12.5 The Pythagorean Theorem and Its Converse

What you should learn

GOAL 1 Use the Pythagorean theorem and its converse.

GOAL 2 Use the Pythagorean theorem and its converse in **real-life** problems such as finding distance on a baseball field in **Example 4**.

Why you should learn it

▼ To solve **real-life** problems such as installing guy wires in **Ex. 44**.

GOAL 1 USING THE THEOREM AND ITS CONVERSE

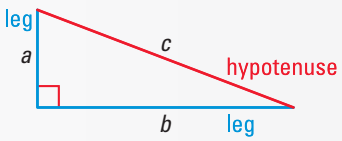
A *theorem* is a statement that can be proven to be true. The activity on page 737 uses area to explore the *Pythagorean theorem*, but does not prove the theorem.

The **Pythagorean theorem** states a relationship among the sides of a right triangle. The **hypotenuse** is the side opposite the right angle. The other two sides are the **legs**. The theorem is named after the Greek mathematician Pythagoras. However, there are records that suggest earlier use of the basic principle in northern Africa, in Babylonia, and in India.

THE PYTHAGOREAN THEOREM

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs a and b equals the square of the length of the hypotenuse c.

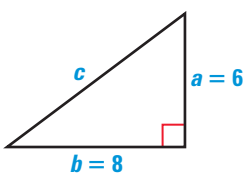
$$a^2 + b^2 = c^2$$



EXAMPLE 1 *Using the Pythagorean Theorem*

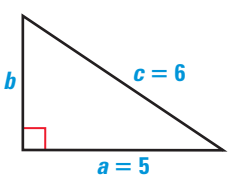
a. Given $a = 6$ and $b = 8$, find c.

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$100 = c^2$$
$$\sqrt{100} = c$$
$$10 = c$$



b. Given $a = 5$ and $c = 6$, find b.

$$a^2 + b^2 = c^2$$
$$5^2 + b^2 = 6^2$$
$$b^2 = 6^2 - 5^2$$
$$b^2 = 11$$
$$b = \sqrt{11}, \text{ or about } 3.32$$



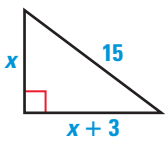
EXAMPLE 2 Using the Pythagorean Theorem

A right triangle has one leg that is 3 inches longer than the other leg. The hypotenuse is 15 inches. Find the missing lengths.

PROBLEM SOLVING STRATEGY

SOLUTION

DRAW A DIAGRAM Sketch a right triangle and label the sides. Let x be the length of the shorter leg. Use the Pythagorean theorem to solve for x.



$$a^2 + b^2 = c^2 \quad \text{Write Pythagorean theorem.}$$

$$x^2 + (x + 3)^2 = 15^2 \quad \text{Substitute for } a, b, \text{ and } c.$$

$$x^2 + x^2 + 6x + 9 = 225 \quad \text{Simplify.}$$

$$2x^2 + 6x - 216 = 0 \quad \text{Write in standard form.}$$

$$2(x - 9)(x + 12) = 0 \quad \text{Factor.}$$

$$x = 9 \text{ or } x = -12 \quad \text{Zero-product property}$$

▶ Length is positive. The sides have lengths 9 inches and $9 + 3 = 12$ inches.

STUDENT HELP

Look Back
For help with if-then statements, see p. 189.

LOGICAL REASONING In mathematics an if-then statement is a statement of the form "If p, then q," where p is the **hypothesis** and q is the **conclusion**.

The **converse** of the statement "If p, then q" is the related statement "If q, then p," in which the hypothesis and conclusion are interchanged. The converse of a true statement may be false. However, the converse of the Pythagorean theorem is true.

CONVERSE OF THE PYTHAGOREAN THEOREM

If a triangle has side lengths a, b, and c such that $a^2 + b^2 = c^2$, then the triangle is a right triangle.

EXAMPLE 3 Determining Right Triangles

Determine whether the given lengths are sides of a right triangle.

a. 11.9, 12.0, 16.9 **b.** 5, 11, 12

SOLUTION Use the converse of the Pythagorean theorem.

a. The lengths are sides of a right triangle because

$$11.9^2 + 12.0^2 = 141.61 + 144 = 285.61 = 16.9^2.$$

b. The lengths are not sides of a right triangle because

$$5^2 + 11^2 = 25 + 121 = 146 \neq 12^2.$$

STUDENT HELP

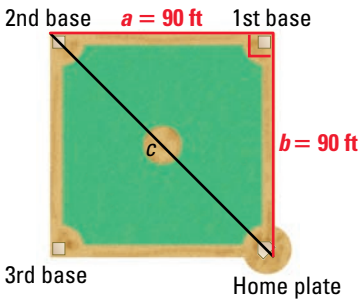
Study Tip
In a right triangle the hypotenuse is always the longest side.

