12.1 Functions Involving Square Roots

What *you should learn*

GOAL 1 Evaluate and graph a function involving square roots.

GOAL 2 Use functions involving square roots to model **real-life** problems, such as an investigation of skid marks in **Ex. 60**.

Why *you should learn it*

▼ To solve **real-life** problems such as estimating a dinosaur's walking speed in **Example 4**.

REAL LIFE

GOAL 1 GRAPHING FUNCTIONS INVOLVING SQUARE ROOTS

ACTIVITY Developing Concepts

Investigating the Square Root Function

1. Copy the table of values for the function $y = \sqrt{x}$. Use a calculator or the Table of Square Roots on page 811 to complete your table. Round to the nearest tenth. Plot the points. Graph the function.

x	0	1	2	3	4	5
y	?	?	?	?	?	?

2. What are the domain and the range of the function?
3. Why are negative values of x not included in the domain?

The **square root function** is defined by the equation $y = \sqrt{x}$. The domain is all nonnegative numbers and the range is all nonnegative numbers. Understanding this function will help you work with other functions involving square roots.

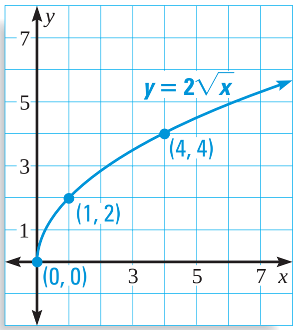
EXAMPLE 1 Graphing $y = a\sqrt{x}$

Sketch the graph of $y = 2\sqrt{x}$. Give the domain and the range.

SOLUTION

The radicand of a square root is always nonnegative, so the domain is the set of all nonnegative numbers. Make a table of values. Then plot the points and connect them with a smooth curve.

x	*y*
0	$y = 2\sqrt{0} = 0$
1	$y = 2\sqrt{1} = 2$
2	$y = 2\sqrt{2} \approx 2.8$
3	$y = 2\sqrt{3} \approx 3.5$
4	$y = 2\sqrt{4} = 4$
5	$y = 2\sqrt{5} \approx 4.5$



Both the domain and the range are all the nonnegative numbers.

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

It is a good idea to find the domain of a function *before* you make a table of values. This will help you choose values of x for the table.

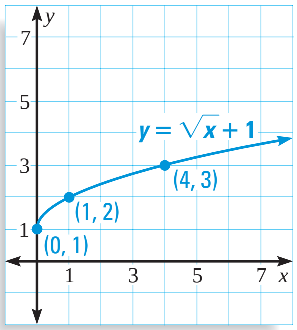
EXAMPLE 2 *Graphing* $y = \sqrt{x} + k$

Find the domain and the range of $y = \sqrt{x} + 1$. Then sketch its graph.

SOLUTION

The domain is the set of all nonnegative numbers. The range is the set of all numbers that are greater than or equal to 1. Make a table of values, plot the points, and connect them with a smooth curve.

x	y
0	$y = \sqrt{0} + 1 = 1$
1	$y = \sqrt{1} + 1 = 2$
2	$y = \sqrt{2} + 1 \approx 2.4$
3	$y = \sqrt{3} + 1 \approx 2.7$
4	$y = \sqrt{4} + 1 = 3$
5	$y = \sqrt{5} + 1 \approx 3.2$



EXAMPLE 3 *Graphing* $y = \sqrt{x - h}$

Find the domain and the range of $y = \sqrt{x - 3}$. Then sketch its graph.

SOLUTION

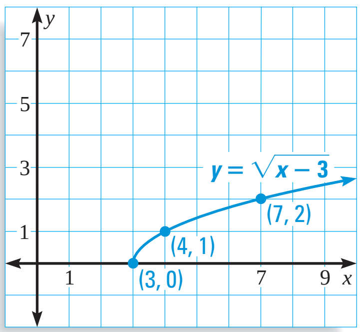
To find the domain, find the values of x for which the radicand is nonnegative.

