2.1 The Real Number Line

What you should learn

GOAL 1 Graph and compare real numbers using a number line.

GOAL 2 Find the opposite and the absolute value of a number in **real-life** applications such as the speed and velocity of an elevator in **Example 8**.

Why you should learn it

▼ To help you understand negative amounts in **real life**, such as "below zero" temperature in the wind-chill table in **Exs. 79–81**.

GOAL 1 GRAPHING REAL NUMBERS

The numbers used in this algebra book are **real numbers**. They can be pictured as points on a horizontal line called a **real number line**.

The point labeled 0 is the **origin**. Points to the left of zero represent **negative numbers** and points to the right of zero represent **positive numbers**. Zero is neither positive nor negative.

REAL NUMBER LINE

The scale marks are equally spaced and represent **integers**. An integer is either negative, zero, or positive.

…, −3, −2, −1,	0,	1, 2, 3, …
Negative integers	Zero	Positive integers

The three dots on each side in the list above indicate that the list continues in both directions without end. For instance, the integer immediately to the left of −3 is −4, read as *negative four*.

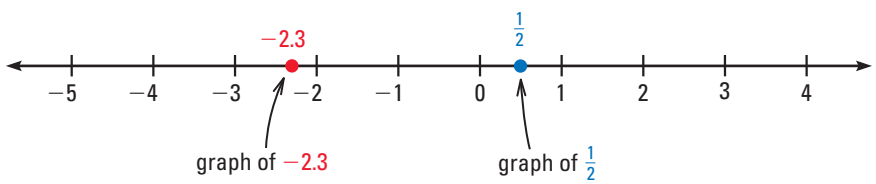
The real number line has points that represent fractions and decimals, as well as integers. The point that corresponds to a number is the **graph** of the number, and drawing the point is called graphing the number or **plotting** the point.

EXAMPLE 1 *Graphing Real Numbers*

Graph the numbers $\frac{1}{2}$ and −2.3 on a number line.

SOLUTION

The point that corresponds to $\frac{1}{2}$ is one half unit to the *right* of zero. The point that corresponds to −2.3 is 2.3 units to the *left* of zero.



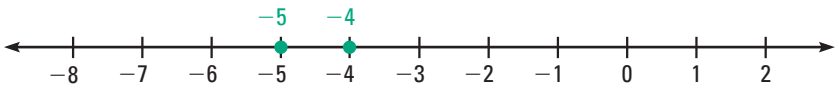
EXAMPLE 2 Comparing Real Numbers

STUDENT HELP

Skills Review
For help with comparing and ordering numbers, see pp. 779–780.

Graph -4 and -5 on a number line. Then write two inequalities that compare the two numbers.

SOLUTION



On the graph, -5 is to the left of -4, so -5 is **less than** -4.

$$-5 < -4$$

On the graph, -4 is to the right of -5, so -4 is **greater than** -5.

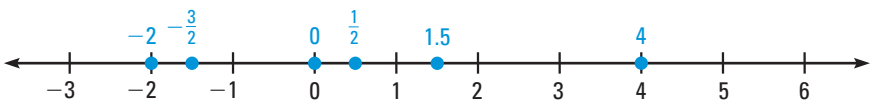
$$-4 > -5$$

EXAMPLE 3 Ordering Real Numbers

Write the following numbers in increasing order: $-2, 4, 0, 1.5, \frac{1}{2}, -\frac{3}{2}$.

SOLUTION

Graph the numbers on a number line.



▶ From the graph, you can see that the order is $-2, -\frac{3}{2}, 0, \frac{1}{2}, 1.5, 4$.

FOCUS ON APPLICATIONS

REAL LIFE NOME, ALASKA
Around December 21, the northern hemisphere has its shortest day and longest night. On that day, Nome has only 3 hours and 54 minutes of daylight.