GOAL 2 USING THE PYTHAGOREAN THEOREM IN REAL LIFE

REAL LIFE Baseball

EXAMPLE 4 *Using the Pythagorean Theorem*

The length of each side of a baseball diamond is 90 feet. What is the distance from home plate to second base?



SOLUTION

The diagonal from home plate to second base is the hypotenuse c of a right triangle. The length of each leg is 90 feet.

$c^2 = a^2 + b^2$ **Write Pythagorean theorem.**

$c^2 = 90^2 + 90^2$ **Substitute 90 for a and for b.**

$c^2 = 8100 + 8100$ **Multiply.**

$c^2 = 16{,}200$ **Add.**

$c = \sqrt{16{,}200}$ **Find positive square root of each side.**

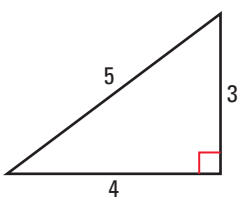
$c = 90\sqrt{2}$ **Simplify.**

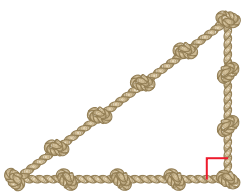
$c \approx 127.3$ **Use a calculator.**

▶ The distance from home plate to second base is about 127.3 feet.

EXAMPLE 5 *Using the Pythagorean Converse*

You can take a rope and tie 12 equally spaced knots in it. You can then use the rope to check that a corner is a right angle. Why does this method work?





SOLUTION

You can use the rope to form a triangle with a hypotenuse length of 5 and leg lengths of 3 and 4. Use the converse of the Pythagorean theorem to check.

$5^2 \stackrel{?}{=} 3^2 + 4^2$

$25 \stackrel{?}{=} 9 + 16$

$25 = 25$ ✓

▶ Because you can use the knots to form the sides of a right triangle, one angle of the triangle must measure 90°. So you can check with the rope that a corner is a right angle.

GUIDED PRACTICE

Vocabulary Check ✓

1. The two sides of a right triangle that are not the hypotenuse are the ? .

Concept Check ✓

2. State the hypothesis and the conclusion of the statement "If x is an even number, then x^2 is an even number."

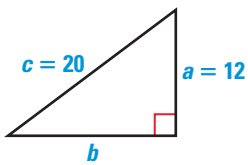
3. Explain how you can use the converse of the Pythagorean theorem to tell whether three given lengths can be sides of a right triangle.

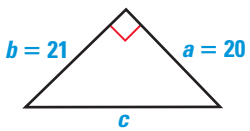
Skill Check ✓

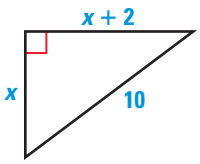
Find the missing length of the right triangle if *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse.

4. $a = 7, b = 24$
5. $a = 5, c = 13$
6. $b = 15, c = 17$
7. $a = 9, c = 41$
8. $b = 11, c = 61$
9. $a = 12, b = 35$

Find each missing length.

10. 

11. 

12. 

PRACTICE AND APPLICATIONS

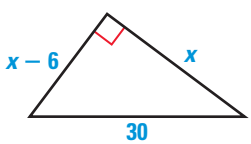
STUDENT HELP

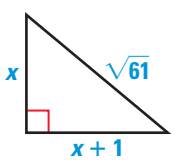
Extra Practice to help you master skills is on p. 808.

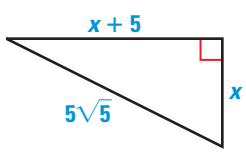
USING THE PYTHAGOREAN THEOREM **Find the missing length of the right triangle if *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse.**

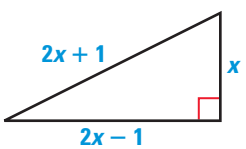
13. $a = 3, b = 4$
14. $a = 5, c = 10$
15. $b = 3, c = 7$
16. $a = 10, b = 24$
17. $b = 9, c = 16$
18. $a = 14, c = 21$

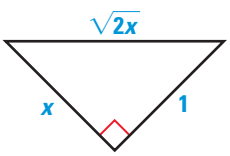
MISSING LENGTH **Find each missing length.**

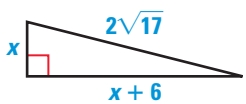
19. 

20. 

21. 

22. 

23. 

