11.2

Percents

GOAL 1 SOLVING PERCENT PROBLEMS

In the percent equation "8 is 40% of 20," you compare 8 to the *base number* 20 by writing $\frac{8}{20} = \frac{40}{100}$. The percent for $\frac{8}{20}$ can be written as 40%, as the fraction $\frac{40}{100}$, or as the decimal 0.4. When you work with percents in equations, you usually write the percent as a fraction or as a decimal. In any percent equation the **base number** is the number you are comparing to. You can write a verbal model to help you solve the equation.

VERBAL MODEL	**Number being compared to base** is a **percent** of **base number**

LABELS	Number compared to base = **a**	(same units as *b*)
	Percent = **p**	(no units)
	Base number = **b**	(assigned units)

ALGEBRAIC MODEL	$a = p \cdot b$
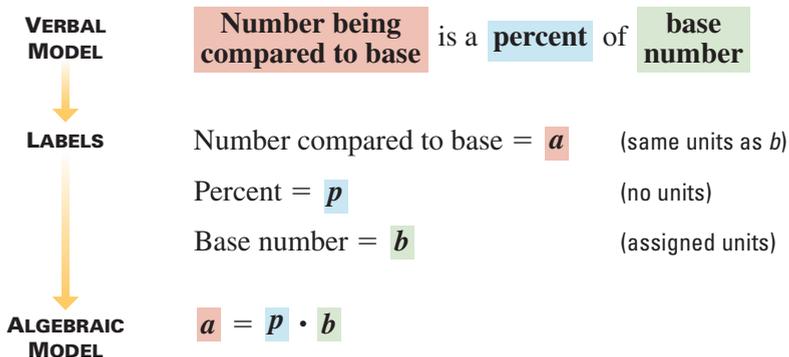
In Examples 1–3 you will see how the basic verbal model above can be applied to the three different cases of percent problems—finding *a*, finding *p*, or finding *b*. As shown in these examples, percents are usually converted to decimal form before performing arithmetic operations.

EXAMPLE 1 *Number Compared to Base is Unknown*

What is 30% of 70 feet?

SOLUTION

VERBAL MODEL	a is p **percent** of b

LABELS	Number compared to base = **a**	(feet)
	Percent = 30% = **0.3**	(no units)
	Base number = **70**	(feet)

ALGEBRAIC MODEL	$a = (0.3)(70)$
	$a = 21$
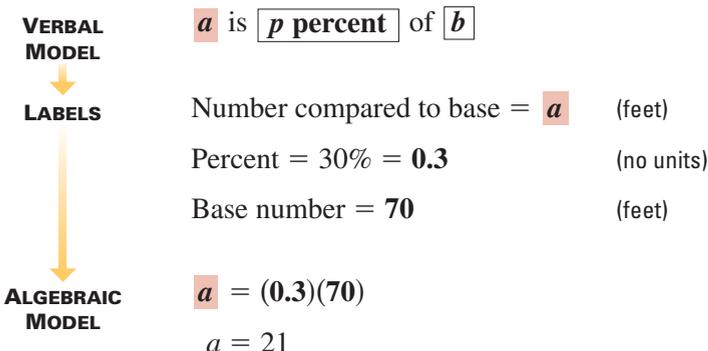
▶ 21 feet is 30% of 70 feet.

EXAMPLE 2   *Base Number is Unknown*

Fourteen dollars is 25% of what amount of money?

SOLUTION

VERBAL MODEL $\boxed{a}$ is $\boxed{p \text{ percent}}$ of $\boxed{b}$

LABELS

Number compared to base $= \mathbf{14}$ (dollars)

Percent $= 25\% = \mathbf{0.25}$ (no units)

Base number $= \boxed{b}$ (dollars)

ALGEBRAIC MODEL

$14 = (\mathbf{0.25})\,\boxed{b}$

$\dfrac{14}{0.25} = b$

$56 = b$

▶ $14 is 25% of $56.

EXAMPLE 3   *Percent is Unknown*

One hundred thirty-five is what percent of 27?

SOLUTION

VERBAL MODEL $\boxed{a}$ is $\boxed{p \text{ percent}}$ of $\boxed{b}$

LABELS

Number compared to base $= \mathbf{135}$ (no units)

Percent $= \boxed{p}$ (no units)

Base number $= \mathbf{27}$ (no units)

ALGEBRAIC MODEL

$135 = \boxed{p}\,(\mathbf{27})$

$\dfrac{135}{27} = p$

$5 = p$ **Decimal form**

$500\% = p$ **Percent form** $\left(5 = \dfrac{500}{100}\right)$

CONCEPT SUMMARY	THREE FORMS OF PERCENT PROBLEMS $a = pb$		
QUESTION	**GIVEN**	**NEED TO FIND**	**EXAMPLE**
What is p percent of b?	b and p	a	Example 1
a is p percent of **what**?	a and p	b	Example 2
a is **what percent** of b?	a and b	p	Example 3

GOAL 2 **USING PERCENTS IN REAL LIFE**

EXAMPLE 4 *Modeling and Using Percents*

SCIENCE CONNECTION
There are about 170,000 species of butterflies and moths world-wide. Butterflies and moths make up about 17% of all classified insect species. Estimate how many insect species have been classified.

