11.4 Simplifying Rational Expressions

What *you should learn*

GOAL 1 Simplify a rational expression.

GOAL 2 Use rational expressions to find geometric probability.

Why *you should learn it*

▼ To model **real-life** situations, such as finding the probability of a meteor strike in **Exs. 36–38**.

REAL LIFE

GOAL 1 Simplifying a Rational Expression

A **rational number** is a number that can be written as the quotient of two integers, such as $\frac{1}{2}$, $\frac{4}{3}$, and $\frac{7}{1}$. A fraction whose numerator, denominator, or both numerator and denominator are nonzero polynomials is a **rational expression**. Here are some examples.

$$\frac{3}{x+4} \qquad \frac{2x}{x^2-9} \qquad \frac{3x+1}{x^2+1}$$

A rational expression is *undefined* when the denominator is equal to zero. For instance, in the first expression x can be any real number except -4.

To simplify a fraction, you factor the numerator and the denominator and then divide out any common factors. A rational expression is **simplified** if its numerator and denominator have no factors in common (other than ± 1).

SIMPLIFYING FRACTIONS

Let a, b, and c be nonzero numbers.

$$\frac{ac}{bc} = \frac{a \cdot \cancel{c}}{b \cdot \cancel{c}} = \frac{a}{b} \qquad \textbf{Example: } \frac{28}{35} = \frac{4 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \frac{4}{5}$$

EXAMPLE 1 *When and When Not to Divide Out*

Simplify the expression.

a. $\frac{2x}{2(x+5)}$ **b.** $\frac{x(x^2+6)}{x^2}$ **c.** $\frac{x+4}{x}$

SOLUTION

a. $\frac{2x}{2(x+5)} = \frac{\cancel{2} \cdot x}{\cancel{2}(x+5)}$ You can divide out the common factor 2.

$= \frac{x}{x+5}$ Simplified form

b. $\frac{x(x^2+6)}{x^2} = \frac{\cancel{x}(x^2+6)}{\cancel{x} \cdot x}$ You can divide out the common factor x.

$= \frac{x^2+6}{x}$ Simplified form

c. $\frac{x+4}{x}$ You cannot divide out the common term x.

STUDENT HELP

Study Tip
When you simplify rational expressions, you can divide out only factors, not terms.

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 *Factoring Numerator and Denominator*

Simplify $\frac{2x^2 - 6x}{6x^2}$.

SOLUTION

$$\frac{2x^2 - 6x}{6x^2} = \frac{2x(x-3)}{2 \cdot 3 \cdot x \cdot x} \qquad \text{Factor numerator and denominator.}$$

$$= \frac{\cancel{2x}(x-3)}{\cancel{2x}(3x)} \qquad \text{Divide out common factor } 2x.$$

$$= \frac{x-3}{3x} \qquad \text{Simplified form}$$

EXAMPLE 3 *Recognizing Opposite Factors*

Simplify $\frac{4 - x^2}{x^2 - x - 2}$.

SOLUTION

$$\frac{4 - x^2}{x^2 - x - 2} = \frac{(2-x)(2+x)}{(x-2)(x+1)} \qquad \text{Factor numerator and denominator.}$$

$$= \frac{-(x-2)(2+x)}{(x-2)(x+1)} \qquad \text{Factor } -1 \text{ from } (2-x).$$

$$= \frac{-\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)} \qquad \text{Divide out common factor } x-2.$$

$$= -\frac{x+2}{x+1} \qquad \text{Simplified form}$$

EXAMPLE 4 *Recognizing when an Expression is Undefined*

In Examples 2 and 3, are the original expression and the simplified expression defined for the same values of the variable?

STUDENT HELP

Look Back
For help with using the zero-product property, see p. 597.

SOLUTION A rational expression is undefined when the denominator is 0. Think of setting the denominator equal to 0 and then solving that equation.

Example 2: Yes, $6x^2 = 0$ and $3x = 0$ have the same solution: $x = 0$. Both expressions are undefined when $x = 0$.

