10.8 Factoring Using the Distributive Property

What you should learn

GOAL 1 Use the distributive property to factor a polynomial.

GOAL 2 Solve polynomial equations by factoring.

Why you should learn it

▼ To solve some types of **real-life** problems, such as modeling the effect of gravity in **Exs. 51–54**.

Shannon Lucid on Space Station *Mir*

GOAL 1 FACTORING AND THE DISTRIBUTIVE PROPERTY

In dealing with polynomials, you have already been using the distributive property to factor out integers common to the various terms of the expression.

$$9x^2 - 15 = 3(3x^2 - 5)$$ **Factor out common factor.**

In many situations, it is important to factor out common *variable* factors. To save steps, you should factor out the *greatest common factor* (GCF).

EXAMPLE 1 Finding the Greatest Common Factor

Factor the greatest common factor out of $14x^4 - 21x^2$.

SOLUTION First find the greatest common factor. It is the product of all the common factors.

$$14x^4 = 2 \cdot 7 \cdot x \cdot x \cdot x \cdot x$$

$$21x^2 = 3 \cdot 7 \cdot x \cdot x$$

$$\text{GCF} = 7 \cdot x \cdot x = 7x^2$$

Then use the distributive property to factor the greatest common factor out of the polynomial.

$$14x^4 - 21x^2 = 7x^2(2x^2 - 3)$$

. . . . . .

In this lesson we restrict the polynomials we consider to polynomials having integer coefficients. A polynomial is **prime** if it is not the product of polynomials having integer coefficients. To **factor a polynomial completely**, write it as the product of these types of factors:

- monomial factors
- prime factors with at least two terms

EXAMPLE 2 Recognizing Complete Factorization

Tell whether the polynomial is factored completely.

a. $2x^2 + 8 = 2(x^2 + 4)$ **b.** $2x^2 - 8 = 2(x^2 - 4)$

SOLUTION

a. This polynomial is factored completely because $x^2 + 4$ cannot be factored using integer coefficients.

b. This polynomial is not factored completely because $x^2 - 4$ can be factored as $(x - 2)(x + 2)$.

STUDENT HELP

Skills Review
For help with finding a greatest common factor, see p. 777.

EXAMPLE 3 Factoring Completely

Factor $4x^3 + 20x^2 + 24x$ completely.

SOLUTION

$$4x^3 + 20x^2 + 24x = 4x(x^2 + 5x + 6)$$
$$= 4x(x + 2)(x + 3)$$

Monomial factor — Prime factors

EXAMPLE 4 Factoring Completely

Factor $45x^4 - 20x^2$ completely.

SOLUTION

$$45x^4 - 20x^2 = 5x^2(9x^2 - 4)$$ Factor out GCF.
$$= 5x^2(3x - 2)(3x + 2)$$ Factor difference of squares.

. . . . . .

GROUPING Another use of the distributive property is in factoring polynomials that have four terms. Sometimes you can factor the polynomial by grouping into two groups of terms and factoring the greatest common factor out of each term.

EXAMPLE 5 Factoring by Grouping

Factor $x^3 + 2x^2 + 3x + 6$ completely.

SOLUTION

$$x^3 + 2x^2 + 3x + 6 = (x^3 + 2x^2) + (3x + 6)$$ Group terms.
$$= x^2(x + 2) + 3(x + 2)$$ Factor each group.
$$= (x + 2)(x^2 + 3)$$ Use distributive property.

EXAMPLE 6 Factoring by Grouping

Factor $x^3 - 2x^2 - 9x + 18$ completely.

SOLUTION

$$x^3 - 2x^2 - 9x + 18 = (x^3 - 2x^2) - (9x - 18)$$ Group terms.
$$= x^2(x - 2) - 9(x - 2)$$ Factor each group.
$$= (x^2 - 9)(x - 2)$$ Use distributive property.
$$= (x - 3)(x + 3)(x - 2)$$ Factor difference of squares.

GOAL 2 SOLVING POLYNOMIAL EQUATIONS

EXAMPLE 7 *Solving a Polynomial Equation*

Solve $8x^3 - 18x = 0$.

