10.6

Factoring $ax^2 + bx + c$

GOAL 1 Factoring a Quadratic Trinomial

In this lesson, you will learn to factor quadratic polynomials whose leading coefficient is not 1. Find the factors of a (m and n) and the factors of c (p and q) so that the sum of the outer and inner products (mq and pn) is b.

$$ax^2 + bx + c = (mx + p)(nx + q) \qquad c = pq,\ a = mn,\ b = mq + pn$$

Example: $6x^2 + 22x + 20 = (3x + 5)(2x + 4)$ $\qquad 20 = 5 \cdot 4,\ 6 = 3 \cdot 2,\ 22 = (3 \cdot 4) + (5 \cdot 2)$

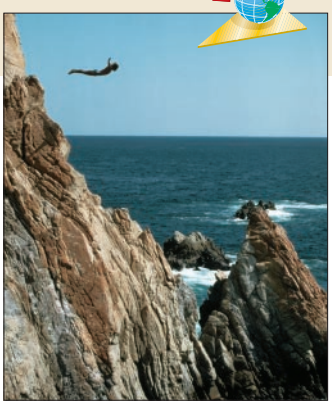
What you should learn

GOAL 1 Factor a quadratic expression of the form $ax^2 + bx + c$.

GOAL 2 Solve quadratic equations by factoring.

Why you should learn it

▼ To help solve **real-life** problems, such as finding the time it takes a cliff diver to reach the water in **Example 6**.



EXAMPLE 1 One Pair of Factors for a and c

Factor $2x^2 + 11x + 5$.

SOLUTION Test the possible factors of a (1 and 2) and c (1 and 5).

Try $a = 1 \cdot 2$ and $c = 1 \cdot 5$.

$(1x + 1)(2x + 5) = 2x^2 + 7x + 5$ **Not correct**

Now try $a = 1 \cdot 2$ and $c = 5 \cdot 1$.

$(1x + 5)(2x + 1) = 2x^2 + 11x + 5$ **Correct**

▶ The correct factorization of $2x^2 + 11x + 5$ is $(x + 5)(2x + 1)$.

EXAMPLE 2 One Pair of Factors for a and c

Factor $3x^2 - 4x - 7$.

SOLUTION

FACTORS OF a AND c	PRODUCT	CORRECT?
$a = 1 \cdot 3$ and $c = (-1)(7)$	$(x - 1)(3x + 7) = 3x^2 + 4x - 7$	No
$a = 1 \cdot 3$ and $c = (7)(-1)$	$(x + 7)(3x - 1) = 3x^2 + 20x - 7$	No
$a = 1 \cdot 3$ and $c = (1)(-7)$	$(x + 1)(3x - 7) = 3x^2 - 4x - 7$	Yes
$a = 1 \cdot 3$ and $c = (-7)(1)$	$(x - 7)(3x + 1) = 3x^2 - 20x - 7$	No

▶ The correct factorization of $3x^2 - 4x - 7$ is $(x + 1)(3x - 7)$.

EXAMPLE 3 Several Pairs of Factors for a and c

Factor $6x^2 - 19x + 15$.

SOLUTION

Both factors of c must be negative, because b is negative and c is positive. Test the possible factors of a and c.

FACTORS OF a AND c	PRODUCT	CORRECT?
$a = 1 \cdot 6$ and $c = (-1)(-15)$	$(x - 1)(6x - 15) = 6x^2 - 21x + 15$	No
$a = 1 \cdot 6$ and $c = (-15)(-1)$	$(x - 15)(6x - 1) = 6x^2 - 91x + 15$	No
$a = 1 \cdot 6$ and $c = (-3)(-5)$	$(x - 3)(6x - 5) = 6x^2 - 23x + 15$	No
$a = 1 \cdot 6$ and $c = (-5)(-3)$	$(x - 5)(6x - 3) = 6x^2 - 33x + 15$	No
$a = 2 \cdot 3$ and $c = (-1)(-15)$	$(2x - 1)(3x - 15) = 6x^2 - 33x + 15$	No
$a = 2 \cdot 3$ and $c = (-15)(-1)$	$(2x - 15)(3x - 1) = 6x^2 - 47x + 15$	No
$a = 2 \cdot 3$ and $c = (-3)(-5)$	$(2x - 3)(3x - 5) = 6x^2 - 19x + 15$	Yes
$a = 2 \cdot 3$ and $c = (-5)(-3)$	$(2x - 5)(3x - 3) = 6x^2 - 21x + 15$	No

▶ The correct factorization of $6x^2 - 19x + 15$ is $(2x - 3)(3x - 5)$.