INTERNET DATA UPDATE
Visit our Web site www.mcdougallittell.com

EXAMPLE 4 Comparing Real Numbers

NOME, ALASKA The table at the right shows the low temperatures recorded in Nome, Alaska, each day for five days in December.

Date	Temp. (°F)
Dec. 18	−10°F
Dec. 19	−11°F
Dec. 20	16°F
Dec. 21	3°F
Dec. 22	2°F

a. Which low temperature reading was the coldest?

b. Which dates had low temperatures above 10°F?

c. Which dates had low temperatures below −5°F?

SOLUTION

a. The coldest temperature was −11°F on December 19.

b. On December 20, the low temperature, 16°F, was above 10°F.

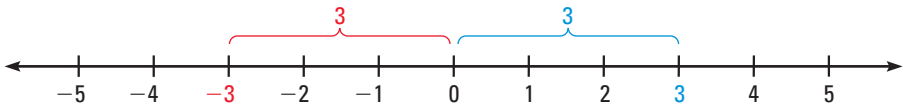
c. Low temperatures were below −5°F on December 18 (−10°F) and December 19 (−11°F).

GOAL 2 FINDING OPPOSITES AND ABSOLUTE VALUES

Two points that are the same distance from the origin but on opposite sides of the origin are **opposites**.

EXAMPLE 5 *Finding the Opposite of a Number*

The numbers -3 and 3 are opposites because each is 3 units from the origin.



The expression -3 can be stated as "negative 3" or "the opposite of 3." You can read the expression $-a$ as "the opposite of a." You should not assume that $-a$ is a negative number. For instance, if $a = -2$, then $-a = -(-2) = 2$.

The **absolute value** of a real number is the distance between the origin and the point representing the real number. The symbol $|a|$ represents the absolute value of a number a.

CONCEPT SUMMARY — THE ABSOLUTE VALUE OF A NUMBER

- If a is a positive number, then $|a| = a$. **Example:** $|3| = 3$
- If a is zero, then $|a| = 0$. **Example:** $|0| = 0$
- If a is a negative number, then $|a| = -a$. **Example:** $|-3| = 3$

The absolute value of a number is *never* negative. If the number a is negative, then its absolute value, $-a$, is positive. For instance, $|-6| = -(-6) = 6$.

EXAMPLE 6 *Finding Absolute Values*

Evaluate the expression.

a. $|2.3|$ **b.** $\left|-\frac{1}{2}\right|$ **c.** $-|-8|$

SOLUTION

a. $|2.3| = 2.3$ If a is positive, then $|a| = a$.

b. $\left|-\frac{1}{2}\right| = -\left(-\frac{1}{2}\right) = \frac{1}{2}$ If a is negative, then $|a| = -a$.

c. $-|-8| = -(8)$ The absolute value of -8 is 8.

$= -8$ Use definition of opposites.

EXAMPLE 7 Solving an Absolute Value Equation

STUDENT HELP

Look Back
For help with the solution of an equation, see p. 24.

Use mental math to solve the equation.

a. $|x| = 7$ **b.** $|x| = -5$

SOLUTION

a. Ask "what numbers are 7 units from the origin?" Both 7 and -7 are 7 units from the origin, so there are two solutions: 7 and -7.

b. The absolute value of a number is never negative, so there is no solution.

. . . . . .

Velocity indicates both speed and direction (up is positive and down is negative). The speed of an object is the absolute value of its velocity.

EXAMPLE 8 Finding Velocity and Speed

FOCUS ON APPLICATIONS

REAL LIFE **SPACE SHUTTLE**
The space shuttle's elevators have been designed to endure the stress of 320,000 gallons of water released at lift off to lessen heat and vibration.

INTERNET APPLICATION LINK
www.mcdougallittell.com

SPACE SHUTTLE ELEVATORS A space shuttle launch pad elevator drops at a rate of 10 feet per second. What are its velocity and speed?

SOLUTION

Velocity $= -10$ feet per second — **Motion is downward.**

Speed $= |-10| = 10$ feet per second — **Speed is positive.**

. . . . . .