Insect Species in North America

Lepidoptera
13% (butterflies and moths)

Hymenoptera
18% (includes bees and ants)

Coleoptera
33% (beetles)

Diptera
20% (true flies)

Other 16%
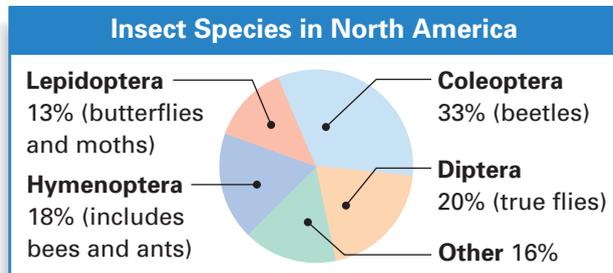
Note: the percents for the world-wide species are slightly different.

SOLUTION

Method 1 Use the percent equation $a = pb$.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

| Number of butterfly and moth species | is | p percent | of | Total number of insect species |

LABELS

Number of butterfly and moth species = **170,000**   (species)

Percent = 17% = **0.17**   (no units)

Total number of insect species = b (species)

ALGEBRAIC MODEL

$170{,}000 = 0.17\,b$

$\dfrac{170{,}000}{0.17} = b$

$1{,}000{,}000 = b$

▶ About 1,000,000 species of insects have been classified.

Method 2 Use a proportion.

Write ratios that compare the part to the whole. Let b represent the total number of insect species that have been classified.

$\dfrac{\text{Number of butterfly and moth species}}{\text{Total number of insect species}} = \dfrac{17}{100}$ **Write proportion.**

$\dfrac{170{,}000}{b} = \dfrac{17}{100}$ **Substitute.**

$17b = 170{,}000 \cdot 100$ **Use cross products.**

$b = \dfrac{17{,}000{,}000}{17}$ **Divide each side by 17.**

$b = 1{,}000{,}000$ **Simplify.**

▶ About 1,000,000 species of insects have been classified.

FOCUS ON APPLICATIONS

CLASSIFYING INSECTS
Butterflies and moths together make up the insect order *Lepidoptera*. About 17,000 species of butterflies and about 153,000 species of moths have been classified.

APPLICATION LINK
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REAL LIFE

Cars

EXAMPLE 5 *Using Percents to Compare*

The circle graph shows the average costs (in percents) of owning an automobile in 1996.

Suppose that in 1996 a car owner spent $650 on gasoline for a car whose total costs were $3750. Was the percent spent on gasoline about the same as the national average?

Automobile Ownership Costs

- **Depreciation and Interest** 52.9%
- **Gasoline** 17.1%
- **Insurance** 14.1%
- **Maintenance** 7.7%
- **Tires** 3.6%
- **Other** 4.6%
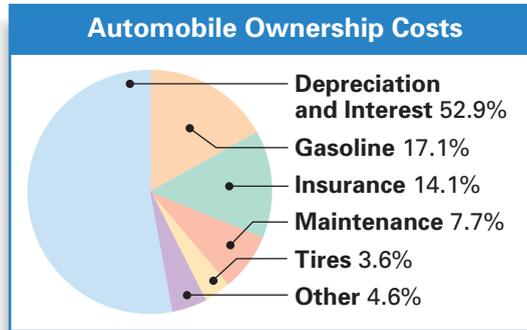
▶ Source: Runzheimer International

SOLUTION Use the percent equation $a = pb$.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

| Amount spent on gasoline | is | p percent | of | Total costs of owning a car |

LABELS

Amount spent on gasoline = **650**   (dollars)

Percent = **p**   (no units)

Total costs of owning a car = **3750**   (dollars)

ALGEBRAIC MODEL

$650 = \boxed{p} \, (3750)$

▶ $p \approx 0.173$ or 17.3%, which is about the national average of 17.1%.

GUIDED PRACTICE

Vocabulary Check ✓

1. Write an equation that represents the statement "10% of 160 is 16." What is the base number?

Concept Check ✓

🌎 **DISCOUNTS** In Exercises 2–4, the sale price of a shirt is $17.25 after a 25% discount is taken.

2. The sale price is what percent of the regular price?

3. You can model the situation with an equation of the form *a* is *p* percent of *b*. Is the base *b* the sale price or the regular price?

