left(-\frac{1}{8}\right)$

FINDING TERMS Find the terms of the expression.

51. $-4 - y$

52. $-x - 7$

53. $-3x + 6$

54. $9 - 28x$

55. $-9 + 4b$

56. $a + 3b - 5$

57. $x - y - 7$

58. $-3x + 5 - 8y$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–35
Example 2: Exs. 36–50
Example 3: Exs. 51–58
Example 4: Exs. 59–64
Example 5: Exs. 71–75
Example 6: Exs. 65–70

EVALUATING FUNCTIONS Evaluate the function for these values of x : -2 , -1 , 0 , and 1 . Organize your results in a table.

59. $y = x - 8$

60. $y = 12 - x$

61. $y = -x + 12.1$

62. $y = -8.5 - (-x)$

63. $y = 27 + x$

64. $y = -x + 13 - x$



EVALUATING EXPRESSIONS Evaluate the expression. Use estimation to check your answer.

65. $5.3 - (-2.5) - 4.7$

66. $8.9 - (-2.1) - 7.3$

67. $-4.89 + 2.69 - (-3.74)$

68. $-7.85 + 5.96 - (-2.49)$

69. $-13.87 - (-13.87) + 5.8$

70. $-15.7 + 0.01 + (-34.44)$

71. **SUBMARINE DEPTH** A submarine is at a depth of 725 feet below sea level. Five minutes later, it is at a depth of 450 feet below sea level. What is the change in depth of the submarine? Did it go up or down?

72. **GASOLINE PRICES** Last month the price of gasoline was \$1.19 per gallon. This month the price of gasoline is \$1.13 per gallon. What was the change in the price per gallon of gasoline?

GOLD PRICES IN LONDON At 9 A.M., an ounce of gold sells for \$287.56. At noon, gold sells for \$286.90 per ounce. At 4 P.M., the final price for the day is \$287.37 per ounce.

73. What is the change in the price per ounce of gold from 9 A.M. to noon?

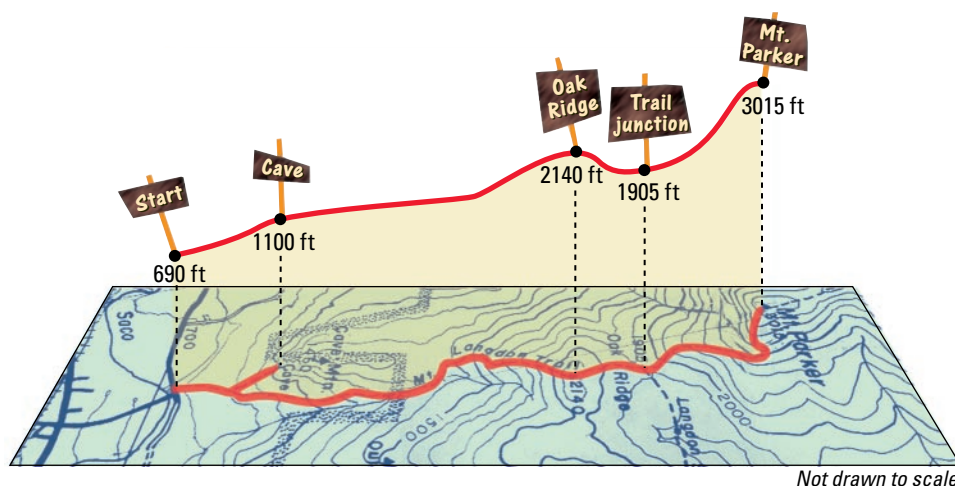
74. What is the change in the price per ounce of gold from noon to 4 P.M.?

75. What is the change in the price per ounce of gold from 9 A.M. to 4 P.M.?

HIKING Use the diagram for Exercises 76–78. It shows the elevation above sea level (in feet) of various points along the Mount Langdon Trail in the White Mountain National Forest in New Hampshire.