In mathematics, to prove that a statement is true, you need to show that it is true for all examples. To prove that a statement is false, you need to show that it is *not* true for only one example, called a **counterexample**.

EXAMPLE 9 Using a Counterexample

Decide whether the statement is *true* or *false*. If it is false, give a counterexample.

a. The opposite of a number is *always* negative.

b. The absolute value of a number is *never* negative.

c. The expression $-a$ is *never* positive.

d. The expression $-a$ is *sometimes* greater than a.

SOLUTION

a. False. Counterexample: The opposite of -5 is 5, which is positive.

b. True, by definition.

c. False. Counterexample: If $a = -2$, then $-a = -(-2) = 2$, which is positive.

d. True. For instance, if $a = -8$, then $-a = -(-8) = 8$, and $8 > -8$.

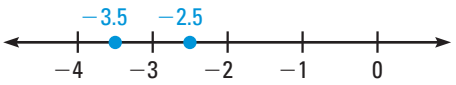
GUIDED PRACTICE

Vocabulary Check ✓

1. Write an inequality for the sentence: *Three is greater than negative five.*

Concept Check ✓

2. Copy the number line. Use it to explain why $-2.5 > -3.5$.



3. Use a counterexample to show that the following statement is false. *The opposite of a number is never positive.*

Skill Check ✓

Complete the statement using > or <.

4. $-3 \underline{?} -5$ **5.** $-3 \underline{?} 0$ **6.** $-2 \underline{?} -\frac{1}{2}$ **7.** $-8 \underline{?} 7$

Graph the numbers on a number line. Then write the numbers in increasing order.

8. 3, −5, 0 **9.** 2, 3, −4 **10.** $-\frac{1}{2}, -\frac{3}{4}, \frac{1}{4}$ **11.** −0.1, −1.1, −1

Evaluate the expression.

12. $|-12|$ **13.** $|4.1|$ **14.** $\left|-\frac{1}{5}\right|$ **15.** $|-103|$

Tell whether you would use a positive number or a negative number to represent the velocity.

16. The velocity of a rising rocket

17. The velocity of a falling raindrop

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 798.

WRITING INEQUALITIES **Graph the numbers on a number line. Then write two inequalities that compare the two numbers.**

18. −6 and 4 **19.** −6.4 and −6.3 **20.** −7 and 2

21. −0.1 and −0.11 **22.** 5.7 and −4.2 **23.** −2.8 and 3.7

24. -2.7 and $\frac{3}{4}$ **25.** -0.5 and $-\frac{1}{3}$ **26.** $-1\frac{5}{6}$ and $-1\frac{7}{9}$

ORDERING NUMBERS **Write the numbers in increasing order.**

27. 4.66, 0.7, 4.6, −1.8, 3, −0.66

28. −0.03, 0.2, 0, 2.0, −0.2, −0.02

29. $4.8, -2.6, 0, -3, \frac{1}{2}, -\frac{1}{2}$

30. $3\frac{1}{2}$, 3.4, 4.1, −5, −5.1, $-4\frac{1}{2}$

31. $7, -\frac{1}{2}, 2.4, -\frac{3}{4}, -5.8, \frac{1}{3}$

32. 6.03, −6.08, −6.1, −6.11, −6.02, 6.07

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 18–26
Example 2: Exs. 18–26
Example 3: Exs. 27–32
Example 4: Exs. 66–70
Example 5: Exs. 33–40

continued on p. 68

OPPOSITES **Find the opposite of the number.**

33. 8 **34.** −3 **35.** 3.8 **36.** −2.5

37. $-\frac{5}{6}$ **38.** $3\frac{4}{5}$ **39.** −2.01 **40.** $-\frac{1}{9}$

STUDENT HELP

HOMEWORK HELP

continued from p. 67

Example 6: Exs. 41–49
Example 7: Exs. 50–53
Example 8: Exs. 76–78
Example 9: Exs. 54–57