Example 3: No, $x^2 - x - 2 = 0$ and $x + 1 = 0$ do not have the same solutions. To see this set the denominator of the original expression equal to zero.

$$x^2 - x - 2 = 0 \qquad \text{Set denominator equal to 0.}$$

$$(x-2)(x+1) = 0 \qquad \text{Factor denominator.}$$

$$x - 2 = 0 \quad \text{or} \quad x + 1 = 0 \qquad \text{Use zero-product property.}$$

$$x = 2 \qquad\qquad x = -1 \qquad \text{Solve for } x.$$

▶ The original expression is undefined when $x = 2$ and $x = -1$. The simplified expression is undefined only when $x = -1$.

GOAL 2 APPLYING RATIONAL EXPRESSIONS

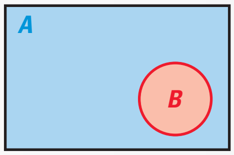
Rational expressions can be useful in modeling situations such as finding averages, ratios, and probabilities. In Chapter 2 you learned to find theoretical probability by comparing the number of favorable outcomes to the total number of outcomes. In some cases, you can give theoretical probabilities a geometric interpretation in terms of areas.

GEOMETRIC PROBABILITY

Region *B* is contained in Region *A*. An object is tossed onto Region *A* and is equally likely to land on any point in the region.

The **geometric probability** that it lands in Region *B* is

$$P = \frac{\text{Area of Region } B}{\text{Area of Region } A}.$$

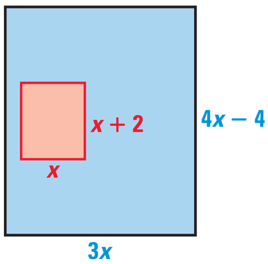


EXAMPLE 5 *Writing and Using a Rational Model*

A coin is tossed onto the large rectangular region shown at the right. It is equally likely to land on any point in the region.

a. Write a model that gives the probability that the coin will land in the small rectangle.

b. Evaluate the model when $x = 10$.



SOLUTION

a. $P = \dfrac{\text{Area of small rectangle}}{\text{Area of large rectangle}}$ — **Formula for geometric probability**

$= \dfrac{x(x + 2)}{3x(4x - 4)}$ — **Find areas.**

$= \dfrac{\cancel{x} \cdot (x + 2)}{3\cancel{x} \cdot 4(x - 1)}$ — **Divide out common factors.**

$= \dfrac{x + 2}{12(x - 1)}$ — **Simplified form**

b. To find the probability when $x = 10$, substitute 10 for x in the model.

$$P = \frac{x + 2}{12(x - 1)} = \frac{10 + 2}{12(10 - 1)} = \frac{12}{108} = \frac{1}{9}$$

▶ The probability of landing in the small rectangle is $\frac{1}{9}$.

Guided Practice

Vocabulary Check ✓

1. Define a rational expression. Then give an example of a rational expression.

Concept Check ✓

Error Analysis **Describe the error.**

2. $$\frac{3+x}{5+2x} = \frac{3+\cancel{x}}{5+2\cancel{x}} = \frac{4}{7}$$

3. $$\frac{\cancel{x+4}}{2\cancel{(x+4)}} = \frac{0}{2} = 0$$

Skill Check ✓

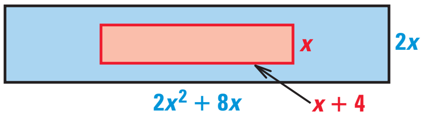
For what values of the variable is the rational expression undefined?

4. $\frac{6}{8x}$ **5.** $\frac{x-1}{x-5}$ **6.** $\frac{2}{x^2-x-2}$

7. Which of the following is the simplified form of $\frac{6+2x}{x^2+5x+6}$?

A. $\frac{2x}{x^2+5x}$ **B.** $\frac{2}{x+5}$ **C.** $\frac{2}{x+2}$

8. Which ratio represents the ratio of the area of the smaller rectangle to the area of the larger rectangle?



A. $\frac{1}{2x}$ **B.** $\frac{1}{4x}$

Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 807.