SOLUTION

$$8x^3 - 18x = 0 \qquad \text{Write original equation.}$$

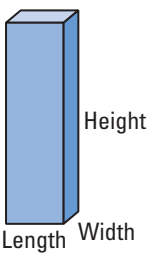
$$2x(4x^2 - 9) = 0 \qquad \text{Factor out GCF.}$$

$$2x(2x - 3)(2x + 3) = 0 \qquad \text{Factor difference of squares.}$$

▶ By setting each variable factor equal to zero, you can find that the solutions are 0, $\frac{3}{2}$, and $-\frac{3}{2}$.

EXAMPLE 8 *Writing and Using a Polynomial Model*

The width of a box is 1 inch less than the length. The height is 4 inches greater than the length. The box has a volume of 12 cubic inches. What are the dimensions of the box?



SOLUTION

VERBAL MODEL

$$\text{Length} \cdot \text{Width} \cdot \text{Height} = \text{Volume}$$

LABELS

Length = x (inches)

Width = $x - 1$ (inches)

Height = $x + 4$ (inches)

Volume = 12 (cubic inches)

ALGEBRAIC MODEL

$$x(x - 1)(x + 4) = 12 \qquad \text{Write model.}$$

$$x^3 + 3x^2 - 4x = 12 \qquad \text{Multiply.}$$

$$(x^3 + 3x^2) - (4x + 12) = 0 \qquad \text{Write in standard form and group terms.}$$

$$x^2(x + 3) - 4(x + 3) = 0 \qquad \text{Factor each group of terms.}$$

$$(x^2 - 4)(x + 3) = 0 \qquad \text{Use distributive property.}$$

$$(x - 2)(x + 2)(x + 3) = 0 \qquad \text{Factor difference of squares.}$$

▶ By setting each factor equal to zero, you can see that the solutions are 2, -2, and -3. The only positive solution is $x = 2$. The dimensions of the box are 2 inches by 1 inch by 6 inches.

CONCEPT SUMMARY — **WAYS TO SOLVE POLYNOMIAL EQUATIONS**

GRAPHING: Can be used to solve any equation, but gives only approximate solutions. Examples 2 and 3, p. 527

THE QUADRATIC FORMULA: Can be used to solve any *quadratic* equation. Examples 1–3, pp. 533–534

FACTORING: Can be used with the zero-product property to solve an equation that is factorable.

- **Factoring $x^2 + bx + c$:** Examples 1–6, pp. 604–606

  $x^2 + 9x + 18 = (x + 3)(x + 6)$

- **Factoring $ax^2 + bx + c$:** Examples 1–5, pp. 611–613

  $3x^2 + 10x + 7 = (3x + 7)(x + 1)$

- **Special Products:** Examples 1–4, pp. 619–620

  $a^2 - b^2 = (a + b)(a - b)$ **Example:** $4x^2 - 36 = (2x + 6)(2x - 6)$

  $a^2 + 2ab + b^2 = (a + b)^2$ **Example:** $x^2 + 18x + 81 = (x + 9)^2$

  $a^2 - 2ab + b^2 = (a - b)^2$ **Example:** $x^2 - 16x + 64 = (x - 8)^2$

- **Factoring Completely:** Examples 1–7, pp. 625–627

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 9 *Solving Quadratic and Other Polynomial Equations*

Solve the equation.

a. $-30x^4 + 58x^3 - 24x^2 = 0$ **b.** $18x^3 - 30x^2 = 60x$

SOLUTION

a. $-30x^4 + 58x^3 - 24x^2 = 0$ **Write original equation.**

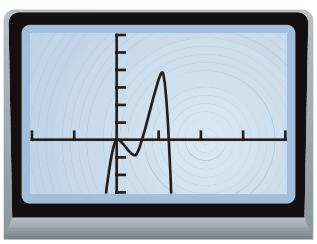
$-2x^2(15x^2 - 29x + 12) = 0$ **Factor out $-2x^2$.**

$-2x^2(5x - 3)(3x - 4) = 0$ **Find correct factorization for trinomial.**

▶ Set each variable factor equal to zero. The solutions are 0, $\frac{3}{5}$, and $\frac{4}{3}$.

