EXPLORING DATA AND STATISTICS

12.7

Trigonometric Ratios

What *you should learn*

GOAL 1 Use the sine, cosine, and tangent of an angle.

GOAL 2 Use trigonometric ratios in **real-life** problems, such as finding cloud height in **Ex. 20**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the height of a parasailer in **Example 3**.

REAL LIFE

GOAL 1 USING TRIGONOMETRIC RATIOS

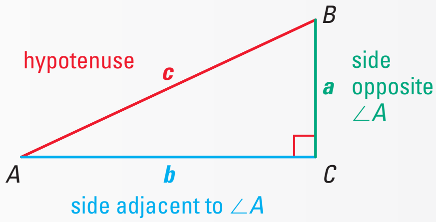
In the ancient Greek language, the word *trigonometry* means *measurement of triangles.* A **trigonometric ratio** is a ratio of the lengths of two sides of a right triangle. The three basic trigonometric ratios are **sine**, **cosine**, and **tangent**. You can abbreviate them as *sin*, *cos*, and *tan*.

TRIGONOMETRIC RATIOS

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$



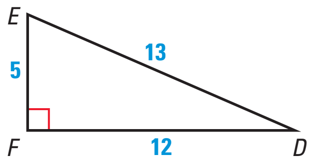
Because all right triangles with a given measure for $\angle A$ are similar, the value of a trigonometric ratio depends only on the measure of $\angle A$. It does not depend on the triangle's size.

EXAMPLE 1 *Finding Trigonometric Ratios*

For $\triangle DEF$, find the sine, the cosine, and the tangent of the angle.

a. $\angle D$

b. $\angle E$



SOLUTION

a. For $\angle D$, the opposite side is 5, and the adjacent side is 12. The hypotenuse is 13.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\tan D = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}$$

b. For $\angle E$, the opposite side is 12, and the adjacent side is 5. The hypotenuse is 13.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\tan E = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{5}$$

STUDENT HELP

Trig Table
For a table of trigonometric ratios, see p. 812.

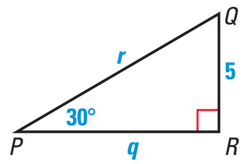
SOLVING RIGHT TRIANGLES If you know the measure of one angle and the length of one side of a right triangle, then you can use trigonometric ratios and a calculator or a table to find the lengths of the other two sides. This is called *solving a right triangle.*

EXAMPLE 2 *Solving a Right Triangle*

For $\triangle PQR$, $p = 5$ and the measure of $\angle P$ is $30°$.

a. Find the length q.

b. Find the length r.



SOLUTION

a. You are given the side opposite $\angle P$, and you need to find the length of the adjacent side.

$\tan P = \dfrac{\text{opposite}}{\text{adjacent}}$	**Definition of tangent**
$\tan 30° = \dfrac{5}{q}$	**Substitute 5 for p and 30° for $\angle P$.**
$q = \dfrac{5}{\tan 30°}$	**Solve for q.**
$q \approx \dfrac{5}{0.5774}$	**Use a calculator or a table.**
$q \approx 8.66$	**Simplify.**

▶ The length q is about 8.66 units.

b. You are given the side opposite $\angle P$ and you need to find the length of the hypotenuse.

$\sin P = \dfrac{\text{opposite}}{\text{hypotenuse}}$	**Definition of sine**
$\sin 30° = \dfrac{5}{r}$	**Substitute 5 for p and 30° for $\angle P$.**
$r = \dfrac{5}{\sin 30°}$	**Solve for r.**
$r = \dfrac{5}{0.5}$	**Use a calculator or a table.**
$r = 10$	**Simplify.**

▶ The length r is 10 units.

STUDENT HELP

Study Tip
When you use a calculator to find trigonometric ratios, be sure that the calculator is set in *degree* mode.

✓**CHECK** You can use the Pythagorean theorem to check that the results are reasonable. Because the value of q was rounded, the check will not be exact.