Simplifying Expressions **Simplify the expression if possible.**

9. $\frac{4x}{20}$ **10.** $\frac{15x}{45}$ **11.** $\frac{-18x^2}{12x}$

12. $\frac{14x^2}{50x^4}$ **13.** $\frac{3x^2-18x}{-9x^2}$ **14.** $\frac{42x-6x^3}{36x}$

15. $\frac{7x}{12x+x^2}$ **16.** $\frac{x+2x^2}{x+2}$ **17.** $\frac{12-5x}{10x^2-24x}$

18. $\frac{x^2+25}{2x+10}$ **19.** $\frac{5-x}{x^2-8x+15}$ **20.** $\frac{2x^2+11x-6}{x+6}$

21. $\frac{x^2+x-20}{x^2+2x-15}$ **22.** $\frac{x^3+9x^2+14x}{x^2-4}$ **23.** $\frac{x^3-x}{x^3+5x^2-6x}$

Student Help

Homework Help
Example 1: Exs. 9–23
Example 2: Exs. 9–23
Example 3: Exs. 9–23
Example 4: Exs. 24–29
Example 5: Exs. 30–33

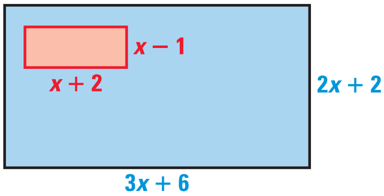
Undefined Values **For what values of the variable is the rational expression undefined?**

24. $\frac{7}{x-3}$ **25.** $\frac{11}{x-8}$ **26.** $\frac{4}{x^2-1}$

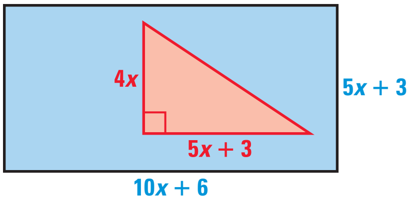
27. $\frac{x+3}{x^2-9}$ **28.** $\frac{x+9}{x^2+x-12}$ **29.** $\frac{x-3}{x^2+5x-6}$

GEOMETRIC PROBABILITY **A coin is tossed onto the large rectangular region shown. It is equally likely to land on any point in the region. Write a model that gives the probability that the coin will land in the red region. Then evaluate the model when $x = 3$.**

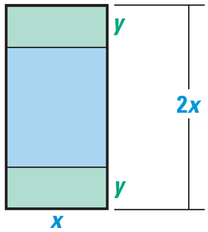
30.



31.



CARNIVAL GAMES **In Exercises 32–34, use the following information.**
You are designing a game for a school carnival. Players will drop a coin into a basin of water, trying to hit a target on the bottom. The water is kept moving randomly, so the coin is equally likely to land anywhere. You use a rectangular basin twice as long as it is wide. You place the blue rectangular target an equal distance from each end.



32. Express the two dimensions of the target in terms of the variables x and y.

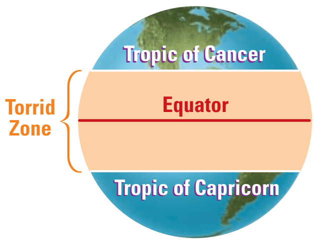
33. Write a model that gives the probability that the coin will land on the target.

34. **CRITICAL THINKING** You want players to win about half the time. Give a set of values you could use for x and y if the basin's area is between 72 and 120 square inches.

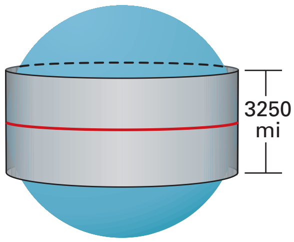
35. *Writing* Create three problems of the form $\dfrac{ax^2 + bx + c}{dx^2 + ex + f}$ in which the numerator and the denominator have a common factor. Describe the process you used to create your problems.