✓ **CHECK** Graph $y = -30x^4 + 58x^3 - 24x^2$. Use your calculator's TRACE feature to estimate the x-intercepts.

The graph appears to confirm the solutions.



b. $18x^3 - 30x^2 - 60x = 0$ **Rewrite equation in standard form.**

$6x(3x^2 - 5x - 10) = 0$ **Factor out GCF.**

▶ $6x$ factors out, so one solution is 0. Because $3x^2 - 5x - 10$ is not factorable, use the quadratic formula to find the other solutions, $x \approx 2.84$ and $x \approx -1.17$.

GUIDED PRACTICE

Vocabulary Check ✓

1. In writing a polynomial as the product of polynomials of lesser degrees, what does it mean to say that a factor is *prime*?

Concept Check ✓

2. To factor a polynomial completely, you must write it as the product of what two types of factors?

3. Name three ways to find the solution(s) of a quadratic equation. Which way do you prefer? Explain.

ERROR ANALYSIS **Describe and correct the factoring error.**

4. $4x^3 + 36x$
$= 4x(x^2 + 9)$
$= 4x(x + 3)(x - 3)$

5. $-2b^3 + 12b^2 - 14b$
$= -2b(b^2 + 6b - 7)$
$= -2b(b + 7)(b - 1)$

Skill Check ✓

Find the greatest common factor and factor it out of the expression.

6. $5n^3 - 20n$
7. $6x^2 + 3x^4$
8. $6y^4 + 14y^3 - 10y^2$

Tell whether the expression is factored completely. If the expression is not factored completely, write the complete factorization.

9. $7x^3 - 11x$
10. $9t(t^2 + 49)$
11. $3w(9w^2 - 16)$

Solve the equation. Tell which method you used.

12. $y^2 - 4y - 5 = 0$
13. $z^2 + 11z + 30 = 0$
14. $5a^2 + 11a + 2 = 0$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

FACTORING OUT THE GCF **Find the greatest common factor and factor it out of the expression.**

15. $6v^3 - 18v$
16. $4q^4 + 12q$
17. $3x - 9x^2$
18. $24t^5 + 6t^3$
19. $4a^5 + 8a^3 - 2a^2$
20. $18d^6 - 6d^2 + 3d$

FACTORING COMPLETELY **Factor the expression completely.**

21. $24x^3 + 18x^2$
22. $-3w^4 + 21w^3$
23. $2y^3 - 10y^2 - 12y$
24. $5s^3 + 30s^2 + 40s$
25. $-7m^3 + 28m^2 - 21m$
26. $2d^4 + 2d^3 - 60d^2$
27. $4t^3 - 144t$
28. $-12z^4 + 3z^2$
29. $c^4 + c^3 - 12c - 12$
30. $x^3 - 3x^2 + x - 3$
31. $6b^4 + 5b^3 - 24b - 20$
32. $3y^3 - y^2 - 21y + 7$
33. $a^3 + 6a^2 - 4a - 24$
34. $t^3 - t^2 - 16t + 16$
35. $3m^3 - 15m^2 - 6m + 30$
36. $7n^5 + 7n^4 - 3n^2 - 6n - 3$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–20
Examples 2–6: Exs. 21–36
Example 7: Exs. 37–50
Example 8: Exs. 55–57
Example 9: Exs. 37–50

SOLVING EQUATIONS **Solve the equation. Tell which solution method you used.**

37. $y^2 + 7y + 12 = 0$

38. $x^2 - 3x - 4 = 0$

39. $b^2 + 4b - 117 = 0$

40. $t^2 - 16t + 65 = 0$

41. $27 + 6w - w^2 = 0$

42. $x^2 - 21x + 84 = 0$

43. $5x^4 - 80x^2 = 0$

44. $-16x^3 + 4x = 0$

45. $10x^3 - 290x^2 + 620x = 0$

46. $34x^4 - 85x^3 + 51x^2 = 0$

47. $8x^2 + 9x - 7 = 0$

48. $18x^2 - 21x + 28 = 0$

49. $24x^3 + 18x^2 - 168x = 0$

50. $-14x^4 + 118x^3 + 72x^2 = 0$

FOCUS ON APPLICATIONS

REAL LIFE **GRAVITY** varies depending on the mass of a planet. This is why acceleration due to gravity is greater on Jupiter than on Earth.