$x - 3 \geq 0$ **Write an inequality for the domain.**

$x \geq 3$ **Add 3 to each side.**

The domain is the set of all numbers that are greater than or equal to 3. The range is the set of all nonnegative numbers. Make a table of values, plot the points, and connect them with a smooth curve.

x	y
3	$y = \sqrt{3 - 3} = 0$
4	$y = \sqrt{4 - 3} = 1$
5	$y = \sqrt{5 - 3} \approx 1.4$
6	$y = \sqrt{6 - 3} \approx 1.7$
7	$y = \sqrt{7 - 3} = 2$
8	$y = \sqrt{8 - 3} \approx 2.2$



STUDENT HELP

Study Tip
When you make a table of values to sketch the graph of a function involving square roots, choose several values to see the pattern of the curve.

FOCUS ON CAREERS

PALEONTOLOGIST
A paleontologist studies fossils of animals and plants to better understand the history of life on Earth.

CAREER LINK
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GOAL 2 USING SQUARE ROOTS IN REAL LIFE FUNCTIONS

EXAMPLE 4 *Using a Model Involving a Square Root*

DINOSAURS In a natural history museum you see leg bones for two species of dinosaurs and want to know how fast they walked. The maximum walking speed S (in feet per second) of an animal can be modeled by the equation below where $g = 32$ ft/sec^2 and L is the length (in feet) of the animal's leg. ▶ Source: *Discover*

Walking speed model: $S = \sqrt{gL}$

a. Use unit analysis to check the units of the model.

b. Sketch the graph of the model.

c. For one dinosaur the length of the leg is 1 foot. For the other dinosaur the length of the leg is 4 feet. Could the taller dinosaur walk four times as fast as the shorter dinosaur?

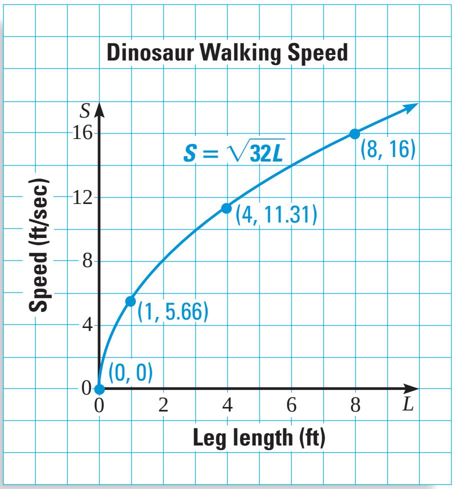
SOLUTION

a. **UNIT ANALYSIS** Substitute the units in the model.

$$\sqrt{\frac{\text{ft}}{\text{sec}^2}} \cdot \text{ft} = \sqrt{\frac{\text{ft}^2}{\text{sec}^2}} = \frac{\text{ft}}{\text{sec}}$$

b. To sketch the graph of $S = \sqrt{32L}$, make a table of values, plot the points, and draw a smooth curve through the points.

L	S
0	$S = \sqrt{32 \cdot 0} = 0$
1	$S = \sqrt{32 \cdot 1} \approx 5.66$
2	$S = \sqrt{32 \cdot 2} = 8$
3	$S = \sqrt{32 \cdot 3} \approx 9.80$
4	$S = \sqrt{32 \cdot 4} \approx 11.31$
5	$S = \sqrt{32 \cdot 5} \approx 12.65$
6	$S = \sqrt{32 \cdot 6} \approx 13.86$
7	$S = \sqrt{32 \cdot 7} \approx 14.97$
8	$S = \sqrt{32 \cdot 8} = 16$



c. Substitute 1 foot and 4 feet into the model.

When $L = 1$ foot: $S = \sqrt{32 \cdot 1} \approx 5.66$ feet/second

When $L = 4$ feet: $S = \sqrt{32 \cdot 4} \approx 11.31$ feet/second

▶ The taller dinosaur could walk about twice as fast as the shorter dinosaur, not four times as fast.

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe the square root function.

Concept Check ✓

2. Does the domain of $y = \sqrt{x+3}$ include negative values of x? Explain.

Skill Check ✓

Evaluate the function for $x = 0, 1, 2, 3,$ and 4. Round your answer to the nearest tenth.

3. $y = 4\sqrt{x}$
4. $y = \frac{1}{2}\sqrt{x}$
5. $y = 3\sqrt{x} + 4$
6. $y = 6\sqrt{x} - 3$
7. $y = \sqrt{x+2}$
8. $y = \sqrt{4x-1}$

Find the domain and the range of the function.