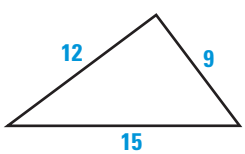
24. 

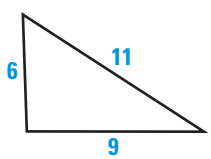
STUDENT HELP

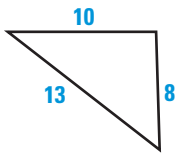
HOMEWORK HELP

Example 1: Exs. 13–18
Example 2: Exs. 19–24
Example 3: Exs. 25–33
Example 4: Exs. 39, 41–44
Example 5: Ex. 40

DETERMINING RIGHT TRIANGLES **Determine whether the given lengths are sides of a right triangle. Explain your reasoning.**

25. 

26. 

27. 

DETERMINING RIGHT TRIANGLES **Determine whether the given lengths are sides of a right triangle. Explain your reasoning.**

28. 2, 10, 11

29. 15, 20, 25

30. 5, 12, 13

31. 11, 60, 61

32. 7, 24, 26

33. 9.9, 2, 10.1

IF-THEN STATEMENTS **In Exercises 34–38, state the hypothesis and the conclusion of the statement.**

34. If today is Tuesday, then yesterday was Monday.

35. If a polygon is a square, then it is a parallelogram.

36. If $\frac{x}{3} = -15$, then $x = -45$.

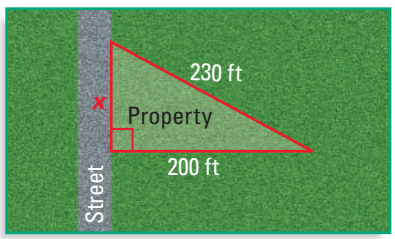
37. If the area of a square is 25 square feet, then the length of a side is 5 feet.

38. If a triangle has sides that are 8 inches and 9 inches long, then the length of the third side is greater than 1 inch and less than 17 inches.

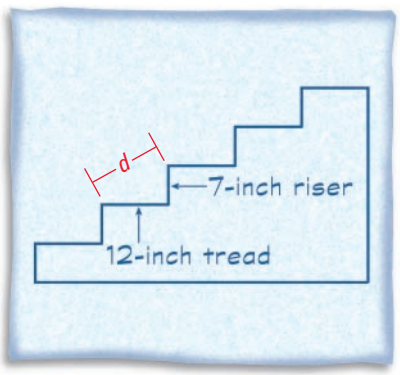
39. **DIAGONAL OF A FIELD** A field hockey field is a rectangle 60 yards by 100 yards. What is the length of the diagonal from one corner of the field to the opposite corner?

40. *Writing* You have a rope with 30 equally spaced knots in it. How can you use the rope to check that a corner is a right angle?

41. **SURVEYING LAND** You are surveying a triangular-shaped piece of land. You have measured and recorded two lengths on a plot plan. What is the length of the property along the street? Round your answer to the nearest hundredth.



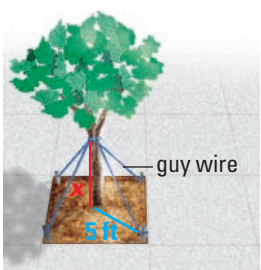
DESIGNING A STAIRCASE **In Exercises 42 and 43, you are building the staircase shown at the right.**



42. Find the distance d between the edges of each step.

43. The staircase will also have a handrail that is as long as the distance between the edge of the first step and the edge of the top step. How long is the handrail?

44. **PLANTING A NEW TREE** You have just planted a new tree. To support the tree in bad weather, you attach guy wires from the trunk of the tree to stakes in the ground. You cut 30 feet of wire into four equal lengths to make the guy wires. You attach four guy wires, evenly spaced around the tree. You put the stakes in the ground five feet from the base of the trunk. Approximately how far up the trunk should you attach the guy wires?



Test Preparation

45. MULTI-STEP PROBLEM Amalia and Cindy leave from the same point at the same time. Cindy bicycles east at a rate that is 2 miles per hour faster than Amalia, who bicycles south. After one hour they are 10 miles apart.

a. Let r represent Amalia's rate in miles per hour. Write an expression for the distance each girl has traveled in one hour.

b. Draw and label a diagram of the situation.

c. Use the Pythagorean theorem to find how fast each person is traveling.

d. *Writing* Which method did you use to solve the quadratic equation? Give a reason for your choice.

Challenge

In Exercises 46–48, use the fact that a Pythagorean triple is a group of three integers, such as 3, 4, and 5, that could be the lengths of the sides of a right triangle.