METEOR STRIKES **In Exercises 36–38, use this information. A meteorite is equally likely to hit anywhere on Earth. The probability that a meteorite lands in the Torrid Zone is $\dfrac{\text{Area of Torrid Zone}}{\text{Total surface area of Earth}}$.**

Let R represent Earth's radius.



36. Write an expression to estimate the area of the Torrid Zone. You can think of the distance between the tropics (about 3250 miles) as the height of a cylindrical belt around Earth at the equator. The length of the belt is Earth's circumference $2\pi R$.



37. The surface area of a sphere with radius R is $4\pi R^2$. Write and simplify an expression for the probability that a meteorite lands in the Torrid Zone.

38. Find the probability in Exercise 37. Use 3963 miles for Earth's radius.

FOCUS ON APPLICATIONS

REAL LIFE **IMPACT CRATER** at Wolfe Creek, Australia. When a rocky object moving in space enters Earth's atmosphere, it starts to burn causing the flash of light called a *meteor*. Most such objects burn up completely, but about 500 strike Earth each year as *meteorites*.

Test Preparation

39. MULTI-STEP PROBLEM In this exercise you will look for a pattern.

a. Copy and complete the table by evaluating the expression for each x-value.

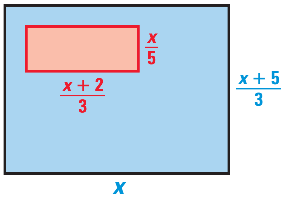
x-values	-2	-1	0	1	2	3	4
$\frac{x^2 - x - 6}{x - 3}$	?	?	?	?	?	?	?
$x + 2$	?	?	?	?	?	?	?

b. Describe the relationship between $\frac{x^2 - x - 6}{x - 3}$ and $x + 2$. What does the table tell you about this relationship?

c. CRITICAL THINKING Write another pair of expressions that have this relationship. Describe the procedure you used to find your example.

★ Challenge

40. RATIOS Write the ratio in simplest form comparing the area of the smaller rectangle to the area of the larger rectangle.



Ex. 40

41. PROPORTIONS Solve the proportion.

$$\frac{x^2 + 5x + 6}{x^2 - 2x - 8} = \frac{x^2 - 4x - 5}{x^2 - 8x + 15}$$

EXTRA CHALLENGE
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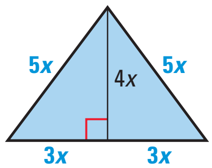
MIXED REVIEW

PRODUCTS AND QUOTIENTS **Simplify.** (Review 2.5 and 2.7 for 11.5)

42. $\left(-\frac{1}{2}\right)\left(\frac{2}{3}\right)$ **43.** $(-15)\left(-\frac{5}{6}\right)$ **44.** $\frac{2}{7} \div \frac{14}{24}$ **45.** $\frac{4}{9} \div (-36)$

46. $\left(-\frac{3}{4}\right)\left(\frac{3y}{-5}\right)$ **47.** $\frac{2m}{3} \cdot 6m^2$ **48.** $\frac{36}{45a} \div \frac{-9a}{5}$ **49.** $-18c^3 \div \frac{-27c}{-4}$

50. GEOMETRY The area of the triangle is 192 square meters. What is the value of x? What is the perimeter? (Review 9.1)



SKETCHING GRAPHS **Sketch the graph of the function.** (Review 9.3)

51. $y = x^2$ **52.** $y = 4 - x^2$ **53.** $y = \frac{1}{2}x^2$

54. $y = 5x^2 + 4x - 5$ **55.** $y = 4x^2 - x + 6$ **56.** $y = -3x^2 - x + 7$

57. POPULATION DENSITY Population density is the number of people per square mile. Suppose the population density of a city decreases by 8% for every mile you travel from the center of the city. In the center of the city, the population density is 2500 people per square mile. Find an exponential decay model for the population density. Copy and complete the table. (Review 8.6)

Distance from center of city (miles)	2	3	4	5	6
Population density (people per square mile)	?	?	?	?	?