$p^2 + q^2 = r^2$	**Pythagorean theorem**
$5^2 + 8.66^2 \stackrel{?}{=} 10^2$	**Substitute for p, q, and r.**
$99.996 \approx 100$	**Side lengths are approximately correct.**

GOAL 2 USING TRIGONOMETRIC RATIOS IN REAL LIFE

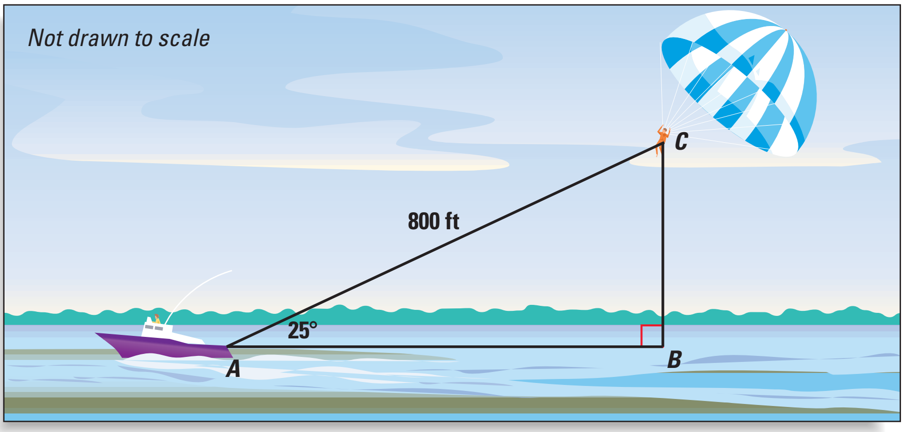
REAL LIFE
Parasailing

EXAMPLE 3 *Solving a Right Triangle*

A boat is pulling a parasailer. The line to the parasailer is 800 feet long. The angle between the line and the water is about 25°.

a. How high is the parasailer?

b. How far back is the parasailer from the boat?



SOLUTION

a. Find the length of the side opposite $\angle A$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$ **Definition of sine**

$$\sin 25° = \frac{a}{800}$$ **Substitute 800 for *c* and 25° for $\angle A$.**

$$800 \cdot \sin 25° \approx a$$ **Multiply each side by 800.**

$$0.4226(800) \approx a$$ **Use a calculator or a table.**

$$338.08 \approx a$$ **Simplify.**

The parasailer is about 338 feet above the water.

b. Find the length of the side adjacent to $\angle A$.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$ **Definition of cosine**

$$\cos 25° = \frac{b}{800}$$ **Substitute 800 for *c* and 25° for $\angle A$.**

$$800 \cdot \cos 25° = b$$ **Multiply each side by 800.**

$$0.9063(800) \approx b$$ **Use a calculator or a table.**

$$725.04 \approx b$$ **Simplify.**

The parasailer is about 725 feet back from the boat.

Guided Practice

Vocabulary Check ✓

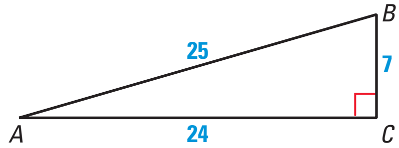
1. Define *trigonometric ratio*.

Concept Check ✓

2. Is it *true* or *false* that for any right triangle with a 30° angle, sin 30° = 0.5? Explain.

Skill Check ✓

In Exercises 3–5, use $\triangle ABC$ at the right.



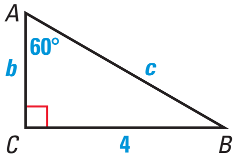
3. Find the sine of $\angle A$.

4. Find the cosine of $\angle A$.

5. Find the tangent of $\angle A$.

Find the missing lengths of the sides of the triangle. Round to the nearest hundredth. Use the Pythagorean theorem to check the result.

6.



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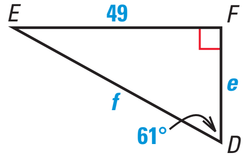

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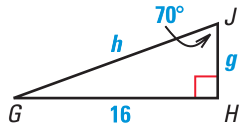


9. **PARASAILING** In Example 3, suppose the angle is 35°. How high is the parasailer? How far back is the parasailer?

