11.5 Multiplying and Dividing Rational Expressions

What *you should learn*

GOAL 1 Multiply and divide rational expressions.

GOAL 2 Use rational expressions as **real-life** models, as when comparing parts of the service industry to the total in **Exs. 38–41**.

Why *you should learn it*

▼ To model **real-life** situations, such as describing the average car sales per dealership in **Example 6**.

GOAL 1 FINDING PRODUCTS AND QUOTIENTS

Because the variables in a rational expression represent real numbers, the rules for multiplying and dividing rational expressions are the same as the rules for multiplying and dividing numerical fractions.

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Let *a, b, c,* and *d* be nonzero polynomials.

TO MULTIPLY, multiply numerators and denominators. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

TO DIVIDE, multiply by the reciprocal of the divisor. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

EXAMPLE 1 *Multiplying Rational Expressions Involving Monomials*

Simplify $\frac{3x^3}{4x} \cdot \frac{8x^2}{15x^4}$.

SOLUTION

$$\frac{3x^3}{4x} \cdot \frac{8x^2}{15x^4} = \frac{24x^5}{60x^5} \quad \text{Multiply numerators and denominators.}$$

$$= \frac{2 \cdot \cancel{12} \cdot \cancel{x^5}}{5 \cdot \cancel{12} \cdot \cancel{x^5}} \quad \text{Factor, and divide out common factors.}$$

$$= \frac{2}{5} \quad \text{Simplified form}$$

STUDENT HELP

Study Tip
When multiplying, you usually factor as far as possible to identify all common factors. Note, however, that you do not need to write the prime factorizations of 24 and 60 in Example 1, if you recognize 12 as their greatest common factor.

EXAMPLE 2 *Multiplying Rational Expressions Involving Polynomials*

Simplify $\frac{x}{3x^2 - 9x} \cdot \frac{x - 3}{2x^2 + x - 3}$.

SOLUTION

$$\frac{x}{3x^2 - 9x} \cdot \frac{x - 3}{2x^2 + x - 3} = \frac{x(x - 3)}{(3x^2 - 9x)(2x^2 + x - 3)} \quad \text{Multiply numerators and denominators.}$$

$$= \frac{\cancel{x}\cancel{(x - 3)}}{3\cancel{x}\cancel{(x - 3)}(x - 1)(2x + 3)} \quad \text{Factor, and divide out common factors.}$$

$$= \frac{1}{3(x - 1)(2x + 3)} \quad \text{Simplified form}$$

STUDENT HELP

Study Tip
When you multiply the numerators and the denominators, leave the products in factored form. At the very end, you may multiply the remaining factors or you may leave your answer in factored form, as in Example 2.

EXAMPLE 3 *Multiplying by a Polynomial*

Simplify $\frac{7x}{x^2 + 5x + 4} \cdot (x + 4)$.

SOLUTION

$$\frac{7x}{x^2 + 5x + 4} \cdot (x + 4) = \frac{7x}{x^2 + 5x + 4} \cdot \frac{x + 4}{1}$$ Write $x + 4$ as $\frac{x+4}{1}$.

$$= \frac{7x(x + 4)}{x^2 + 5x + 4}$$ Multiply numerators and denominators.

$$= \frac{7x\cancel{(x + 4)}}{(x + 1)\cancel{(x + 4)}}$$ Factor, and divide out common factors.

$$= \frac{7x}{x + 1}$$ Simplified form

EXAMPLE 4 *Dividing Rational Expressions*

Simplify $\frac{4n}{n + 5} \div \frac{n - 9}{n + 5}$.

SOLUTION

$$\frac{4n}{n + 5} \div \frac{n - 9}{n + 5} = \frac{4n}{n + 5} \cdot \frac{n + 5}{n - 9}$$ Multiply by reciprocal.

$$= \frac{4n(n + 5)}{(n + 5)(n - 9)}$$ Multiply numerators and denominators.

$$= \frac{4n\cancel{(n + 5)}}{\cancel{(n + 5)}(n - 9)}$$ Divide out common factors.

$$= \frac{4n}{n - 9}$$ Simplified form

EXAMPLE 5 *Dividing by a Polynomial*

Simplify $\frac{x^2 - 9}{4x^2} \div (x - 3)$.