INTERNET **APPLICATION LINK**
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VERTICAL MOTION **In Exercises 51–54, use the vertical motion models, where *h* is the initial height (in feet), *v* is the initial velocity (in feet per second) and *t* is the time (in seconds) the object spends in the air. (Note that the acceleration due to gravity on the Moon is $\frac{1}{6}$ that of Earth.)**

Model for vertical motion on Earth: $h = 16t^2 - vt$

Model for vertical motion on the Moon: $h = \frac{16}{6}t^2 - vt$

51. **EARTH** On Earth, you toss a tennis ball from a height of 96 feet with an initial upward velocity of 16 feet per second. How long will it take the tennis ball to reach the ground?

52. **MOON** On the Moon, you toss a tennis ball from a height of 96 feet with an initial upward velocity of 16 feet per second. How long will it take the tennis ball to reach the surface of the moon?

53. *Writing* Do objects fall faster on Earth or on the Moon? Use your results from Exercises 51 and 52 and the vertical motion models shown above to support your answer.

54. **CRITICAL THINKING** The coefficient of the t^2-term in the vertical motion models is one-half the acceleration of a falling object due to gravity. On the surface of Jupiter, the acceleration due to gravity is about 2.4 times that on the surface of Earth. Write a vertical motion model for Jupiter.

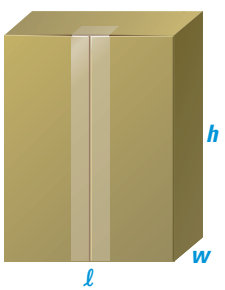
PACKAGING **In Exercises 55–57, use the following information. Refer to the diagram at the right.**

The length ℓ of a box is 3 inches less than the height h. The width w is 9 inches less than the height. The box has a volume of 324 cubic inches.

55. Copy and complete the diagram by labeling the dimensions.

56. Write a model that you can solve to find the length, height, and width of the box.

57. What are the dimensions of the box?



EXTENSION: BRITISH METHOD **Another method for finding the binomial factors of factorable trinomials of the form $ax^2 + bx + c$ is sometimes called the *British method.* To factor $6x^2 - x - 2$ by the British method, use the following steps.**

1. Find the "magic" number ac.
   $6 \cdot (-2) = -12$
2. Write the magic number as the product of two factors whose sum is b.
   $-12 = 3 \cdot (-4)$
3. Rewrite the trinomial using the factors.
   $6x^2 - x - 2 = 6x^2 + 3x - 4x - 2$
4. Factor using the distributive property.
   $6x^2 + 3x - 4x - 2 = 3x(2x + 1) - 2(2x + 1)$
   $= (3x - 2)(2x + 1)$

Use the British method to factor the trinomials.

58. $8x^2 - 2x - 3$ **59.** $3x^2 + 13x + 14$ **60.** $5x^2 + 27x - 18$

Test Preparation

61. MULTIPLE CHOICE Which of the following is the complete factorization of $x^3 - 5x^2 + 4x - 20$?

Ⓐ $(x + 2)(x + 2)(x - 5)$ Ⓑ $(x + 2)(x - 2)(x - 5)$

Ⓒ $(x^2 + 4)(x - 5)$ Ⓓ $(x - 4)(x - 1)(x - 20)$

62. MULTIPLE CHOICE Which of the following equations have real solutions?

I. $6x^2 + x - 12 = 0$ **II.** $6x^2 + 7x - 12 = 0$ **III.** $6x^2 + 7x - 13 = 0$

Ⓐ I only Ⓑ II only Ⓒ III only Ⓓ All

63. MULTIPLE CHOICE The length l of a box is 3 centimeters more than half the height h. The width w is 2 centimeters more than one fifth of the height. The box has a volume of 3780 cubic centimeters. Which of the following equations can be used to find the height of the box?