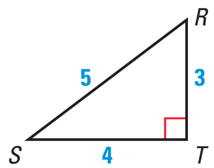
Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 808.

FINDING TRIGONOMETRIC RATIOS **Find the sine, the cosine, and the tangent of $\angle R$ and of $\angle S$.**

10.



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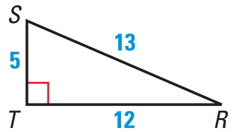

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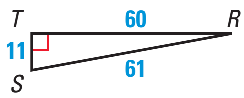
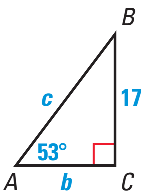


SOLVING RIGHT TRIANGLES **Find the missing lengths of the sides of the triangle. Round your answers to the nearest hundredth. Use the Pythagorean theorem to check.**

13.



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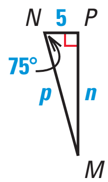

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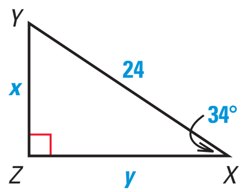

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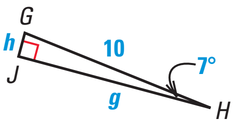

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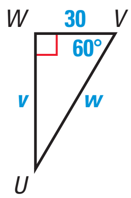

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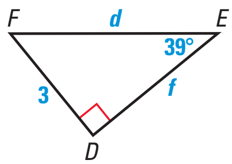
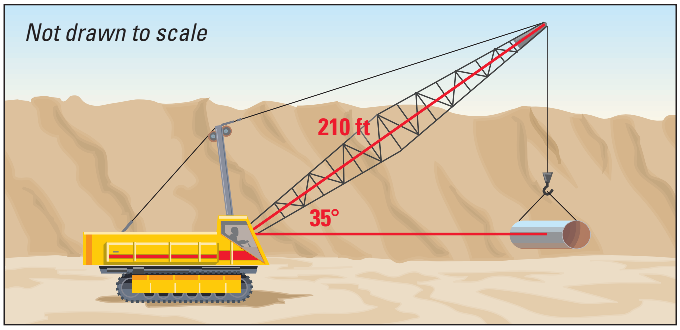


STUDENT HELP

HOMEWORK HELP

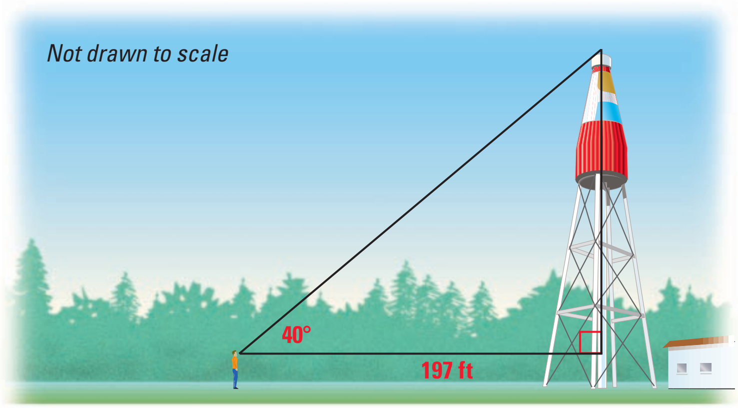
Example 1: Exs. 10–12
Example 2: Exs. 13–18
Example 3: Exs. 19, 20

19. **CRANE OPERATOR** You are a crane operator using a crane with a 210-foot boom. You lift a load so that the load is directly in front of you. You estimate that the angle between the boom and the line between you and the load is 35°. About how far are you from the load?

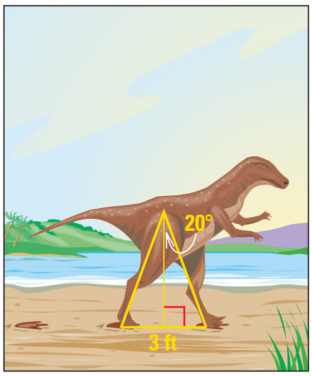


20. **CLOUD HEIGHT** You are doing an experiment for your science class. You shine a spotlight from the ground straight up onto a cloud to measure the height of the cloud. Your friend stands 500 feet from the spotlight. She estimates that the angle formed between the ground and the line from her feet to the cloud is 25°. Draw a sketch to model the situation. Then find the height of the cloud.