SOLUTION

$$\frac{x^2 - 9}{4x^2} \div (x - 3) = \frac{x^2 - 9}{4x^2} \cdot \frac{1}{x - 3}$$ Multiply by reciprocal.

$$= \frac{x^2 - 9}{4x^2(x - 3)}$$ Multiply numerators and denominators.

$$= \frac{(x + 3)\cancel{(x - 3)}}{4x^2\cancel{(x - 3)}}$$ Factor, and divide out common factors.

$$= \frac{x + 3}{4x^2}$$ Simplified form

STUDENT HELP

INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 USING RATIONAL MODELS IN REAL LIFE

REAL LIFE Car Sales

EXAMPLE 6 *Writing and Using a Rational Model*

The models below can be created using data collected by the National Automobile Dealers Association in the United States. Five-year intervals from 1975–1995 were used. Let t represent the number of years since 1975.

Number of new-car dealerships: $D = \dfrac{30{,}000 + 300t}{1 + 0.03t}$

Total sales (in billions of dollars) of new-car dealerships: $S = \dfrac{80 + 10t}{1 - 0.02t}$

a. Find a model for the average sales per new-car dealership.

b. Use the model to predict the average sales in 2005.

SOLUTION

PROBLEM SOLVING STRATEGY

a. **VERBAL MODEL**

$\boxed{\text{Average sales per dealership}} = \text{Total sales of dealerships} \div \text{Number of dealerships}$

LABELS

Average sales per dealership $= \boxed{A}$

Total sales of dealerships $= \dfrac{80 + 10t}{1 - 0.02t}$

Number of dealerships $= \dfrac{30{,}000 + 300t}{1 + 0.03t}$

ALGEBRAIC MODEL

$$\boxed{A} = \frac{80 + 10t}{1 - 0.02t} \div \frac{30{,}000 + 300t}{1 + 0.03t}$$ **Write algebraic model.**

$$= \frac{80 + 10t}{1 - 0.02t} \cdot \frac{1 + 0.03t}{30{,}000 + 300t}$$ **Multiply by reciprocal.**

$$= \frac{(80 + 10t)(1 + 0.03t)}{(1 - 0.02t)(30{,}000 + 300t)}$$ **Multiply numerators and denominators.**

$$= \frac{(\cancel{10})(8 + t)(1 + 0.03t)}{(1 - 0.02t)(\cancel{10})(3000 + 30t)}$$ **Divide out common factor 10.**

▶ The equation $A = \dfrac{(8 + t)(1 + 0.03t)}{(1 - 0.02t)(3000 + 30t)}$ is a model for the average sales (in billions of dollars) per new-car dealership.

b. In the year 2005, $t = 30$, so substitute 30 for t in the model for A.

$$\frac{(8 + 30)(1 + 0.03 \cdot 30)}{(1 - 0.02 \cdot 30)(3000 + 30 \cdot 30)} = \frac{38 \cdot 1.9}{0.4 \cdot 3900} = \frac{72.2}{1560} \approx 0.04628$$

▶ The model predicts that the average sales in 2005 will be about \$0.0463 billion. Because 1 billion is 1000 million, you can express \$0.0463 billion as $0.0463 \cdot 1000$ million, or \$46.3 million.

GUIDED PRACTICE

Concept Check ✓

1. Describe the steps used to multiply two rational expressions.

2. Describe the steps used to divide two rational expressions.

3. **ERROR ANALYSIS** Describe the error in the problem at the right. Then do the division correctly.

$$\frac{x+3}{x-3} \div \frac{4x}{x^2-9} =$$
$$\frac{\cancel{x+3}}{x-3} \cdot \frac{4x}{(\cancel{x+3})(x-3)} = \frac{4x}{(x-3)^2}$$

Ex. 3

Skill Check ✓

Simplify the expression.

4. $\frac{3x}{8x^2} \cdot \frac{4x^3}{3x^4}$

5. $\frac{x^2-1}{x} \cdot \frac{2x}{3x-3}$

6. $\frac{x}{x^2-25} \cdot \frac{x-5}{x+5}$

7. $\frac{3x}{x^2-2x-15} \cdot (x+3)$

8. $\frac{x}{8-2x} \div \frac{2x}{4-x}$

9. $\frac{4x^2-25}{4x} \div (2x-5)$

10. $\frac{x^2-4x+3}{2x} \div \frac{x-1}{2}$

11. $\frac{9x^2+6x+1}{x+5} \div \frac{3x+1}{x^2+5x}$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 807.