Ⓐ $h\left(\frac{1}{5}h + 2\right)\left(\frac{1}{2}h + 3\right) = 3780$ Ⓑ $h\left(\frac{1}{2}h + 2\right)\left(\frac{1}{5}h + 3\right) = 3780$

Ⓒ $\frac{1}{10}h(h + 2)(h + 3) = 3780$ Ⓓ $h\left(\frac{1}{2}w + 2\right)\left(\frac{1}{5}l + 3\right) = 3780$

Challenge

CUBIC EQUATIONS **Sums and differences of *cubes* can be factored using the following patterns.**

Sum of cubes pattern: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of cubes pattern: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

In Exercises 64–67, use the patterns above to factor the cubic expression completely. Use the distributive property to verify your results.

64. $y^3 - 125$ **65.** $b^3 + 27$ **66.** $\frac{1}{8} + 8x^3$ **67.** $216 - 343t^3$

EXTRA CHALLENGE
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68. Set each expression in Exercises 64–67 equal to zero and solve the equation.

MIXED REVIEW

SOLVING AND GRAPHING INEQUALITIES **Solve the inequality and graph its solution.** (Review 6.1, 6.2)

69. $7 + x \leq -9$ **70.** $-3 > 2x - 5$ **71.** $-x + 6 \leq 13$

SOLVING EQUATIONS **Solve the equation.** (Review 6.4)

72. $|x| = 3$ **73.** $|x - 5| = 7$ **74.** $|x + 6| = 13$

75. $|x + 8| - 2 = -12$ **76.** $|2x - 5| + 7 = 16$ **77.** $\left|x + \frac{3}{4}\right| = \frac{9}{4}$

SKETCHING GRAPHS **Sketch the graph of the inequality.** (Review 6.5)

78. $x + y < 9$ **79.** $y - 3x \geq 2$ **80.** $y - 4x \leq 10$

81. **HEIGHT** One of the tallest people in the world is Ri Myong-hun of North Korea, who is about 93 inches tall. The average height of a United States male is about 69 inches. Compare Ri Myong-hun's height to the average U.S. male height using a ratio and a percent. (Review 3.8 for 11.1)

82. **COMPARING ROOMS** One bedroom in a house is 10 feet by 12 feet. The living room is 24 feet by 15 feet. Compare the areas of the two rooms using a ratio. (Review 3.8 for 11.1)

QUIZ 3

Self-Test for Lessons 10.7 and 10.8

Factor the expression. Tell which special product factoring pattern you used. (Lesson 10.7)

1. $49x^2 - 64$ **2.** $121 - 9x^2$ **3.** $4t^2 + 20t + 25$

4. $72 - 50y^2$ **5.** $9y^2 + 42y + 49$ **6.** $3n^2 - 36n + 108$

Use factoring to solve the equation. (Lesson 10.7)

7. $3x^2 - 192 = 0$ **8.** $4x^2 + 32x + 64 = 0$ **9.** $9x^2 + 96x + 256 = 0$

10. $\frac{1}{2}x^2 - 4x + 8 = 0$ **11.** $216x^2 - 96 = 0$ **12.** $-3x^2 - 2x - \frac{1}{3} = 0$

Find the greatest common factor and factor it out of the expression. (Lesson 10.8)

13. $3x^3 + 12x^2$ **14.** $6x^2 + 3x$ **15.** $18x^4 - 9x^3$ **16.** $8x^5 + 4x^2 - 2x$

Factor the expression completely. (Lesson 10.8)

17. $2x^3 - 6x^2 + 4x$ **18.** $48x^3 - 75x$ **19.** $x^3 + 3x^2 + 4x + 12$

Solve the equation. Tell which solution method you used. (Lesson 10.8)

20. $12x^4 - 27x^2 = 0$ **21.** $-3x^2 - 4x + 15 = 0$ **22.** $3x^3 - 6x^2 + 5x - 10 = 0$

23. $56x^3 + 98x = 170x^2$ **24.** $17x^6 = 17x^5 + 204x^4$ **25.** $6x^3 + 45x^2 + 15x = 0$