EXAMPLE 4 A Common Factor for a, b, and c

Factor $6x^2 - 2x - 8$.

SOLUTION

Begin by factoring out the common factor 2.

$$6x^2 - 2x - 8 = 2(3x^2 - x - 4)$$

Now factor $3x^2 - x - 4$ by testing the possible factors of a and c.

FACTORS OF a AND c	PRODUCT	CORRECT?
$a = 1 \cdot 3$ and $c = (-1)(4)$	$(x - 1)(3x + 4) = 3x^2 + x - 4$	No
$a = 1 \cdot 3$ and $c = (4)(-1)$	$(x + 4)(3x - 1) = 3x^2 + 11x - 4$	No
$a = 1 \cdot 3$ and $c = (1)(-4)$	$(x + 1)(3x - 4) = 3x^2 - x - 4$	Yes
$a = 1 \cdot 3$ and $c = (-4)(1)$	$(x - 4)(3x + 1) = 3x^2 - 11x - 4$	No
$a = 1 \cdot 3$ and $c = (-2)(2)$	$(x - 2)(3x + 2) = 3x^2 - 4x - 4$	No
$a = 1 \cdot 3$ and $c = (2)(-2)$	$(x + 2)(3x - 2) = 3x^2 + 4x - 4$	No

▶ The correct factorization is $6x^2 - 2x - 8 = 2(x + 1)(3x - 4)$.

. . . . . .

In Examples 1 through 4, each test is shown for the sake of completeness. But when you factor, you can stop testing once you find the correct factorization.

STUDENT HELP

Study Tip

You may find it easier to factor quadratic trinomials that have positive leading coefficients. If the leading coefficient is negative, factor out -1 first. For instance, $-2x^2 + 3x - 1 = (-1)(2x^2 - 3x + 1)$.

GOAL 2 SOLVING QUADRATIC EQUATIONS BY FACTORING

EXAMPLE 5 *Solving a Quadratic Equation*

$$21n^2 + 14n + 7 = 6n + 11$$ **Write equation.**

$$21n^2 + 8n - 4 = 0$$ **Write in standard form.**

$$(3n + 2)(7n - 2) = 0$$ **Factor left side.**

$$(3n + 2) = 0 \quad \text{or} \quad (7n - 2) = 0$$ **Use zero-product property.**

$$3n + 2 = 0 \qquad 7n - 2 = 0$$

$$n = -\frac{2}{3} \qquad n = \frac{2}{7}$$

▶ The solutions are $-\frac{2}{3}$ and $\frac{2}{7}$.

✓**CHECK** Substitute $-\frac{2}{3}$ and $\frac{2}{7}$ for n in $21n^2 + 14n + 7 = 6n + 11$.

$$21\left(-\frac{2}{3}\right)^2 + 14\left(-\frac{2}{3}\right) + 7 = 6\left(-\frac{2}{3}\right) + 11$$

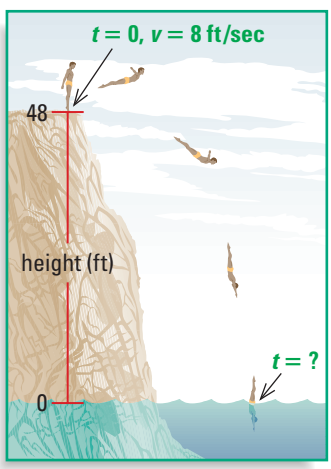
$$7 = 7$$

$$21\left(\frac{2}{7}\right)^2 + 14\left(\frac{2}{7}\right) + 7 = 6\left(\frac{2}{7}\right) + 11$$

$$\frac{89}{7} = \frac{89}{7}$$

EXAMPLE 6 *Writing a Quadratic Model*

When a diver jumps from a ledge, the vertical component of his motion can be modeled by the vertical motion model. Suppose the ledge is 48 feet above the ocean and the initial upward velocity is 8 feet per second. How long will it take until the diver enters the water?



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STUDENT HELP

Look Back
For help with using a vertical motion model, see p. 535.

SOLUTION Use a vertical motion model.
Let $v = 8$ and $s = 48$.

$$h = -16t^2 + vt + s$$ **Vertical motion model**

$$= -16t^2 + 8t + 48$$ **Substitute values.**

To find the time when the diver enters the water, let $h = 0$ and solve the resulting equation for t.

$$0 = -16t^2 + 8t + 48$$ **Write quadratic model.**

$$0 = (-8)(2t^2 - t - 6)$$ **Factor out −8.**

$$0 = (-8)(t - 2)(2t + 3)$$ **Factor.**

▶ The solutions are 2 and $-\frac{3}{2}$. Negative values for time do not make sense, so the only reasonable solution is $t = 2$. It will take the diver 2 seconds.