4. Write and solve an equation to find the regular price of the shirt.

Skill Check ✓

In Exercises 5–8, solve the percent problem.

5. 35 is what percent of 20?

6. 12% of 5 is what number?

7. 18 is 37.5% of what number?

8. 13.2 is 120% of what number?

9. 🌎 **SALES TAX** The price of a book without tax is $5.99 and the sales tax rate is 6%. Find the amount of the tax by using an equation of the form $a = pb$ and by using a proportion. How are the two methods similar?

PRACTICE AND APPLICATIONS

UNDERSTANDING PERCENT EQUATIONS Match the percent problem with the equation that represents it.

A. $a = (0.39)(50)$ **B.** $39 = p(50)$ **C.** $39 = 0.50b$

10. 39 is 50% of what number?

11. 39% of 50 is what number?

12. $39 is what percent of $50?

SOLVING PERCENT PROBLEMS Solve the percent problem.

13. What number is 25% of 80?   **14.** 85% of 300 is what number?

15. 18 is what percent of 60?   **16.** 52 is 12.5% of what number?

17. 14% of 220 feet is what distance?   **18.** How much money is 35% of $750?

19. 42 feet is 50% of what length?   **20.** What distance is 24% of 710 miles?

21. 16% of what number is 8?   **22.** $4 is 2.5% of what amount?

23. 33 grams is 22% of what weight?   **24.** 55 years is what percent of 20 years?

25. How much is 8.2% of 800 tons?   **26.** 9 people is what percent of 60 people?

27. 62 hours is what percent of 3 days?   **28.** 30 inches is what percent of 40 feet?

29. $240 is what percent of $50?   **30.** 2 percent of what amount is $200?

GEOMETRY CONNECTION In Exercises 31 and 32, what percent of the region is shaded blue? What percent is shaded yellow? All figures are rectangles.

31.

32.
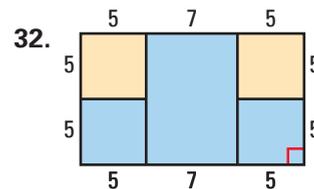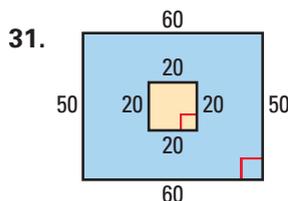
🌎 **OIL CHANGES** The histogram shows how 861 people answered a survey question about when they usually change the oil in their cars.

33. How many of the people change their oil between 3001 and 4000 miles?

34. How many of the people change their oil between 4001 and 6000 miles?

35. If you surveyed 2500 people, about how many people do you expect to answer "2001 to 3000 miles?"
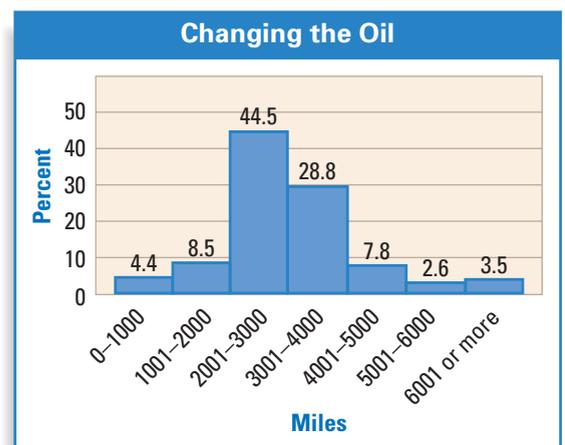
▶ Source: Maritz Marketing Research Inc.

11.2 *Percents*   **653**

🌎 **CHOOSING A COLLEGE** In Exercises 36–39, use the graph. It shows the responses of 3500 seniors from high schools around the United States.

36. What percent of the seniors said location was the reason for their choice?

37. What percent of the seniors said academic reputation was the reason for their choice?

Reason for Choosing a College

Other 280
Size 175
Academic Reputation 630
Cost 945
Availability of Major 735
Location 735
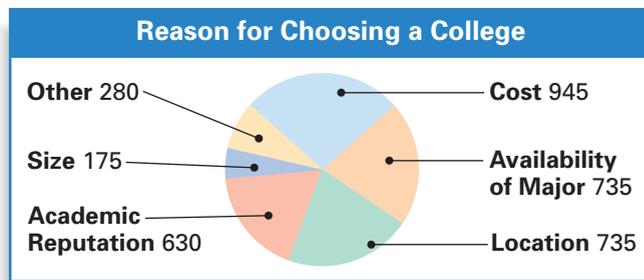
▶ Source: Careers and Colleges

38. What percent of the seniors said size or cost most influences their choice?

39. Use the survey results to predict the number of seniors in a class of 2000 students who would say that availability of major most influences their choice.