FINDING ABSOLUTE VALUES **Evaluate the expression.**

41. $|7|$ **42.** $|-4|$ **43.** $|-4.5|$

44. $\left|\frac{2}{3}\right|$ **45.** $\left|-\frac{4}{5}\right|$ **46.** $|0| + 2$

47. $|6.3| - 2$ **48.** $-\left|-\frac{8}{9}\right|$ **49.** $|-6.1| - 6.01$

SOLVE AN EQUATION **Use mental math to solve the equation.**

50. $|x| = 4$ **51.** $|x| = 0$ **52.** $x = |-3.8|$ **53.** $|-x| = 1$

COUNTEREXAMPLES **Decide whether the statement is *true* or *false*. If it is false, give a counterexample.**

54. The absolute value of a negative number is always negative.

55. The opposite of $-a$ is always positive.

56. The opposite of $|a|$ is never positive.

57. The value of $|-a|$ is sometimes negative.

LOGICAL REASONING **Complete the statement using $>$, $<$, $\geq$, or $\leq$.**

58. If $x > -4$, then -4 ? x.

59. If $3 \leq y$, then y ? 3.

60. If $m \leq 8$, then 8 ? m.

61. If $-7 \geq w$, then w ? -7.

ELEVATION **In Exercises 62–65, write a positive number, a negative number, or zero to represent the elevation of the location.**
Elevation is represented by comparing a location to sea level, which is given a value of zero. A location above sea level has a positive elevation, and a location below sea level has a negative elevation.

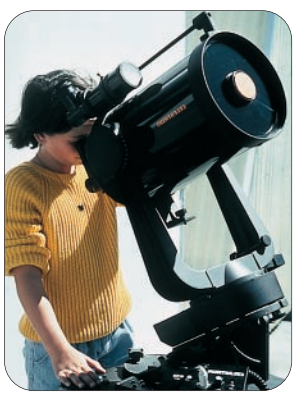
62. Granite Peak, Montana, 12,799 feet above sea level

63. New Orleans, Louisiana, 8 feet below sea level

64. Death Valley, California, 282 feet below sea level

65. Long Island Sound, Connecticut, sea level

FOCUS ON APPLICATIONS



REAL LIFE ASTRONOMY
A star may *appear* to be dim because of its great distance from Earth. Such a star may burn brighter than another star that looks very bright because it is closer to Earth.

ASTRONOMY **In Exercises 66–70, use the table showing the apparent magnitude of several stars.**
A star's brightness as it appears to a person on Earth is measured by its *apparent magnitude*. A bright star has a lesser apparent magnitude than a dim star.

66. Which star looks the brightest?

67. Which star looks the dimmest?

68. Which stars look dimmer than Altair?

69. Which stars look brighter than Procyon?

70. Write the stars in order from dimmest to brightest.

Star	Apparent magnitude
Canopus	−0.72
Procyon	0.38
Pollux	1.14
Altair	0.77
Spica	0.98
Vega	0.03
Regulus	1.35
Sirius	−1.46
Arcturus	−0.04
Deneb	1.25

FOCUS ON APPLICATIONS

REAL LIFE GOLF SCORES If you complete a round of golf in 68 strokes at a course with a par of 71 strokes, you have shot "3 under par," or −3.

GOLF In Exercises 71–75, use the table, which lists several players and their final scores at a 1998 Ladies Professional Golf Association tournament.

In golf the total score is given as the number of strokes above or below *par*, the expected score. If you are at "even par," your score is zero. The player with the lowest score wins.

Player	Score
Luciana Bemvenuti	+6
Liz Early	+15
Michelle Estill	−9
Hiromi Kobayashi	+1
Jenny Lidback	−4
Joan Pitcock	−11
Nancy Scranton	+4
Annika Sörenstam	−19

▶ Source: Ladies Professional Golf Association

71. Which player scored closest to par?

72. Which player scored farthest from par?

73. Which players scored above par?

74. Which players scored below par?

75. One of the players listed won the tournament. Who was it?

SPEED AND VELOCITY Find the speed and the velocity of the object.