GUIDED PRACTICE

Concept Check ✓

1. Tell whether the following statement is *true* or *false*. *If* $(5x - 1)(x + 3) = 1$, *then* $5x - 1 = 1$ *or* $x + 3 = 1$. Explain.

2. **ERROR ANALYSIS** Describe the error at the right. Then solve the equation by factoring correctly.

$$2x^2 - 3x + 1 = 10$$
$$2x^2 - 3x + 1 = (2x - 1)(x - 1)$$
$$2x - 1 = 0$$
$$x = \frac{1}{2}$$
$$x - 1 = 0$$
$$x = 1$$

3. Can a quadratic expression be factored if its discriminant is 1? Explain.

4. The sum of a number and its square is zero. Write and solve an equation to find numbers that fit this description.

Skill Check ✓

Match the trinomial with the correct factorization.

5. $3x^2 - 17x - 6$
6. $3x^2 + 7x - 6$
7. $3x^2 + 11x + 6$
8. $3x^2 + 17x - 6$

A. $(3x + 2)(x + 3)$
B. $(3x + 1)(x - 6)$
C. $(3x - 1)(x + 6)$
D. $(3x - 2)(x + 3)$

Use the discriminant to decide whether the expression can be factored. If it can be factored, factor the expression.

9. $2x^2 - 3x - 2$
10. $-3y^2 - 4y + 7$
11. $6t^2 - 4t - 5$

Solve the equation.

12. $3b^2 + 26b + 35 = 0$
13. $2z^2 + 15z = 8$
14. $-7n^2 - 40n = -12$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

FACTORIZATIONS **Choose the correct factorization. If neither is correct, find the correct factorization.**

15. $3x^2 + 2x - 8$
A. $(3x - 4)(x + 2)$
B. $(3x - 4)(x - 2)$

16. $6y^2 - 29y - 5$
A. $(2y + 1)(3y - 5)$
B. $(6y - 1)(y + 5)$

17. $4w^2 - 14w - 30$
A. $(2w + 3)(2w - 10)$
B. $(4w + 15)(w - 2)$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–32
Example 2: Exs. 15–32
Example 3: Exs. 15–32
Example 4: Exs. 15–32
Example 5: Exs. 33–47
Example 6: Exs. 64–66

FACTORING TRINOMIALS **Factor the trinomial if possible. If it cannot be factored, write *not factorable*.**

18. $3t^2 + 16t + 5$
19. $6b^2 - 11b - 2$
20. $4n^2 - 26n - 42$
21. $5w^2 - 9w - 2$
22. $4x^2 + 27x + 35$
23. $6y^2 - 11y - 10$
24. $6x^2 - 21x - 9$
25. $3c^2 - 37c + 44$
26. $10x^2 + 17x + 6$
27. $14y^2 - 15y + 4$
28. $4z^2 + 32z + 63$
29. $6t^2 + t - 70$
30. $8b^2 + 2b - 3$
31. $2z^2 + 19z - 10$
32. $12m^2 + 48m + 96$

SOLVING EQUATIONS **Solve the equation by factoring.**

33. $2x^2 - 9x - 35 = 0$ **34.** $7x^2 - 10x + 3 = 0$ **35.** $3x^2 + 34x + 11 = 0$

36. $4x^2 - 21x + 5 = 0$ **37.** $2x^2 - 17x - 19 = 0$ **38.** $5x^2 - 3x - 26 = 0$

39. $2x^2 + 19x = -24$ **40.** $4x^2 - 8x = -3$ **41.** $6x^2 - 23x = 18$

42. $8x^2 - 34x + 24 = -11$ **43.** $10x^2 + x - 10 = -2x + 8$

44. $28x^2 - 9x - 1 = -4x + 2$ **45.** $24x^2 + 39x + 15 = 0$

46. $30x^2 - 80x + 50 = 7x - 4$ **47.** $18x^2 - 30x - 100 = 67x + 30$

STUDENT HELP

Look Back
For help with using square roots to solve quadratic equations, see p. 505.

For help with the quadratic formula, see p. 533.