40. ERROR ANALYSIS Find and correct the mistake in the restaurant bill. The tax rate is 8%.

41. 🌎 **TIPPING** Use the corrected bill from Exercise 40. In the United States, the standard tip for a waiter or waitress is 15%–20%. You leave a $4.75 tip. Is your tip within the standard range if the tip is figured before tax is added? if the tip is figured on the total including tax?

Food	$24.93
Beverages	$5.25
Subtotal	$30.18
Tax	$4.22
Total	$34.40

🌎 **OZONE LAYER SURVEY** In Exercises 42–44, use the graph.

42. In the survey 572 people said they think the ozone layer is getting worse. What was the total number of people surveyed?

43. Use the result from Exercise 42. About how many of the people surveyed think the ozone layer is staying the same?

Do You Believe the Ozone Layer Is…

Getting worse 55%
Staying the same 38%
Improving 4%
Don't know 3%
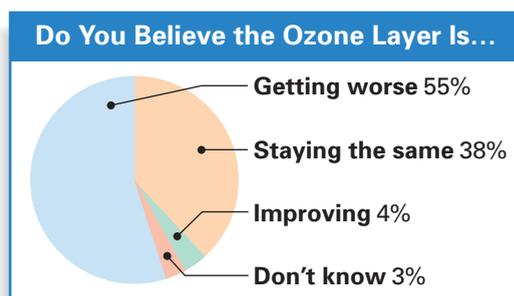
▶ Source: Worthlin Worldwide

44. CRITICAL THINKING In this survey the researchers tried to use a representative sample of people 18 years old and over in the United States. Would this sample be reasonable to use in predicting the responses of scientists? Explain.

45. 🌎 **THE BETTER BUY** You are shopping and find a coat that is on sale for 30% off. It is regularly priced at $80. Your friend tells you that she saw the same coat for $80 in another store, but it was 20% off plus an additional 10% off. Will you save money by going to the other store? Explain why or why not.

ALTERNATIVE MODELS In Exercises 46 and 47, the charge for a cab ride is $11.50, and you give a 20% tip. Using the model, find the total cost of the cab ride. Describe what the variable a represents.

46. Model 1: a is 20% of $11.50.

47. Model 2: a is 120% of $11.50.

QUANTITATIVE COMPARISON In Exercises 48–50, choose the statement below that is true about the given numbers.

(A) The number in column A is greater.

(B) The number in column B is greater.

(C) The two numbers are equal.

(D) The relationship cannot be determined from the given information.

	Column A	Column B
48.	104% of 150	100% of 150 + 4% of 150
49.	The solution of the equation 24% of $x = 450$	The solution of the equation 12% of $x = 225$
50.	The solution of the equation 16% of $x = 28$	The solution of the equation $\dfrac{16}{100} = \dfrac{x}{28}$

★ **Challenge**

51. CRITICAL THINKING You earn 10% more money at your summer job than your sister earns at her summer job. Does this mean that your sister earns 10% less money than you? Explain your answer.

52. CRITICAL THINKING A student claims that if a price is now 220% more than it was before, then it is 320% of what it was before, and what it was before is 31.25% of what it is now. Do you agree? Explain your answer.

EXTRA CHALLENGE

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MIXED REVIEW

FINDING EQUATIONS The variables *x* and *y* vary directly. Use the given values of the variables to write an equation that relates *x* and *y*. **(Review 4.5 for 11.3)**

53. $x = 4, y = 8$ **54.** $x = 33, y = 9$ **55.** $x = -2, y = -1$

56. $x = 6.3, y = 1.5$ **57.** $x = 5\frac{1}{3}, y = 8$ **58.** $x = 9.8, y = 3.6$

CHECKING SOLUTIONS Decide whether the ordered pair is a solution of the inequality. **(Review 9.7)**

59. $y < x^2 + 6x + 12; (-1, 4)$ **60.** $y \le x^2 - 7x + 9; (-1, 2)$

61. $y > 2x^2 - 7x - 15; (2, 5)$ **62.** $y \ge x^2 + 6x + 12; (1, -4)$

FACTORING EXPRESSIONS Completely factor the expression. **(Review 10.8)**

63. $x^2 + 5x - 14$ **64.** $7x^2 + 8x + 1$ **65.** $5x^2 - 51x + 54$

66. $4x^2 - 28x + 49$ **67.** $6x^2 + 16x$ **68.** $36x^5 - 90x^3$

69. $3x^3 + 21x^2 + 30x$ **70.** $36x^3 - 9x$ **71.** $15x^4 - 50x^3 - 40x^2$

72. TEXAS POPULATION The population *P* of Texas (in thousands) for 1995 projected through 2025 can be modeled by $P = 18{,}870(1.0124)^t$, where $t = 0$ represents 1995. Find the ratio of the population in 2025 to the population in 2000. **(Review 8.3)** ▶ Source: U.S. Bureau of the Census