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

12. $\frac{4x}{3} \cdot \frac{1}{x}$

13. $\frac{9x^2}{4} \cdot \frac{8}{18x}$

14. $\frac{7x^2}{6x} \cdot \frac{12x^2}{2x}$

15. $\frac{16x^2}{8x} \div \frac{4x^2}{16x}$

16. $\frac{25x^2}{10x} \div \frac{5x}{10x}$

17. $\frac{13x^4}{7x} \div \frac{x^3}{7x}$

18. $\frac{5-2x}{-2} \cdot \frac{24}{10-4x}$

19. $\frac{4x}{x^2-9} \cdot \frac{x-3}{8x^2+12x}$

20. $\frac{-3}{x-4} \cdot \frac{x-4}{12(x-7)}$

21. $\frac{3x^2}{10} \div \frac{9x^3}{25}$

22. $\frac{x}{x+2} \div \frac{x+5}{x+2}$

23. $\frac{5x+15}{3x} \div \frac{x+3}{9x}$

24. $\frac{2(x+2)}{5(x-3)} \div \frac{4(x-2)}{5x-15}$

25. $\frac{x^2-36}{-5x^2} \div (x-6)$

26. $\frac{8}{2+3x} \cdot (8+12x)$

27. $\frac{3x}{x^2-2x-24} \cdot \frac{x-6}{6x^2+9x}$

28. $\frac{x}{3x^2+2x-8} \cdot (3x-4)$

29. $\frac{x+1}{x^3(3-x)} \div \frac{5}{x(x-3)}$

30. $(4x^2+x-3) \cdot \frac{1}{(4x+3)(x-1)}$

31. $\frac{x^2-8x+15}{x^2-3x} \div (3x-15)$

32. $\frac{6x^2+7x-33}{x+4} \div (6x-11)$

33. $\left(\frac{x^2}{5} \cdot \frac{x+2}{2}\right) \div \frac{x}{30}$

34. $\left(\frac{2x^2}{3} \cdot \frac{5}{x}\right) \div \frac{6x^2}{25}$

STUDENT HELP

HOMEWORK HELP
Examples 1–5: Exs. 12–34
Example 6: Exs. 35–40

FOCUS ON CAREERS

SERVICE INDUSTRY CAREERS The service industry includes a wide range of careers. Fields of service include health care, automobile and other repair services, legal assistance, education, and recreation.

CAREER LINK
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RAILROAD TRAVEL In Exercises 35–37, the models are based on data about train travel from 1990 to 1996 in the United States. Let *t* represent the number of years since 1990. ▶ Source: *Statistical Abstract of the United States*

Miles (in millions) traveled by passengers: $M = \dfrac{6300 - 800t}{1 - 0.12t}$

Passengers (in millions) who traveled by train: $P = \dfrac{222 - 24t}{10 - t}$

35. Find a model for the average number of miles traveled per passenger.

36. Use the model found in Exercise 35 to estimate the average number of miles traveled per passenger in 1995.

37. Use the model to predict the average number of miles traveled per passenger in 2005.

SERVICE INDUSTRY In Exercises 38–41, the models below are based on data collected by the Bureau of Economic Analysis from 1990 to 1997 in the United States. Let *t* represent the number of years since 1990.

Total sales (in billions of dollars) of services: $S = \dfrac{1055 + 23t}{1 - 0.04t}$

Total sales (in billions of dollars) of hotel services: $H = \dfrac{46 + 0.7t}{1 - 0.04t}$

Total sales (in billions of dollars) of auto repair services: $A = \dfrac{48 - t}{1 - 0.06t}$

38. Find the total sales given by each model in 1990.

39. Find a model for the ratio of hotel service sales to total service industry sales. Was this ratio increasing or decreasing from 1990 to 1997? Explain.

40. Find a model for the ratio of auto service sales to total service industry sales. Was this ratio increasing or decreasing from 1990 to 1997? Explain.