21. **COLLINSVILLE CATSUP BOTTLE** You are standing 197 feet from the base of the world's largest catsup bottle located in Collinsville, Illinois. You estimate that the angle between your eye level and the line from your eyes to the top of the bottle is 40°. If you are 5 feet tall, about how high is the top of the bottle?



22. **DINOSAUR TRACKS** You are a paleontologist. You are estimating the length of a dinosaur's hind legs based on its track marks. You measure the pace length as three feet. The pace length represents the base of an isosceles triangle, where the equal sides are the dinosaur's hind legs that enclose the walking angle. Assuming the walking angle is 40°, what is the length of the dinosaur's hind legs?



23. **VIEWING ANGLE** A car is traveling on a level road toward a mountain two kilometers high. The angle of elevation from the car to the top of the mountain changes from 6° to 15°. How far has the car traveled?

Test Preparation

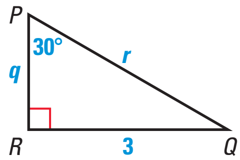
24. **MULTI-STEP PROBLEM** Use the triangle below.

a. Find the length q.

b. Describe two methods for finding the length r.

c. Find the length r.

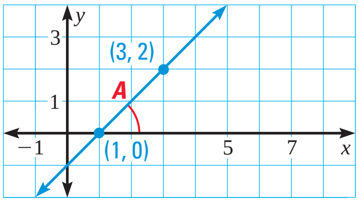
d. *Writing* Which method did you use in part (c)? Tell why you chose that method.



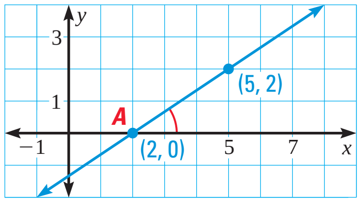
★ Challenge

EXTENSION: SLOPE AND ANGLE **In Exercises 25 and 26, find the slope of the line that passes through the points. Explain how the slope of the line is related to $\angle A$.**

25.



26.



EXTRA CHALLENGE
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27. **LOGICAL REASONING** A line with a positive slope passes through the origin, making a 60° angle with the positive x-axis. Write an equation of the line.

MIXED REVIEW

COUNTEREXAMPLES **Decide whether the statement is *true* or *false*. If it is false, give a counterexample.** (Review 2.1 for 12.8)

28. The absolute value of a number is always positive.

29. The opposite of a number is always positive.

USING THE DISTRIBUTIVE PROPERTY **Use the distributive property to simplify the expression.** (Review 2.6 for 12.8)

30. $6(w - 3)$

31. $-p(p + 1)$

32. $-(x - 8)$

33. $(x + 3)x$

34. $(x - 2)5x$

35. $(4 + x)(-6x)$

SUBTRACTING VERTICALLY **Use a vertical format to subtract the second polynomial from the first polynomial.** (Review 10.1)

36. $6x^2 - 3x + 2,\ 2x^2 + x + 7$

37. $4x^3 + 3x^2 + 8x + 6,\ 2x^3 - 3x^2 - 7x$

38. $10x^3 + 15,\ 17x^3 - 4x + 5$

39. $-2x^3 + 5x^2 - x + 8,\ -2x^3 + 3x - 4$

SIMPLIFYING RATIONAL EXPRESSIONS **Simplify the expression.** (Review 11.6)

40. $\dfrac{3}{x} + \dfrac{x + 9}{x}$

41. $\dfrac{8}{4a + 1} - \dfrac{5}{4a + 1}$

42. $\dfrac{2}{2x} + \dfrac{12}{x}$

43. $\dfrac{5}{4x} - \dfrac{7}{3x}$

44. $\dfrac{2x}{x + 1} + \dfrac{5}{x + 3}$

45. $\dfrac{6x}{x + 1} - \dfrac{2x + 4}{x + 1}$