9. $y = 3\sqrt{x}$
10. $y = \sqrt{x}$
11. $y = \sqrt{x} - 10$
12. $y = \sqrt{x} + 6$
13. $y = \sqrt{x+5}$
14. $y = \sqrt{x-10}$

Sketch the graph of the function.

15. $y = 3\sqrt{x}$
16. $y = \sqrt{x} + 5$
17. $y = 3\sqrt{x+1}$

FIRE HOSES **For a fire hose with a nozzle that has a diameter of 2 inches, the flow rate f (in gallons per minute) can be modeled by $f = 120\sqrt{p}$ where p is the nozzle pressure in pounds per square inch.**

18. Sketch a graph of the model.
19. If the flow rate is 1200 gallons per minute, what is the nozzle pressure?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 808.

EVALUATING FUNCTIONS **Evaluate the function for the given value of x. Round your answer to the nearest tenth.**

20. $y = 3\sqrt{x}$; 9
21. $y = \frac{1}{2}\sqrt{x} - 1$; 16
22. $y = \sqrt{x-7}$; 15
23. $y = \sqrt{3x-5}$; 7
24. $y = 6\sqrt{15-x}$; -1
25. $y = \sqrt{21-2x}$; -2
26. $y = \sqrt{\frac{x}{2} - 2}$; 22
27. $y = \sqrt{8x^2 + \frac{3}{2}}$; $\frac{1}{4}$
28. $y = \sqrt{\frac{2x}{3} + 5}$; 6
29. $y = \sqrt{36x - 2}$; $\frac{1}{2}$

FINDING THE DOMAIN **Find the domain of the function.**

30. $y = 6\sqrt{x}$
31. $y = \sqrt{x-17}$
32. $y = \sqrt{3x-10}$
33. $y = \sqrt{x+5}$
34. $y = 4 + \sqrt{x}$
35. $y = \sqrt{x} - 3$
36. $y = 5 - \sqrt{x}$
37. $y = 4\sqrt{x}$
38. $y = 2\sqrt{4x}$
39. $y = 0.2\sqrt{x}$
40. $y = x\sqrt{x}$
41. $y = \sqrt{x+9}$
42. $y = 8\sqrt{\frac{5}{2}x}$
43. $y = \frac{\sqrt{x}}{5}$
44. $y = \frac{\sqrt{4-x}}{x}$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 20–56
Example 2: Exs. 20–56
Example 3: Exs. 20–56
Example 4: Exs. 57, 58, 61, 62

GRAPHING FUNCTIONS **Find the domain and the range of the function. Then sketch the graph of the function.**

45. $y = 7\sqrt{x}$ **46.** $y = 4\sqrt{x}$ **47.** $y = 5\sqrt{x}$

48. $y = \sqrt{x} - 2$ **49.** $y = \sqrt{x} + 4$ **50.** $y = \sqrt{x} - 3$

51. $y = \sqrt{x - 4}$ **52.** $y = \sqrt{x + 1}$ **53.** $y = \sqrt{x - 6}$

54. $y = 2\sqrt{4x + 10}$ **55.** $y = \sqrt{2x + 5}$ **56.** $y = x\sqrt{8x}$

57. **MEDICINE** A doctor may use a person's body surface area (*BSA*) as an index to prescribe the correct amount of medicine. You can use the model below to approximate a person's *BSA* (in square meters) where *h* represents height (in inches) and *w* represents weight (in pounds.)

$$BSA = \sqrt{\frac{h \cdot w}{3131}}$$

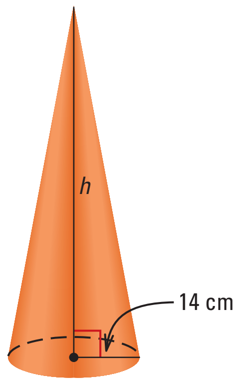
Find the *BSA* of a person who is 5 feet 2 inches tall and weighs 100 pounds. Round your answer to the nearest hundredth.

58. **PENDULUMS** The period T (in seconds) of a pendulum is the time it takes for the pendulum to swing back and forth. The period is related to the length L (in inches) of the pendulum by the model $T = 2\pi\sqrt{\frac{L}{384}}$. Find the length of a pendulum with a period of eight seconds. Give your answer to the nearest tenth.