41. *Writing* What do your answers in Exercises 38 and 39 tell you about how the sales of the service industry were changing in the period from 1990 to 1997?

PROOF In Exercises 42 and 43, use the *proof* shown below.

Statement	Explanation
1. $\dfrac{ac}{bc} = \dfrac{a}{b} \cdot \dfrac{?}{?}$	**1.** Apply the rule for multiplying rational expressions.
2. $\underline{\ ?\ } = \dfrac{a}{b} \cdot \underline{\ ?\ }$	**2.** Any nonzero number divided by itself is 1.
3. $\underline{\ ?\ } = \dfrac{a}{b}$	**3.** Any nonzero number multiplied by 1 is itself.

42. LOGICAL REASONING Copy and complete the *proof* to show why you can divide out common factors.

43. Use the method from Exercise 42 to show that $\dfrac{2x - 4}{x^2 - 4} = \dfrac{2}{x + 2}$.

Test Preparation

44. MULTIPLE CHOICE Which of the following represents the expression $\frac{x^2 - 3x}{x^2 - 5x + 6} \cdot \frac{(x-2)^2}{2x}$ in simplified form?

Ⓐ $\frac{x(x-3)}{2}$ Ⓑ $\frac{x}{2}$ Ⓒ $\frac{x-2}{2}$

Ⓓ $\frac{x(x-3)}{x-2}$ Ⓔ $\frac{x^2 - 4x + 4}{x-2}$

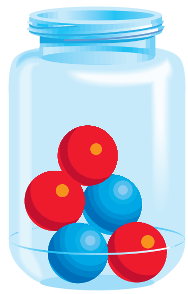
45. MULTIPLE CHOICE Which product equals the quotient $(2x + 2) \div \frac{x^2 + x}{4}$?

Ⓐ $\frac{1}{2x+2} \cdot \frac{x^2+x}{4}$ Ⓑ $\frac{2x+2}{1} \cdot \frac{x^2+x}{4}$ Ⓒ $\frac{1}{2x+2} \cdot \frac{4}{x^2+x}$

Ⓓ $\frac{2x+2}{1} \cdot \frac{4}{x^2+x}$ Ⓔ $\frac{2x+2}{2x+2} \cdot \frac{4}{x^2+x}$

★ Challenge

INDEPENDENT EVENTS In Exercises 46–47, use the following information.
Two events are *independent* if the probability that one event will occur is not affected by whether or not the other event occurs. For independent events A and B, the probability that A *and* B will occur equals the probability of A times the probability of B.



For example, if you draw a marble from the jar at the right, put it back, and then draw another one, the probability that both marbles are red is $\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$.

46. A bag contains n marbles. There are r blue marbles and the rest of the marbles are yellow. Find the probability of drawing a yellow marble followed by a blue marble if the first one is put back before drawing again.

EXTRA CHALLENGE
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47. Look back at the carnival game in Exercises 32–34 on page 668. Find the probability of hitting the target two times in a row.

MIXED REVIEW

FINDING THE LCD Find the least common denominator. **(Skills Review, pp. 781–783)**

48. $\frac{3}{4}, \frac{2}{5}$ **49.** $\frac{2}{9}, \frac{3}{18}$ **50.** $\frac{1}{16}, \frac{9}{20}$ **51.** $\frac{14}{54}, \frac{31}{81}$

QUADRATIC FORMULA Solve the equation. **(Review 9.5)**

52. $2x^2 + 12x - 6 = 0$ **53.** $x^2 - 6x + 7 = 0$ **54.** $3x^2 + 11x + 10 = 0$

POLYNOMIALS Add or subtract. **(Review 10.1 for 11.6)**

55. $(4t^2 + 5t + 2) - (t^2 - 3t - 8)$ **56.** $(16p^3 - p^2 + 24) + (12p^2 - 8p - 16)$

57. $(a^4 - 12a) + (4a^3 + 11a - 1)$ **58.** $(-5x^2 + 2x - 12) - (6 - 9x - 7x^2)$

59. COMPOUND INTEREST After two years, an investment of \$1000 compounded annually at an interest rate r will grow to the amount $1000(1 + r)^2$ in dollars. Write this product as a trinomial. **(Review 10.3)**