White Mountain National Forest, NH

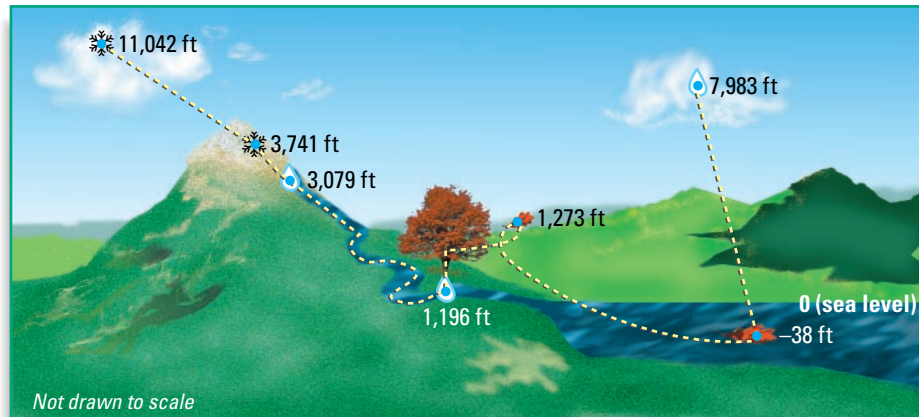


76. How much higher is the cave than the start of the trail?

77. Determine the change in elevation from each point to the next in the diagram.

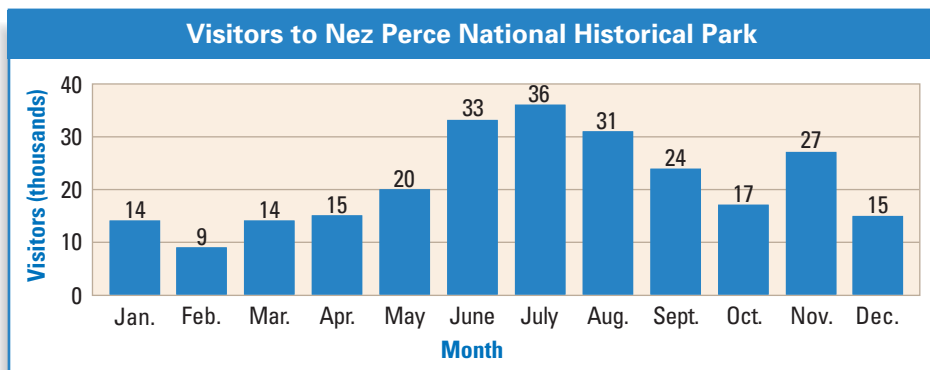
78. You hike along the trail, visit a cave, and then climb Oak Ridge. From there, you reach a trail junction and continue to the top of Mt. Parker. Find the total of these changes to determine your overall change in elevation.

SCIENCE CONNECTION In Exercises 79 and 80, use the diagram, which shows the journey of a water molecule from left to right.



79. Find the change in elevation from each point to the next point.
80. Write an expression using addition and subtraction that models the journey of the water molecule. Then evaluate the expression.
81. **MULTI-STEP PROBLEM** Use the bar graph, which shows the number of visitors to Nez Perce National Historical Park during 1997.

Test Preparation



DATA UPDATE of National Park Service data at www.mcdougallittell.com

- a. Find the change in the number of visitors from each month to the next month. Organize your results in a table.
- b. What does a negative value for a monthly change represent? What does a positive value represent?
- c. **Writing** Write a paragraph describing the pattern of visitors to Nez Perce National Historical Park during the year. Use mathematically descriptive words like *most*, *least*, *increase*, and *decrease*.

★ Challenge

LOGICAL REASONING Decide whether the statement is *true* or *false*. Use the subtraction rule or a number line to support your answer.

82. If you subtract a negative number from a positive number, the result is always a positive number.
83. If you subtract a positive number from a negative number, the result is always a negative number.


EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW


EVALUATING NUMERIC EXPRESSIONS Evaluate the expression. (Review 1.3)

84. $89 - 8 \cdot 5 - 27$ 85. $\frac{10}{3} - \frac{2}{3} \cdot 4 + 5$ 86. $12 \cdot 9 \div 6 - 13.5$
 87. $17 + 100 \div 25 - 5$ 88. $5 \cdot \frac{8}{9} - \frac{6}{9} + 51 \div 3$ 89. $13 + 11 \cdot 7 - 6 \div 3$
 90. $25 - \left[\frac{3}{10}(6 \cdot 5) - 2 \right]$ 91. $(27 \div 9) \div (7 - 5)$ 92. $[(12 \cdot 9) \div 6] - 13.5$

 **SPORTS** In Exercises 93 and 94, use the table, which shows the number of male and female participants in high school sports for three years. Based on the table, decide whether the statement is *true* or *false*. (Review 1.6)

High School Sports Participants (millions)		
Year	Male	Female
1994–95	3.54	2.24
1995–96	3.63	2.37
1996–97	3.71	2.24

► Source: National Federation of State High School Associations

93. There were more than six million total participants during 1994–1995.
 94. During the three-year period, female participation grew more than male participation.
 95.  **PROFIT AND LOSS** A landscaping company had a loss of \$5,126.55 in March. It then had a profit of \$2,943.21 in April, a profit of \$4,988.97 in May, and a loss of \$1,807.81 in June. Did the company make a profit during the four-month period? Explain. (Review 2.2 for 2.4)

QUIZ 1

Self-Test for Lessons 2.1–2.3


Write the numbers in increasing order. (Lesson 2.1)

1. 5.31, 5.04, -5.32 , -6.2 , 6.3, 5.3 2. -1.07 , 1.06, 1.16, -1.6 , 0.18, -0.28
 3. 7.3, -7.5 , $7\frac{2}{3}$, $-7\frac{1}{3}$, 7.5, $7\frac{1}{3}$ 4. $-6\frac{2}{5}$, 6.42, $\frac{33}{5}$, -6.3 , $-\frac{33}{5}$, 6.05

Evaluate the expression. (Lesson 2.1)

5. $|3.76|$ 6. $|-75|$ 7. $-|345|$
 8. $|-27.5|$ 9. $14 - |-7|$ 10. $|-75| - 7.6$

Evaluate the expression. (Lessons 2.2 and 2.3)

11. $-11 + 35$ 12. $29 - 501$ 13. $-17 - (-14)$
 14. $-8 + 12 + (-5)$ 15. $-32 - (-27) - 9$ 16. $35 - 0 - (-19)$
 17. $12 + (-1.2) + (-7)$ 18. $-143 - (-60) - 8$ 19. $14.1 + (-75.2) + 60.7$
 20.  **POOL DEPTH** The water in a pool is 47.3 inches deep on Monday. On Tuesday, 2.1 inches of depth is splashed out. On Wednesday, the depth decreases 11.3 inches due to a leak. On Thursday night, the leak is fixed and 12.9 inches of depth is added overnight. Write and evaluate an expression to find the depth of the water in the pool on Friday morning. (Lesson 2.3)