76. A helicopter is descending for a landing at a rate of 6 feet per second.

77. An elevator in the Washington Monument in Washington, DC, climbs about 429 feet per minute. ▶ Source: National Park Service

78. A diver plunges to the ocean floor at a rate of 3 meters per second.

WIND CHILL The faster the wind blows, the colder you feel. In Exercises 79–81, use the wind-chill index table to identify the combination of temperature and wind speed that feels colder.

To use the table, find the temperature in the top row. Read down until you are in the row for the appropriate wind speed. For example, when the temperature is 0°F and the wind speed is 10 mi/h, the temperature feels like −22°F.

Wind-Chill Index					
Wind speed (mi/h)	Temperature (°F)				
0	−10	−5	0	5	10
5	−15	−10	−5	0	6
10	−34	−27	−22	−15	−9
15	−45	−38	−31	−25	−18
20	−53	−46	−39	−31	−24
25	−59	−51	−44	−36	−29
30	−64	−56	−49	−41	−33

79. A temperature of 10°F with a wind speed of 25 mi/h, or a temperature of −5°F with a wind speed of 10 mi/h

80. A temperature of 5°F with a wind speed of 15 mi/h, or a temperature of −5°F with a wind speed of 5 mi/h

81. A temperature of −10°F with a wind speed of 5 mi/h, or a temperature of 10°F with a wind speed of 20 mi/h

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 79–81.

82. *Writing* You are writing a quiz for this lesson. Write a problem to test whether students can write a set of real numbers in increasing order. Include positive and negative decimals and fractions. Show the answer to your problem on a number line.

Test Preparation

QUANTITATIVE COMPARISON In Exercises 83–85, choose the statement that is true about the given numbers.

(A) The number in column A is greater.

(B) The number in column B is greater.

(C) The two numbers are equal.

(D) The relationship cannot be determined from the given information.

	Column A	Column B
83.	$-\left\|\frac{2}{3}\right\|$	$\left\|-\frac{2}{3}\right\|$
84.	$x + \|2\|$	$x + \|-2\|$
85.	$\|x\| + 8$	$x + 8$

Challenge

86. LOGICAL REASONING Is the opposite of the absolute value of a number ever the same as the absolute value of the opposite of the number? In other words, is it ever true that $-|x| = |-x|$? Explain.

EXTRA CHALLENGE
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87. LOGICAL REASONING Is it *always*, *sometimes*, or *never* true that $|x| = |-x|$?

MIXED REVIEW

FRACTION OPERATIONS Find the sum. (Skills Review, p. 781)

88. $\frac{2}{4} + \frac{3}{8}$ **89.** $\frac{2}{3} + \frac{1}{6}$ **90.** $\frac{2}{5} + \frac{1}{4}$

91. $\frac{5}{8} + \frac{1}{3}$ **92.** $3\frac{2}{7} + 4\frac{1}{2}$ **93.** $1\frac{7}{9} + 4\frac{3}{7}$

EVALUATING EXPRESSIONS Evaluate the expression for the given value(s) of the variable(s). (Review 1.1)

94. $5x + 3$ when $x = 2$ **95.** $2a - 7$ when $a = 6$

96. $3y + 12$ when $y = 0$ **97.** $6b - 39 + c$ when $b = 15$ and $c = 2$

98. $\frac{5}{6}a - b$ when $a = 6$ and $b = 5$ **99.** $\frac{x}{5} - 2y$ when $x = 12$ and $y = \frac{4}{5}$

TRANSLATING VERBAL SENTENCES Write the verbal sentence as an equation or an inequality. (Review 1.5)

100. Five less than z is eight.

101. Eight more than r is seventeen.

102. Nine more than three fourths of y is less than six times w.