CHOOSING A METHOD **Solve the equation by factoring, by finding square roots, or by using the quadratic formula.**

48. $\frac{1}{16}s^2 = 4$ **49.** $x^2 - 10x = -25$ **50.** $y^2 - 7y + 6 = -6$

51. $4n^2 + 2n = 0$ **52.** $12t^2 = 0$ **53.** $35b^2 - 61b + 24 = 0$

54. $9x^2 - 19 = -3$ **55.** $20d^2 - 10d = 100$ **56.** $8c^2 - 27c - 24 = -6$

57. $56w^2 - 61w = 22$ **58.** $3n^2 + 12n + 9 = -1$ **59.** $24z^2 + 46z - 55 = 10$

MULTIPLY AND SOLVE **Multiply each side of the equation by an appropriate power of ten to obtain integer coefficients. Then solve by factoring.**

60. $0.8x^2 + 3.2x + 2.40 = 0$ **61.** $0.23t^2 - 0.54t + 0.16 = 0$

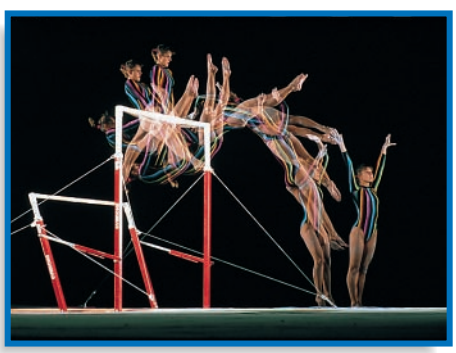
62. $0.3n^2 - 2.2n + 8.4 = 0$ **63.** $0.119y^2 - 0.162y + 0.055 = 0$

VERTICAL COMPONENT OF MOTION **In Exercises 64–66, use the vertical motion model $h = -16t^2 + vt + s$, where h is the height (in feet), t is the time in motion (in seconds), v is the initial velocity (in feet per second), and s is the initial height (in feet). Solve by factoring.**

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 64–66.

64. **GYMNASTICS** A gymnast dismounts the uneven parallel bars at a height of 8 feet with an initial upward velocity of 8 feet per second. Find the time t (in seconds) it takes for the gymnast to reach the ground. Is your answer reasonable?

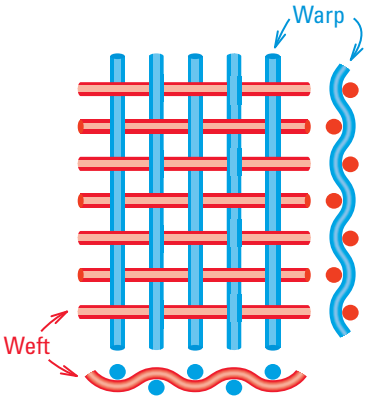


65. **CIRCUS ACROBATS** An acrobat is shot out of a cannon and lands in a safety net that is 10 feet above the ground. Before being shot out of the cannon, she was 4 feet above the ground. She left the cannon with an initial upward velocity of 50 feet per second. Find the time t (in seconds) it takes for her to reach the net. Explain why only one of the two solutions is reasonable.

66. **T-SHIRT CANNON** At a basketball game, T-shirts are rolled-up into a ball and shot from a "T-shirt cannon" into the crowd. The T-shirts are released from a height of 6 feet with an initial upward velocity of 44 feet per second. If you catch a T-shirt at your seat 30 feet above the court, how long was it in the air before you caught it? Is your answer reasonable?

WARP AND WEFT **In Exercises 67 and 68, use the following information.**

In every square inch of sailcloth, the warp (lengthwise threads) intersects the weft (crosswise threads) about 9000 times. The density (number of threads per inch) of the weft threads to the warp threads is about 5 to 2.



67. Write an equation that you can solve to find the number of threads per square inch. Let x represent the number of warp threads.

68. How many threads are there in each square inch of sailcloth?

EXTENSION: WRITING EQUATIONS **In Exercises 69–72, you are tutoring a friend and want to create some quadratic equations that can be solved by factoring. Find a quadratic equation that has the given solutions and explain the procedure you used to obtain the equation.**

69. 4 and -3 **70.** 8 and -8 **71.** $-\frac{1}{2}$ and $\frac{1}{3}$ **72.** $-\frac{5}{4}$ and $-\frac{8}{3}$

Test Preparation

73. MULTIPLE CHOICE Which one of the following equations *cannot* be solved by factoring with integer coefficients?

Ⓐ $12x^2 - 15x - 63 = 0$ Ⓑ $12x^2 + 46x - 8 = 0$

Ⓒ $6x^2 - 38x - 28 = 0$ Ⓓ $8x^2 - 49x - 68 = 0$

74. MULTIPLE CHOICE Which one of the following is a correct factorization of the expression $-16x^2 + 36x + 52$?

Ⓐ $(-16x + 26)(x - 2)$ Ⓑ $(-4x - 13)(4x + 4)$

Ⓒ $-1(4x - 13)(4x + 4)$ Ⓓ $-1(4x - 4)(4x + 13)$

75. MULTIPLE CHOICE Which one of the following is a solution of the equation $7x^2 - 11x - 6 = 0$?

Ⓐ -2 Ⓑ $-\frac{3}{7}$ Ⓒ $-\frac{2}{7}$ Ⓓ $\frac{3}{7}$

76. MULTIPLE CHOICE A pebble is thrown upward from the edge of a building 132 feet above the ground with an initial upward velocity of 4 feet per second. How long does it take to reach the ground?