46. Find two other Pythagorean triples that are not multiples of 3, 4, 5 or of each other.

47. Notice that $3 \cdot 4 \cdot 5 = 60$. Is the product of the three numbers in each Pythagorean triple evenly divisible by 3? by 4? by 5?

EXTRA CHALLENGE
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48. Do your observations in Exercise 47 suggest a statement about Pythagorean triples that might be true? Explain.

MIXED REVIEW

PLOTTING ORDERED PAIRS **Plot the ordered pairs in a coordinate plane.** (Review 4.1 for 12.6)

49. $(2, 5), (0, -1), (3, 1)$

50. $(2, -5), (2, 4), (-3, 0)$

51. $(-1, -2), (-4, 5), (0, 2)$

52. $(1, 4), (-2, -1), (3, -1)$

53. $(2, 3), (-2, -3), (4, -2)$

54. $(1, 3), (-3, 1), (3, -4)$

FINDING INTERCEPTS **Find the x-intercepts of the graph of the equation.** (Review 9.5)

55. $y = x^2 + 2x + 15$

56. $y = x^2 + 8x + 12$

57. $y = x^2 + x - 10$

58. $y = x^2 + 8x + 16$

59. $y = x^2 + 3x + 1$

60. $y = x^2 - 8x - 11$

61. $y = x^2 + 8x - 10$

62. $y = 3x^2 + 20x + 1$

63. $y = -x^2 + 4x + 1$

FACTORING SPECIAL PRODUCTS **Factor the expression.** (Review 10.7)

64. $x^2 - 64$

65. $16x^2 - 25$

66. $x^2 + 18x + 81$

67. $7x^2 - 28x + 28$

68. $45x^2 - 60x + 20$

69. $-48x^2 + 216x - 243$

70. GEOMETRIC MEAN The geometric mean of 16 and a is 32. What is the value of a? (Review 12.3)

71. POSTAGE STAMPS In 1960, a first-class United States postage stamp cost \$.04. In 1999, a first-class United States postage stamp cost \$.33. Write a compound inequality that represents the different prices that a postage stamp could have cost between 1960 and 1999. (Review 6.3)

QUIZ 2

Self-Test for Lessons 12.4 and 12.5

Solve the equation by completing the square. (Lesson 12.4)

1. $2x^2 - 6x - 15 = 5$ **2.** $4x^2 + 4x - 9 = 0$ **3.** $x^2 + 2x = 2$

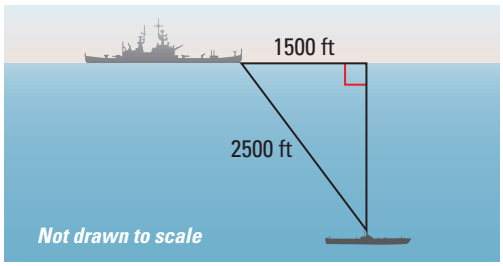
Tell whether the given lengths are sides of a right triangle. (Lesson 12.5)

4. 6, 7, 8 **5.** 9, 40, 41 **6.** 12, 35, 37

Find the missing length of the right triangle if *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse. (Lesson 12.5)

7. $a = 5, b = 12$ **8.** $a = 11, c = 61$ **9.** $b = 63, c = 65$

10. DEPTH OF A SUBMARINE The sonar of a Navy cruiser detects a submarine that is 2500 feet away. The point on the water directly above the submarine is 1500 feet away from the front of the cruiser. What is the depth of the submarine? (Lesson 12.5)



MATH & History — History of Pythagorean Theorem

APPLICATION LINK www.mcdougallittell.com

THEN

THE ANCIENT BABYLONIAN TABLET below suggests that the Babylonians were aware of the theorem now known as the Pythagorean theorem as early as 1650 B.C.

1. The table gives two of three numbers found on the tablet that could be sides of a right triangle. Find the length of the other leg.

Leg	Leg	Hypotenuse
119	?	169
56	?	106
161	?	289
65	?	97
2700	?	4500
1771	?	3229

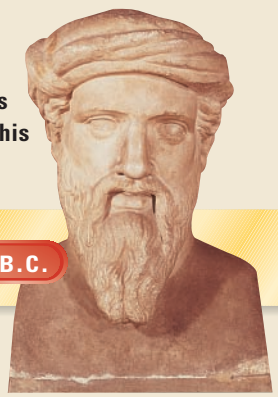
NOW

TODAY architects often use right triangles, whose sides show the relationships stated in the Pythagorean theorem.

c.1650 B.C.

Babylonian tablet lists Pythagorean triples.

Pythagoras discovers his theorem.



c. 540 B.C.

Now

The National Aquarium in Baltimore, Maryland, was designed using right triangles.