59. **GEOMETRY CONNECTION** The lateral surface area S of a cone with a radius of 14 centimeters can be found using the formula

$$S = \pi \cdot 14\sqrt{14^2 + h^2}$$

where h is the height (in centimeters) of the cone.



a. Use unit analysis to check the units of the formula.

b. Sketch the graph of the formula.

c. Find the lateral surface area of a cone with a height of 30 centimeters.

60. **INVESTIGATING ACCIDENTS** An accident reconstructionist is responsible for finding how fast cars were going before an accident. To do this, a reconstructionist uses the model below where S is the speed of the car in miles per hour, d is the length of the tires' skid marks in feet, and f is the coefficient of friction for the road.

Car speed model: $S = \sqrt{30df}$

a. In an accident, a car makes skid marks 74 feet long. The coefficient of friction is 0.5. A witness says that the driver was traveling faster than the speed limit of 45 miles per hour. Can the witness's statement be correct? Explain your reasoning.

b. How long would the skid marks have to be in order to know that the car was traveling faster than 45 miles per hour?

FOCUS ON CAREERS

REAL LIFE **ACCIDENT RECONSTRUCTIONIST**

An accident reconstructionist is a police officer who examines physical evidence to find the cause of an accident.

INTERNET CAREER LINK
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COLLECTING DATA In Exercises 61 and 62, use the information from Example 4 on page 711.

61. Measure the length of your leg and calculate your maximum walking speed.

62. Mark a distance of 80 feet. Time yourself to see how long it takes you to walk the distance. Compare your speed from Exercise 61 with your actual speed. What did you find?

Test Preparation

63. MULTI-STEP PROBLEM The relationship between a roller coaster's velocity v (in feet per second) at the bottom of a drop and the height of the drop h (in feet) can be modeled by the formula $v = \sqrt{2gh}$ where g represents acceleration due to gravity.

a. Use the fact that $g = 32$ ft/sec^2 to show that $v = \sqrt{2gh}$ can be simplified to $v = 8\sqrt{h}$.

b. Sketch the graph of $v = 8\sqrt{h}$.

c. *Writing* Use the formula or the graph to explain why doubling the height of a drop does not double the velocity of a roller coaster.

★ **Challenge**

64. CRITICAL THINKING Find the domain of $y = \dfrac{3}{\sqrt{x} - 2}$.

MIXED REVIEW

SIMPLIFYING RADICAL EXPRESSIONS Simplify the radical expression. (Review 9.2 for 12.2)

65. $\sqrt{24}$ **66.** $\sqrt{60}$ **67.** $\sqrt{175}$ **68.** $\sqrt{9900}$

69. $\sqrt{\dfrac{20}{25}}$ **70.** $\dfrac{1}{2}\sqrt{80}$ **71.** $\dfrac{3\sqrt{7}}{\sqrt{9}}$ **72.** $4\sqrt{\dfrac{11}{16}}$

SOLVING QUADRATIC EQUATIONS Solve the equation. (Review 9.5)

73. $x^2 + 4x - 8 = 0$ **74.** $x^2 - 2x - 4 = 0$ **75.** $x^2 - 6x + 1 = 0$

76. $x^2 + 3x - 10 = 0$ **77.** $2x^2 + x = 3$ **78.** $4x^2 - 6x + 1 = 0$

MULTIPLYING EXPRESSIONS Find the product. (Review 10.2 for 12.2)

79. $(x - 2)(x + 11)$ **80.** $(x + 4)(3x - 7)$ **81.** $(2x - 3)(5x - 9)$

82. $(x - 5)(x - 4)$ **83.** $(6x + 2)(x^2 - x - 1)$ **84.** $(2x - 1)(x^2 + x + 1)$

SIMPLIFYING EXPRESSIONS Simplify the expression. (Review 11.5)

85. $\dfrac{8x}{3} \cdot \dfrac{1}{x}$ **86.** $\dfrac{8x^2}{3} \cdot \dfrac{9}{16x}$ **87.** $\dfrac{x}{x + 6} \div \dfrac{x + 1}{x + 6}$

88. MOUNT RUSHMORE Four American Presidents' faces are carved on Mount Rushmore: Washington, Jefferson, Roosevelt, and Lincoln. Before the faces were carved on the cliff, scale models were made. The ratio of the faces on the cliff to the models was 12 to 1. If the scale model of President Washington's face was 5 feet tall, how tall is his face on Mount Rushmore? (Review 11.1)