Ⓐ $2\frac{1}{2}$ seconds Ⓑ $2\frac{3}{4}$ seconds Ⓒ 3 seconds Ⓓ 6 seconds

Challenge

SAVING FOR A TRIP **You are saving money for a trip to Europe that costs $3600. You make an investment of $1000 two years before the trip and an investment of $1800 one year before the trip.**

77. Use $(1 + r)$ as a growth factor to write an equation that can be solved to find the growth rate r that you need in order to have $3600 at the time of the trip to Europe.

78. Can the equation be solved by factoring? Solve the equation. What growth rate do you need in order to have $3600 at the time of the trip to Europe?

EXTRA CHALLENGE
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MIXED REVIEW

SOLVING SYSTEMS **Use linear combinations to solve the system.** (Review 7.3)

79. $4x + 5y = 7$
$6x - 2y = -18$

80. $6x - 5y = 3$
$-12x + 8y = 5$

81. $2x + y = 120$
$x + 2y = 120$

SOLVING INEQUALITIES **Decide whether the ordered pair is a solution of the inequality.** (Review 9.7)

82. $y > 2x^2 - x + 7$; (2, 15)

83. $y \geq 4x^2 - 64x + 92$; (1, 30)

SPECIAL PRODUCT PATTERNS **Find the product.** (Review 10.3 for 10.7)

84. $(4t - 1)^2$

85. $(b + 9)(b - 9)$

86. $(3x + 5)(3x + 5)$

87. $(2a - 7)(2a + 7)$

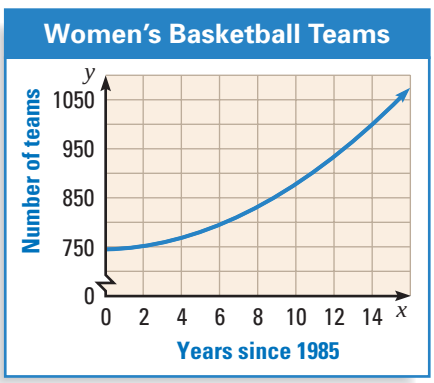
88. $(11 - 6x)^2$

89. $(100 + 27x)^2$

90. BASKETBALL The graph shows the number of women's college basketball teams from 1985 projected through 2000. Which type of model best fits the data?

▶ Source: National Collegiate Athletic Association

(Review 9.8)



QUIZ 2

Self-Test for Lessons 10.4–10.6

Solve the equation. (Lesson 10.4)

1. $(x + 5)(2x + 10) = 0$

2. $(4x - 6)(5x - 20) = 0$

3. $(2x + 7)(3x - 12) = 0$

4. $\left(4x + \frac{1}{3}\right)^2 = 0$

5. $(2x + 8)^2 = 0$

6. $(x - 4)(x + 7)(x + 1) = 0$

Name the *x*-intercepts and the vertex of the graph of the function. Then sketch the graph of the function. (Lesson 10.4)

7. $y = \left(x - \frac{1}{2}\right)(x + 2)$

8. $y = (x + 4)(x - 4)$

9. $y = (3x - 1)(x + 2)$

Factor the trinomial. (Lessons 10.5 and 10.6)

10. $y^2 + 3y - 4$

11. $w^2 + 13w + 22$

12. $n^2 + 16n - 57$

13. $x^2 + 17x + 66$

14. $t^2 - 41t - 86$

15. $-45 + 14z - z^2$

16. $12b^2 - 17b - 99$

17. $2t^2 + 63t + 145$

18. $18d^2 - 54d + 28$

Solve the equation by factoring. (Lessons 10.5 and 10.6)

19. $y^2 + 5y - 6 = 0$

20. $n^2 + 26n + 25 = 0$

21. $t^2 - 11t = -18$

22. $w^2 - 29w = 170$

23. $x^2 + 35x + 3 = 77$

24. $2a^2 + 33a + 136 = 0$

25. $15n^2 + 41n = -14$

26. $18b^2 - 89b = -36$

27. $6x^2 + x - 96 = 80$